



University of
Southampton

EVOLUTIONARY ROUTES

L1: GRAPHS AND PATHS

LESSON OUTCOMES

- LO1: Give a definition for a **graph**, **vertices**, and **edges**.
- LO2: Construct **adjacency matrices** and find the **degrees** of the vertices.
- LO3: Find the **shortest path** in a graph

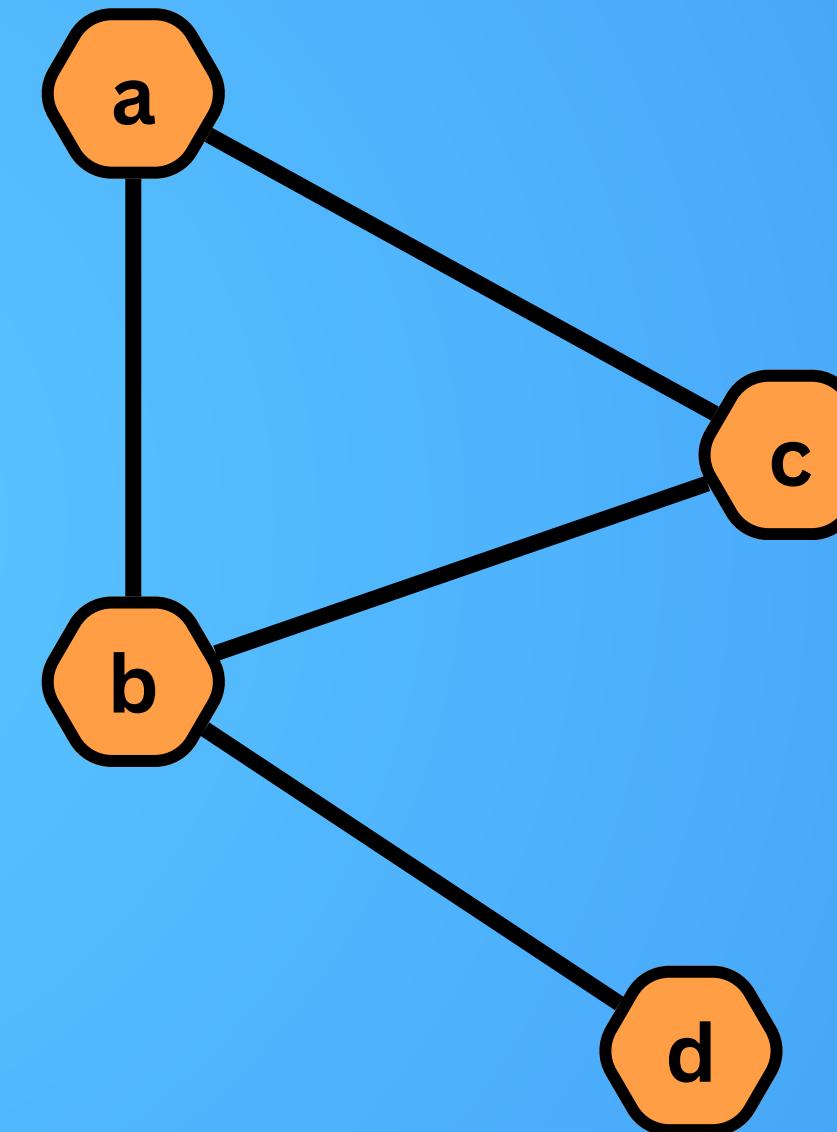
This a graph



It has vertices

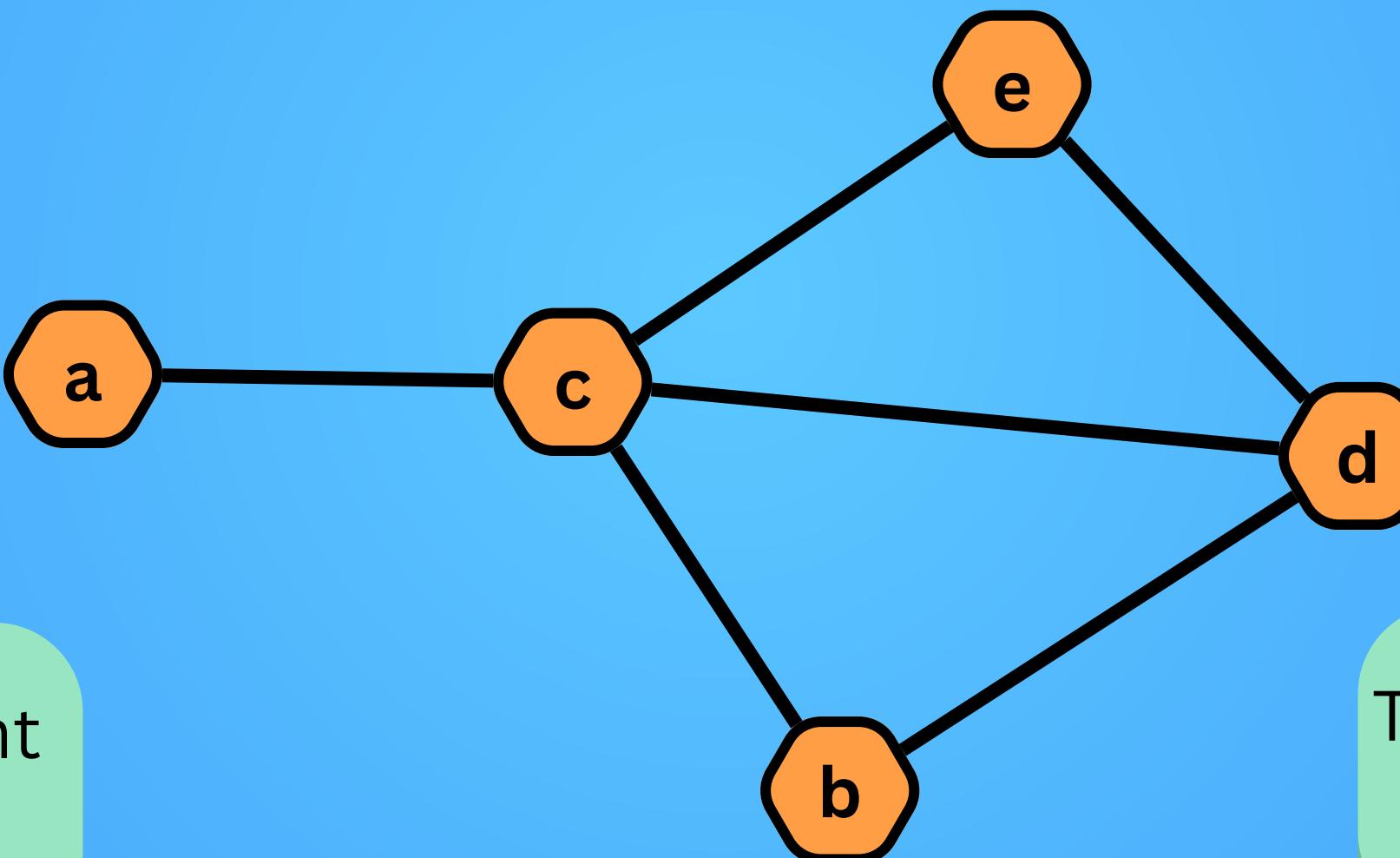


and edges



DEFINITION

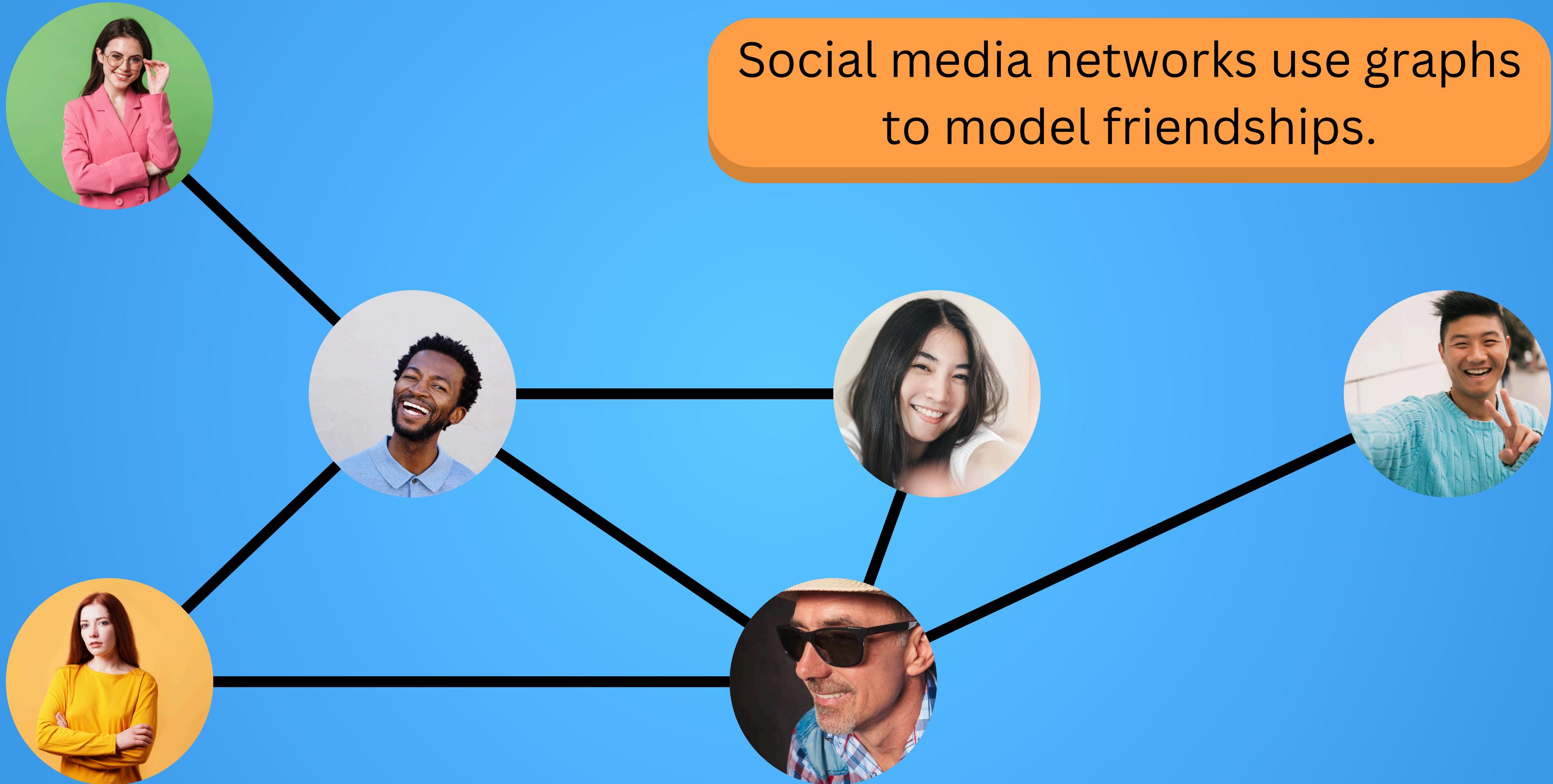
A **graph** is a model of how entities (people, places, things) are related.

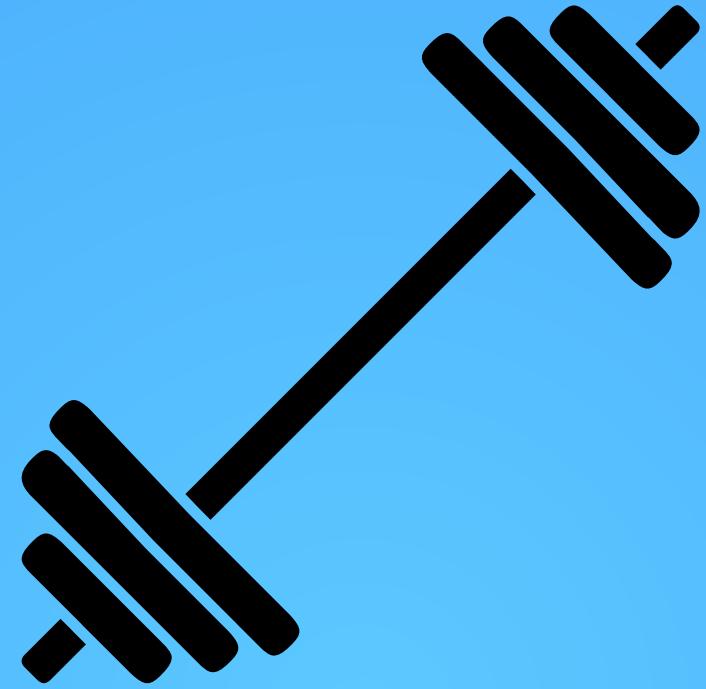


The **vertices** represent the entities.

The **edges** represent the relationships.

Social media networks use graphs to model friendships.





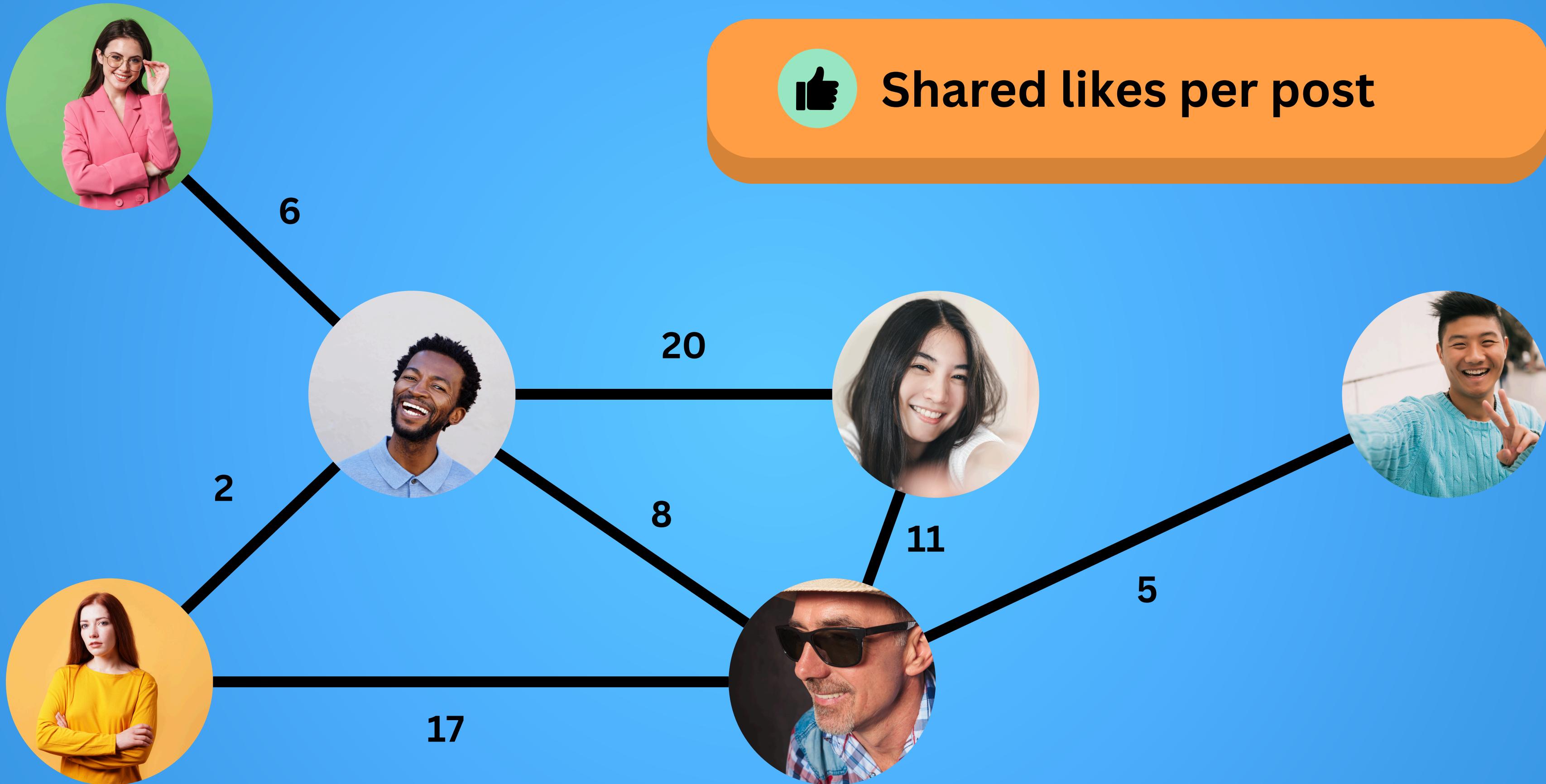
IDEA

The edges of a graph can be given a value called a “**weight**”.

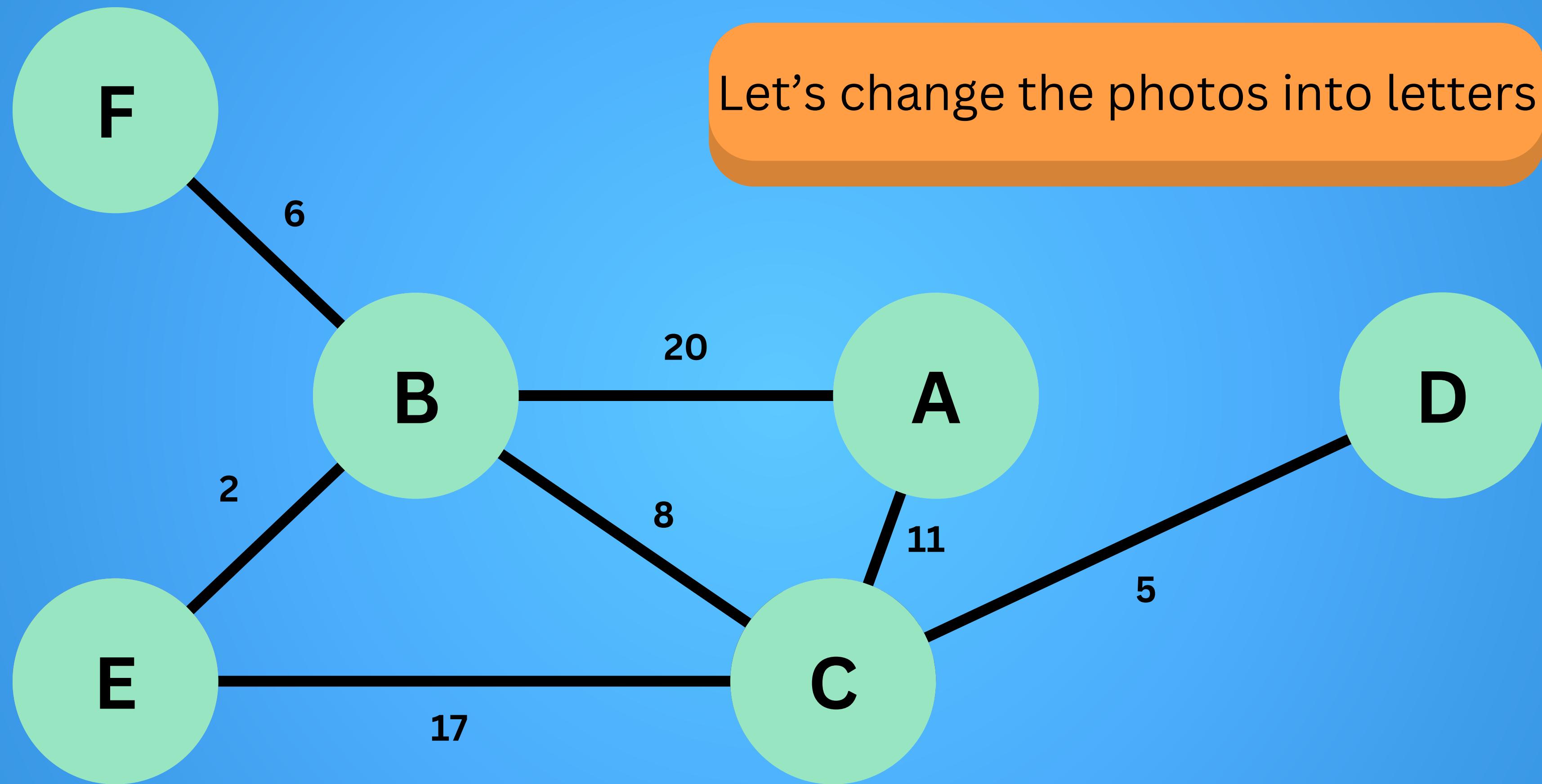
The weight of an edge indicates how strong the relationship is.



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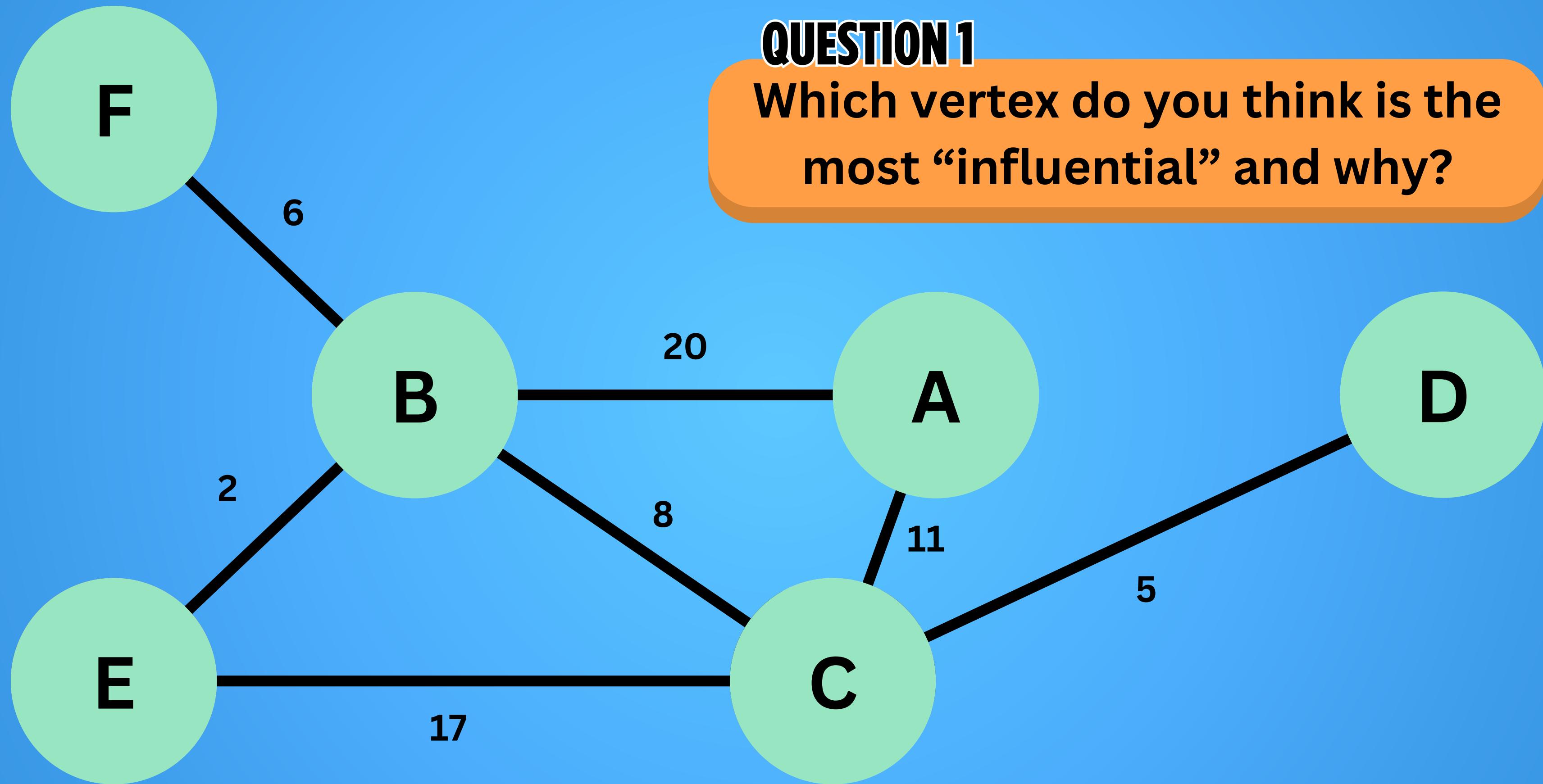


Let's change the photos into letters



QUESTION 1

Which vertex do you think is the most “influential” and why?





	A	B	C	D	E	F
A	0	20	11	0	0	0
B	20	0	8	0	2	6
C	11	8	0	5	17	0
D	0	0	5	0	0	0
E	0	2	17	0	0	0
F	0	6	0	0	0	0

ANSWER 1

All the vertices that have edges between them are “**adjacent**”.

We write down the edge weights in an “**adjacency matrix**”.



A	0	20	11	0	0	0
B	20	0	8	0	2	6
C	11	8	0	5	17	0
D	0	0	5	0	0	0
E	0	2	17	0	0	0
F	0	6	0	0	0	0

ANSWER 1

Next we add up each value in each row to find the “**degree**” of the vertex.

$$\deg(A) = 20 + 11 = 31$$



A	0	20	11	0	0	0
B	20	0	8	0	2	6
C	11	8	0	5	17	0
D	0	0	5	0	0	0
E	0	2	17	0	0	0
F	0	6	0	0	0	0

$\deg(A) = 31$

$\deg(B) = 36$

$\deg(C) = 41$

$\deg(D) = 5$

$\deg(E) = 19$

$\deg(F) = 6$

A B C D E F

A	0	20	11	0	0	0
B	20	0	8	0	2	6
C	11	8	0	5	17	0
D	0	0	5	0	0	0
E	0	2	17	0	0	0
F	0	6	0	0	0	0

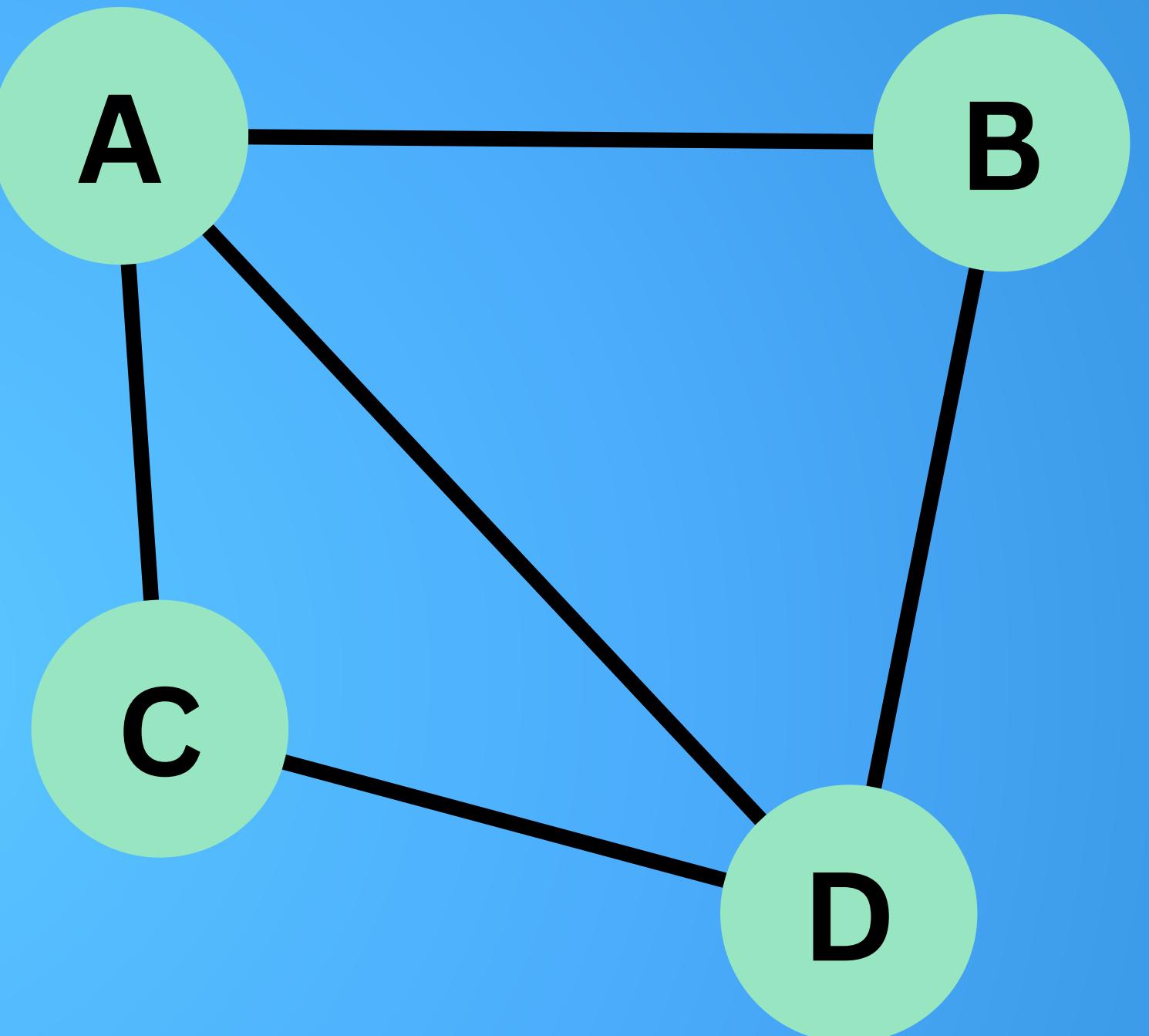
ANSWER 1

Then we pick the highest value!

$$\deg(C) = 41$$

This is called “degree centrality”. It’s one of many kinds of “centrality” metrics.

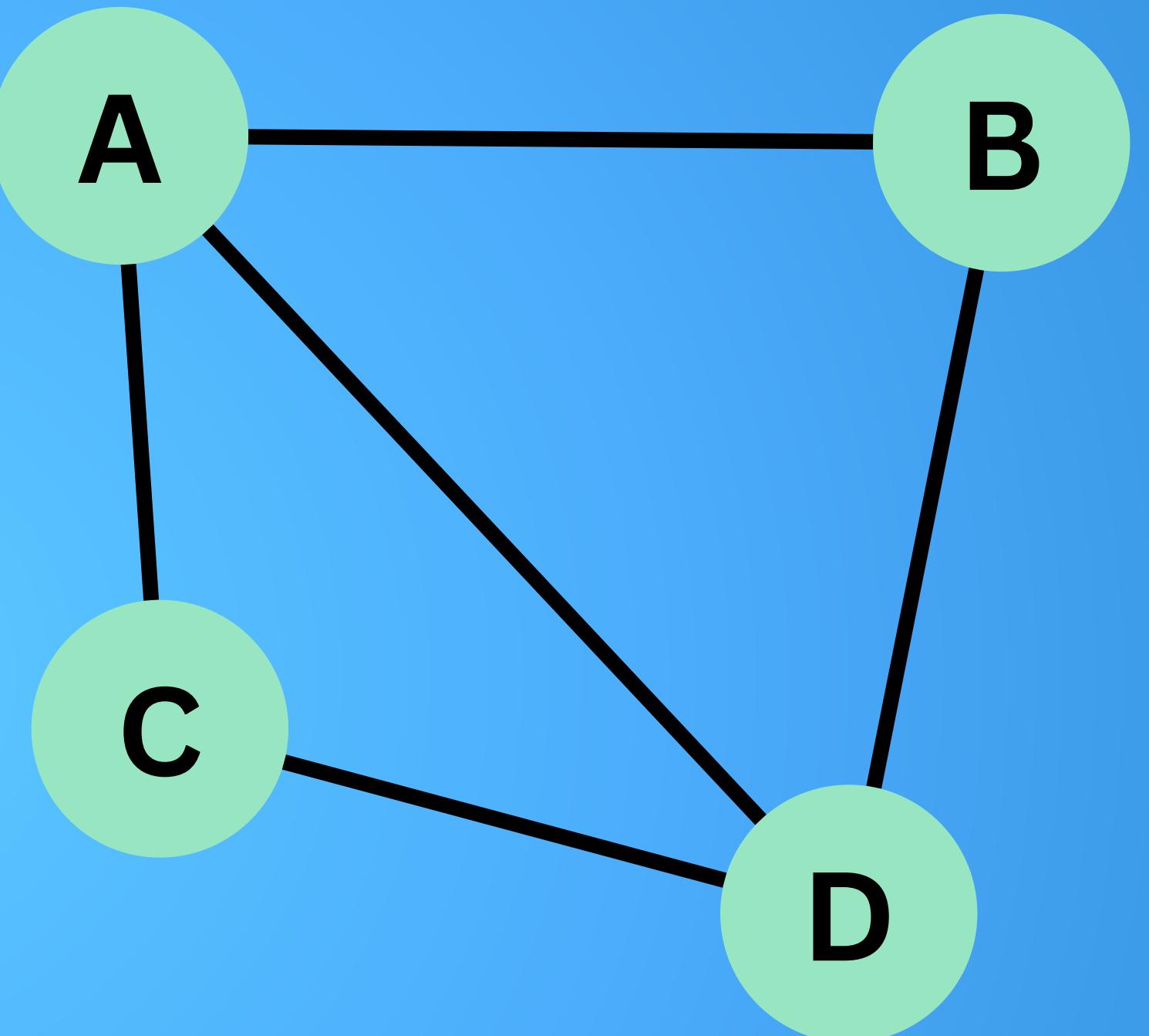
	A	B	C	D
A	0	1	1	1
B	1	0	0	1
C	1	0	0	1
D	1	1	1	0



QUESTION 2

What is $\deg(A)$, $\deg(B)$, $\deg(C)$, and $\deg(D)$?

	A	B	C	D
A	0	1	1	1
B	1	0	0	1
C	1	0	0	1
D	1	1	1	0



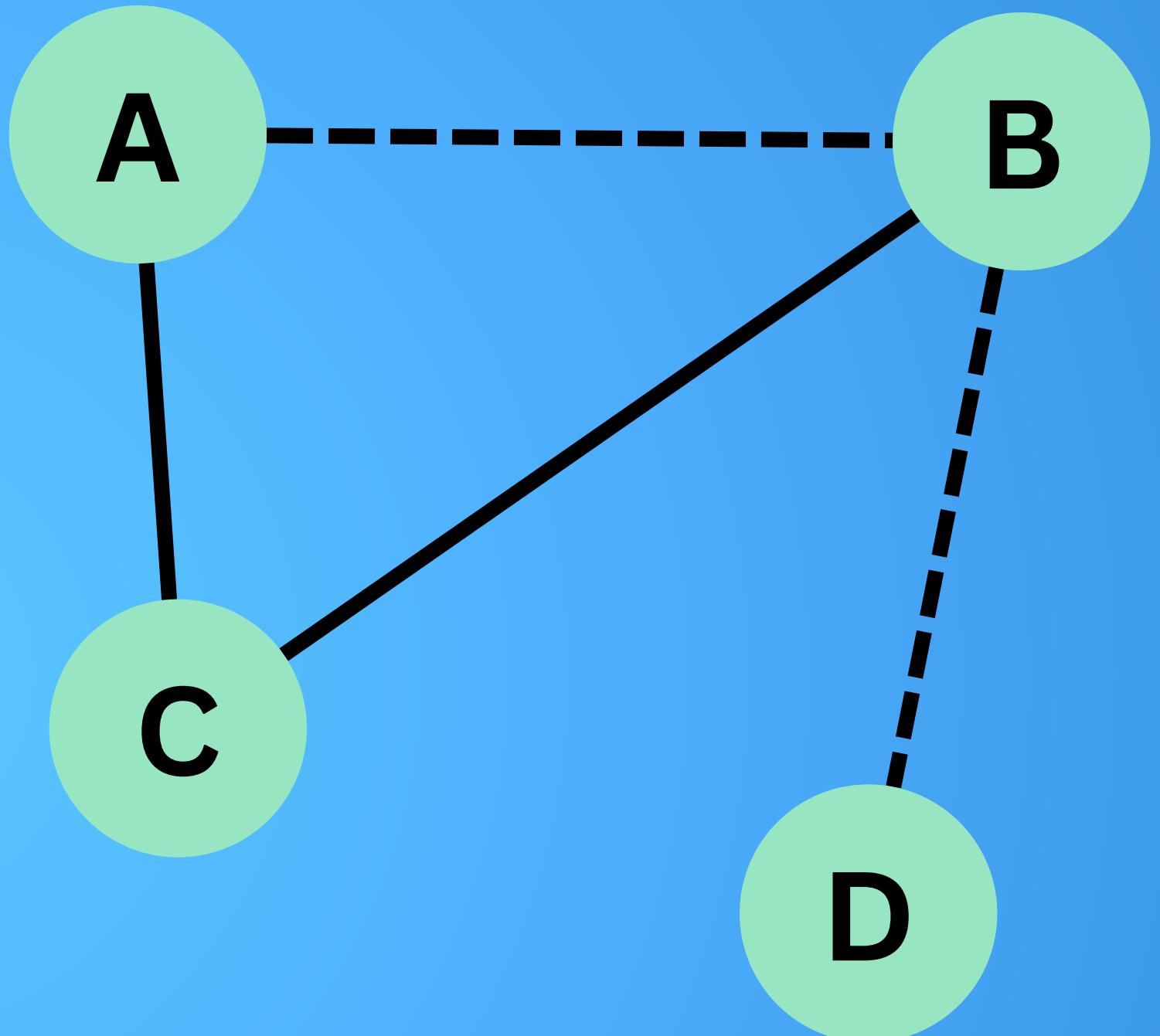
ANSWER 2

$\deg(A) = 3$, $\deg(B) = 2$, $\deg(C) = 2$, and $\deg(D) = 3$

IDEA

Even though two vertices aren't "adjacent", they can be **connected**.

A connects to D through B.

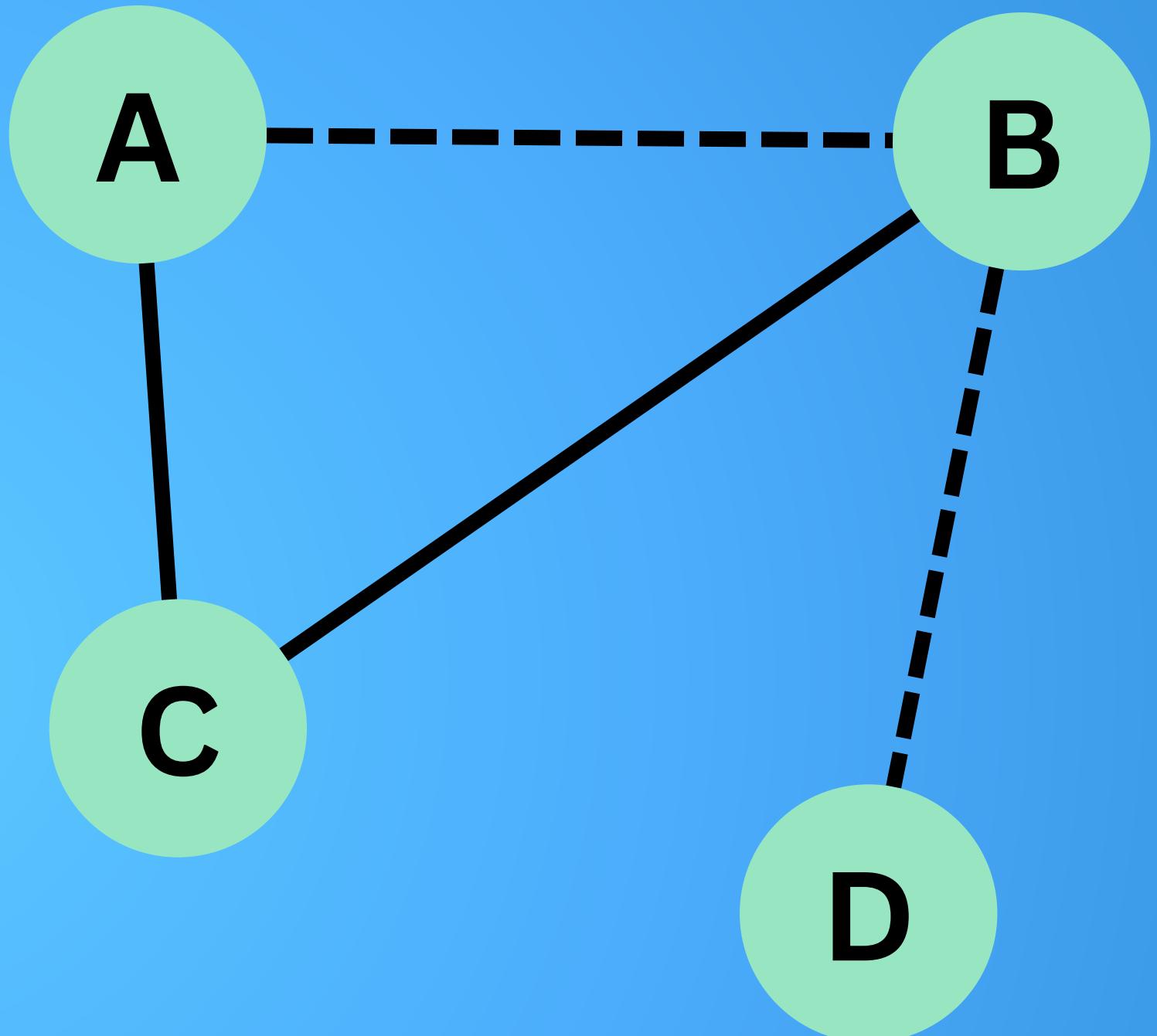


IDEA

If we don't repeat the vertices we travel along we form a “path”.

A > B > D

A > C > B > D

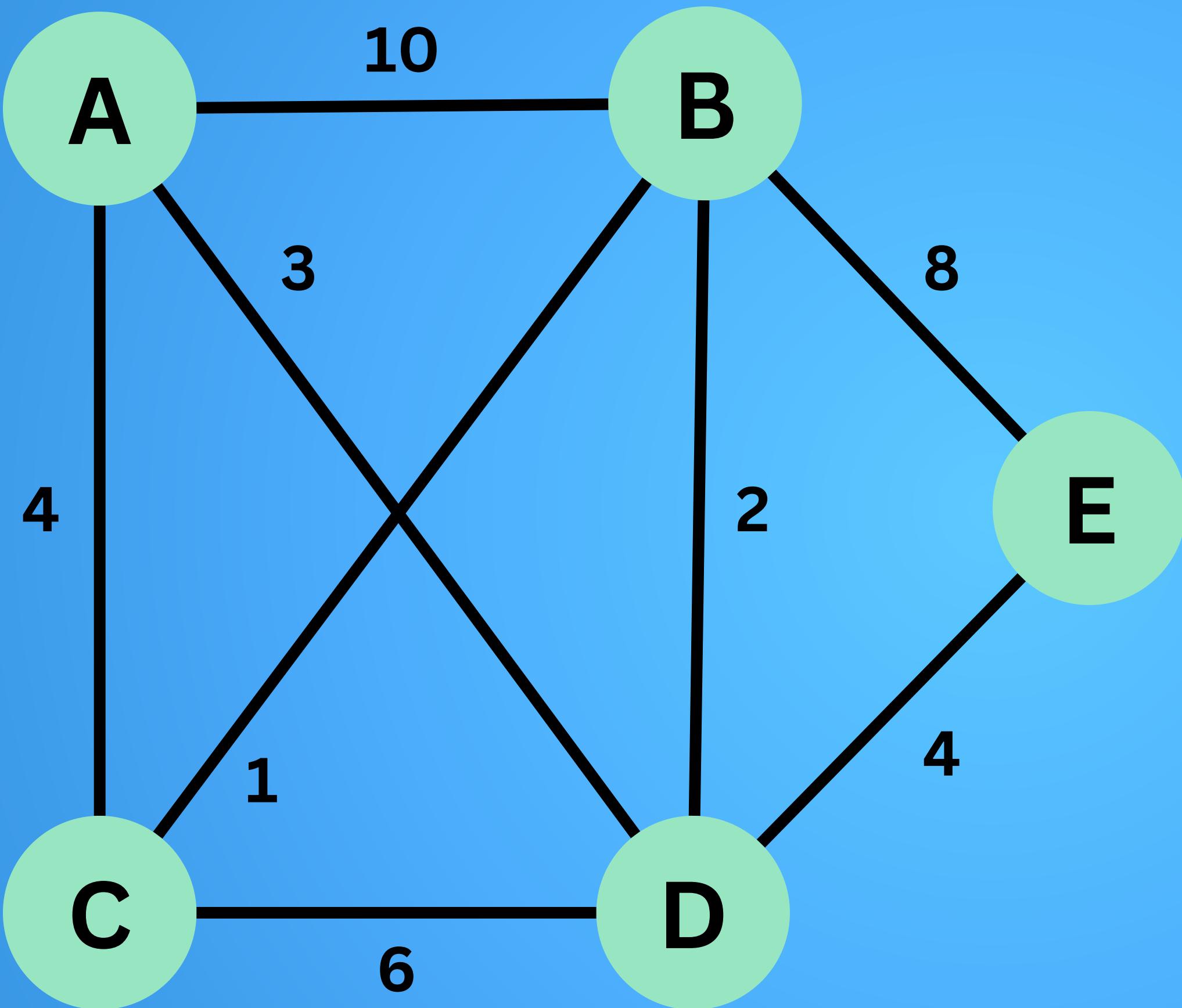


IDEA

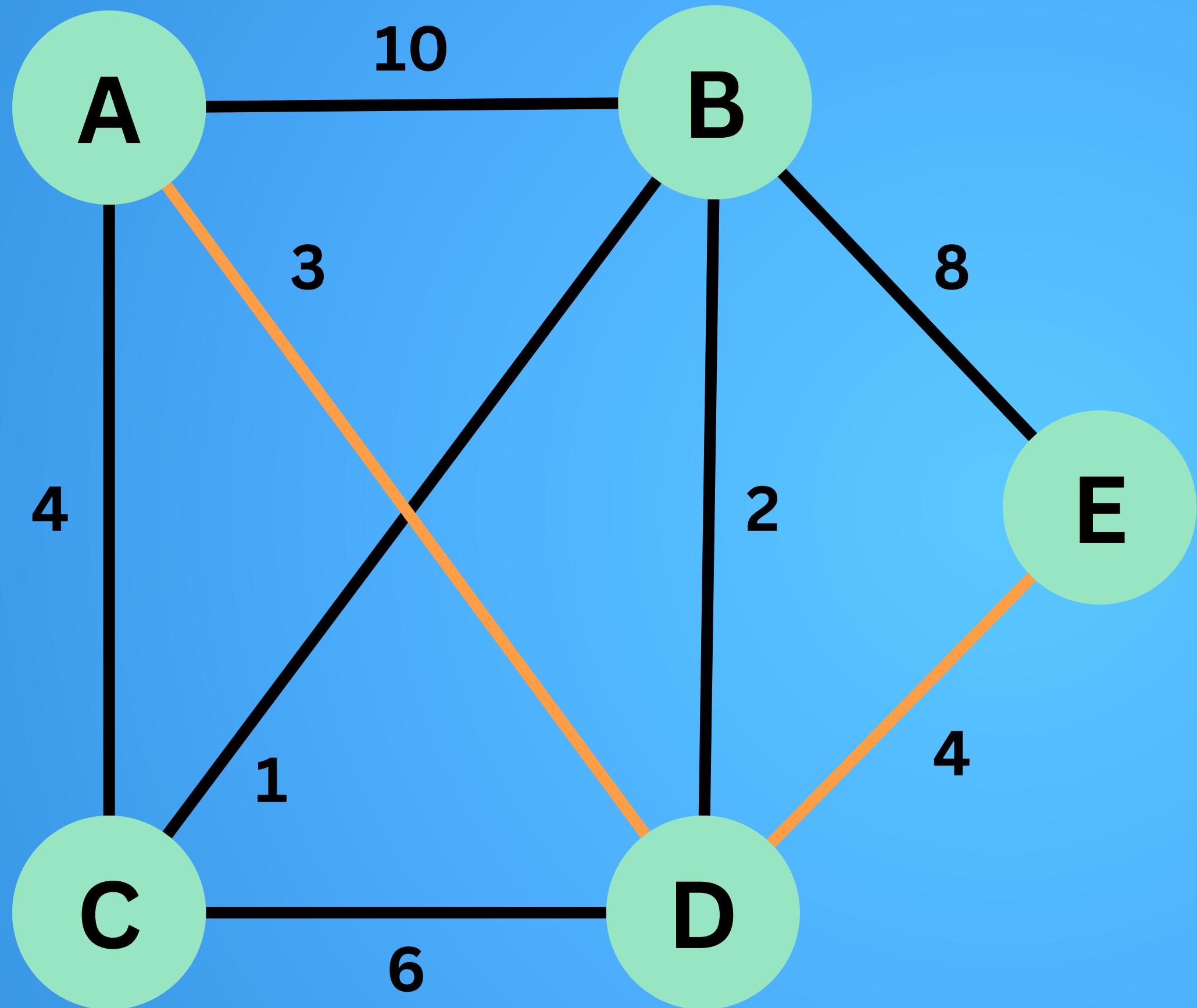


When the edges have weights we are often interested in finding out the “**shortest path**”.

QUESTION 3



What is the shortest path from A to E?



ANSWER 3

What is the shortest path from A to E?

$$A > D > E = 3 + 4 = 7$$

RECAP

A **graph** is a model of how entities (people, places, things) are related.

Graphs have **vertices** that represent people, places, or things, and **edges**, that show the relationships between entities.

Edges can have **weights**.

Vertices have a **degree** which tells us how connected it is within the graph.

A **path** is a list of vertex connections with no repeats.