ECON2125/4021/8013*
Week 10 Tutorial Questions (1/5/2015)

Semester 1 2015

***Question* 1**

1. Suppose there is an arbitrary constant $a > 1$. Show that: $\lim_{n\to\infty} \sqrt[n]{a} = 1$.

2. Show that: $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

3. Prove that: if $\lim_{n\to\infty} a_n = a$, then $\lim_{n\to\infty} \frac{a_1+a_2+...+a_n}{n} = a$.

Hint: using the ε-language that you have learned in the lecture. (i.e.: given $\varepsilon > 0$, find the suitable N). For example, to show that $\lim_{n\to\infty} \sqrt[n]{a} = 1$, you need to pick an arbitrary $\varepsilon > 0$ firstly, then try to find an $N = N(\varepsilon)$ such that $n \geq N \Rightarrow |\sqrt[n]{a} - 1| < \varepsilon$. Also, you probably need to use the following fact that $\forall a, b \in \mathbb{R}$, $(a+b)^n = \sum_{i=0}^{n} C_n^i a^i b^{n-i}$, where $C_n^i := \frac{n!}{i!(n-i)!}$ for $1 \leq i \leq n$.

***Question* 2**

Prove that: $\lim_{n\to\infty} (a_1^n + a_2^n + ... + a_p^n)^{1/n} = \max_{1\leq i\leq p} \{a_i\}$, where $a_i \geq 0$ $(i = 1, 2, 3, ..., p)$.

Hint: using Squeeze Theorem.

***Question* 3**

If a sequence of closed intervals $\{[a_n,\, b_n]\}$ satisfies:
(1) $[a_{n+1},\, b_{n+1}] \subset [a_n,\, b_n]$, $n = 1, 2, 3, ...$;
(2) $\lim_{n\to\infty} (b_n - a_n) = 0$.

Then, show that there exists a **unique** real number $\xi \in \mathbb{R}$, such that:
1. $\xi \in [a_n,\, b_n]$, for all $n \in \mathbb{N}^+$;

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2. $\xi = \lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$.