ECON2125/4021/8013*
Week 8 Tutorial Questions (24/4/2015)

Semester 1 2015

***Question* 1**

A, B, C and D are events over sample space. Prove the following statement:

1. $\mathbb{P}(A \cap B) \geq \mathbb{P}(A) + \mathbb{P}(B) - 1$.

2. If $B \subset A$, then $\mathbb{P}(A|B) = 1$.

3. If $C \subset B$ and $D \subset B$, then $\mathbb{P}(C|B)/\mathbb{P}(D|B) = \mathbb{P}(C)/\mathbb{P}(D)$.

4. If A and B are two disjoint events, then are A and B independent or not?

***Question* 2**

If α and β are constants, and X and Y are random variables, then verify the following equalities:

1. $\text{var}[\alpha + \beta X] = \beta^2 \text{var}[X]$

2. $\text{var}[\alpha X + \beta Y] = \alpha^2 \text{var}[X] + \beta^2 \text{var}[Y] + 2\alpha\beta \text{cov}[X, Y]$

3. $\text{var}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

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***Question* 3**

Given two random variables X and Y, and positive constants α and β, try to verify following properties:

1. $-1 \leq \text{corr}[X, Y] \leq 1$

2. $\text{corr}[\alpha X,\, \beta Y] = \text{corr}[X,\, Y]$

***Question* 4**

Let X be a random variable. Prove the following results:

1. (**Markov Inequality**) If X takes only nonnegative values, then for any value $a > 0$,

$$\mathbb{P}\{X \geq a\} \leq \frac{\mathbb{E}[X]}{a}$$

2. (**Chebyshev's Inequality**) If X has mean μ and variance σ^2, then for any value $k > 0$,

$$\mathbb{P}\{|X - \mu| \geq k\sigma\} \leq \frac{1}{k^2}$$

***Question* 5**

1. A random variable X takes on one of the values $0, 1, 2, ...$ is said to be a **Poisson random variable** with parameter λ, if its probability mass function is given by

$$p_i = \mathbb{P}\{X = i\} = e^{-\lambda}\frac{\lambda^i}{i!},\ i = 0, 1, ...$$

Calculate the mean and variance of the Poisson random variable X.

2. Consider independent trials, each of which is a success with probability p. If the random variable X represents the number of the first trial that is a success, then X is said to be a **geometric random variable** with parameter p, the probability mass function of which will be given by

$$\mathbb{P}\{X = n\} = p(1-p)^{n-1}, n \geq 1$$

Calculate the mean and variance of the geometric random variable X.

***Question* 6**

A continuous random variable X having probability density function

$$f(x) = \lambda e^{-\lambda x}, \ 0 < x < \infty$$

for some λ is said to be an exponential random variable with parameter λ. Calculate the mean and variance of the **exponential random variable** X. Try to find the cumulative distribution function of X.

***Bonus Question* 1**

A random variable X is said to be a **gamma random variable** with parameters $\alpha > 0$ and $\beta > 0$ if the density of X is

$$f(x) = \begin{cases} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}, & 0 \leq x \leq \infty \\ 0, & \text{otherwise} \end{cases}$$

where

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1}e^{-x}dx$$

1. Calculate the mean and variance of the gamma random variable X.

2. Let v be a positive integer. A random variable Y is called a **chi-square** (or χ^2) **random variable** with v degrees of freedom if Y is gamma random variable with parameters $\alpha = v/2$ and $\beta = 2$. Find the mean and variance of the chi-square (or χ^2) random variable Y.

3. What kind of random variable is yielded if X is a gamma random variable with parameter $\alpha = 1$?

***Bonus Question* 2**

A random variable X is said to be a **beta random variable** with parameters $\alpha > 0$ and $\beta > 0$ if the density of X is

$$f(x) = \begin{cases} \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

where

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

Calculate the mean and variance of the beta random variable X.