

# Introduction to Bayesian Statistics

Bayesian Modeling in brms

---

Julia Haaf

September, 2022

- Bayesian statistics is complicated.

# Disclaimer

- Bayesian statistics is complicated.
- Hierarchical modeling is complicated.

# Disclaimer

- Bayesian statistics is complicated.
- Hierarchical modeling is complicated.
- Cognitive modeling is complicated.

# Disclaimer

- Bayesian statistics is complicated.
- Hierarchical modeling is complicated.
- Cognitive modeling is complicated.
- This is an extremely brief primer.

- Bayesian statistics is complicated.
- Hierarchical modeling is complicated.
- Cognitive modeling is complicated.
- This is an extremely brief primer.
- Check out resources here:  
<https://github.com/jstbcs/ESCOP2022-WS/blob/main/resources.txt>.

# An Example

- This is Frank.



## An Example

- This is Frank.
- Frank likes to eat but he might be a tad picky.





# An Example

- This is Frank.
- Frank likes to eat but he might be a tad picky.
- We want to model how often Frank eats his food in a month.



## An Example

Is Frank a picky eater?

- $Y \sim \text{Binomial}(\theta, 30)$ , modeling how often out of 30 Frank eats his food.



## An Example

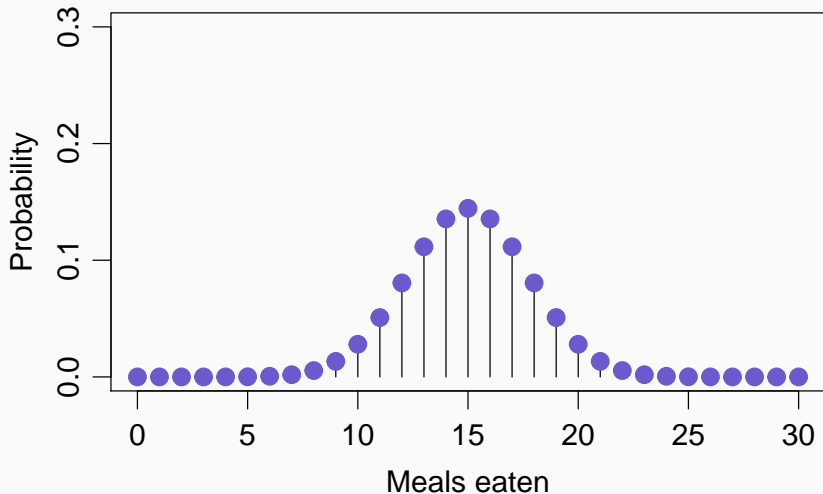
Is Frank a picky eater?

- $Y \sim \text{Binomial}(\theta, 30)$ , modeling how often out of 30 Frank eats his food.
- $\theta = .5$ , assuming the probability of eating is 50/50.



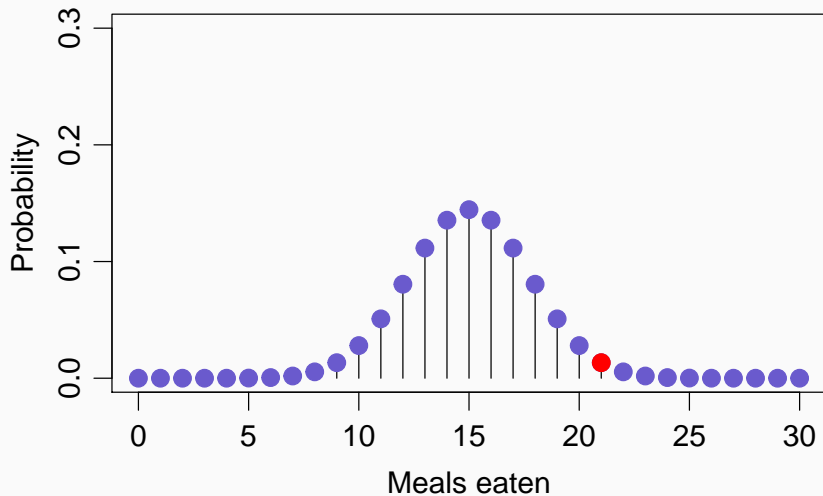
## Models, an Example

Predictions on data, based on the model  $Y \sim \text{Binomial}(\theta, 30)$ ,  
 $\theta = .5$ .



# Data

Let's say he ate 21 out of 30 meals.  $Y = 21$ .



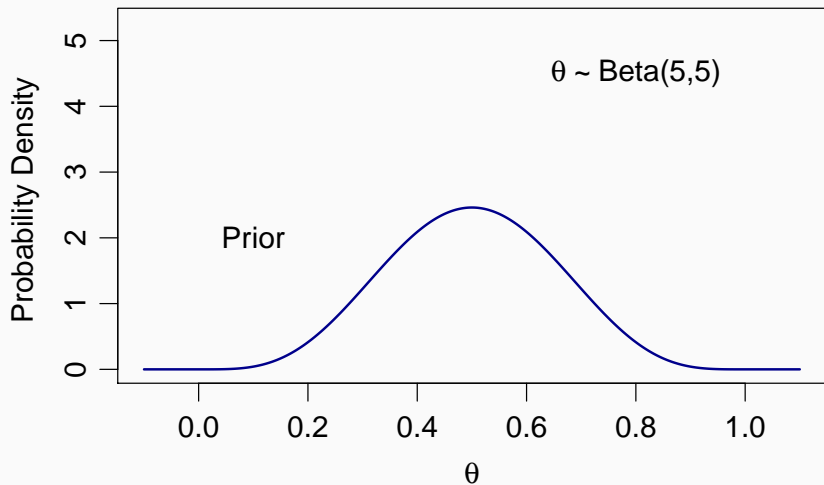
In Bayesian statistical analysis we typically would use a prior *distribution* for parameters.

$$Y|\theta \sim \text{Binomial}(\theta, N),$$
$$\theta \sim \text{Beta}(a, b).$$

If we assume Frank will most likely eat 5 out of 10 meals we may use  $a = 5$  and  $b = 5$ .

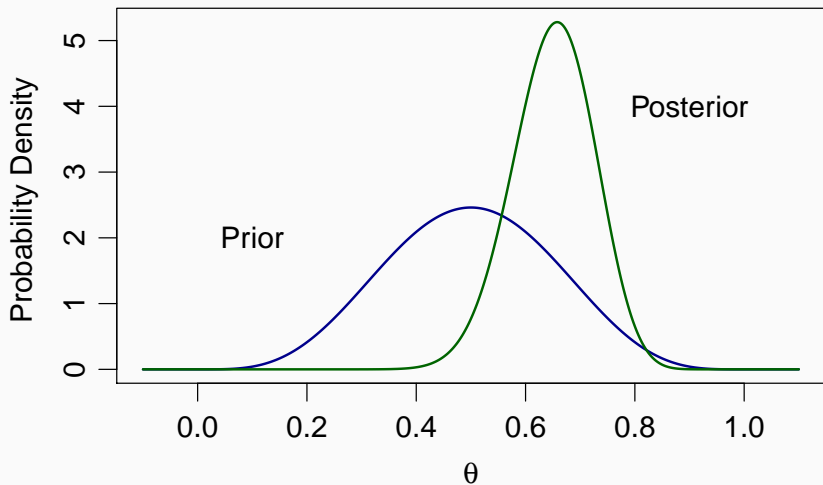
# Posterior Updating

from  $Pr(\theta) \dots$



# Posterior Updating

from  $Pr(\theta) \dots$  to  $Pr(\theta|Y)$ .





## Bayes' Rule:

$$Pr(\theta|Y) = Pr(\theta) \frac{Pr(Y|\theta)}{Pr(Y)},$$

- $Pr(\theta|Y)$  is the *posterior distribution* of  $\theta$ .

## Bayes' Rule:

$$Pr(\theta|Y) = Pr(\theta) \frac{Pr(Y|\theta)}{Pr(Y)},$$

- $Pr(\theta|Y)$  is the *posterior distribution* of  $\theta$ .
- $Pr(\theta)$  is the *prior distribution* of  $\theta$ .

## Bayes' Rule:

$$Pr(\theta|Y) = Pr(\theta) \frac{Pr(Y|\theta)}{Pr(Y)},$$

- $Pr(\theta|Y)$  is the *posterior distribution* of  $\theta$ .
- $Pr(\theta)$  is the *prior distribution* of  $\theta$ .
- $Pr(Y|\theta)$  is the probability distribution of the data.

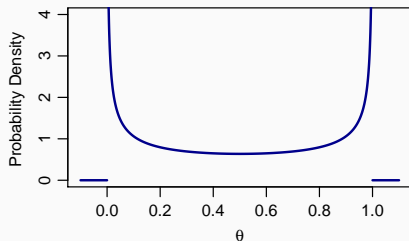
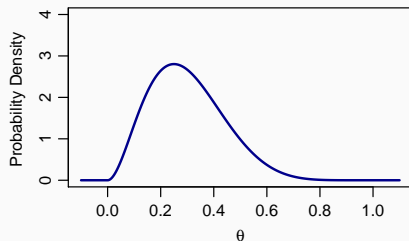
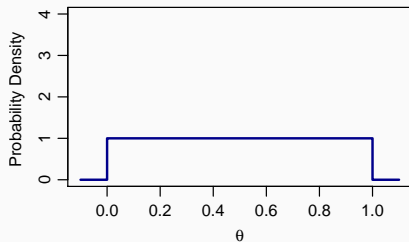
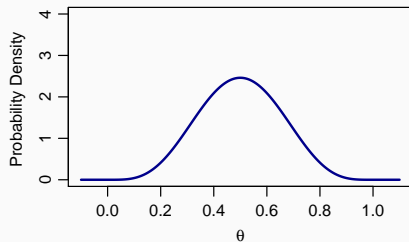
## Bayes' Rule:

$$Pr(\theta|Y) = Pr(\theta) \frac{Pr(Y|\theta)}{Pr(Y)},$$

- $Pr(\theta|Y)$  is the *posterior distribution* of  $\theta$ .
- $Pr(\theta)$  is the *prior distribution* of  $\theta$ .
- $Pr(Y|\theta)$  is the probability distribution of the data.
- $Pr(Y)$  is the prediction for the data.

# Did we choose a good prior?

What should the prior on Frank's eating habit look like?



# Matching priors to goals of analysis

- There are priors that are most suitable for estimation.



# Matching priors to goals of analysis

- There are priors that are most suitable for estimation.
- And there are priors most suitable for model comparison.



# Matching priors to goals of analysis

- There are priors that are most suitable for estimation.
- And there are priors most suitable for model comparison.
- And there are priors that are pretty good for both.





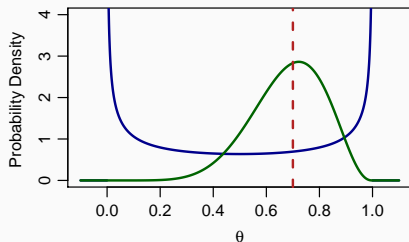
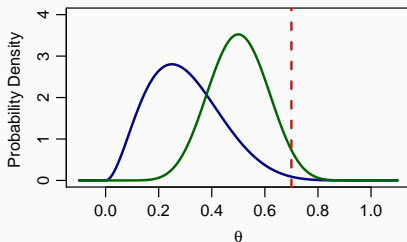
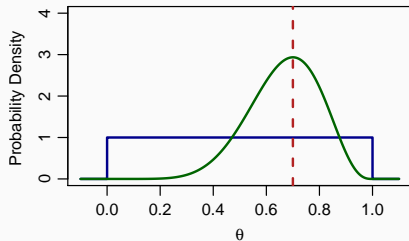
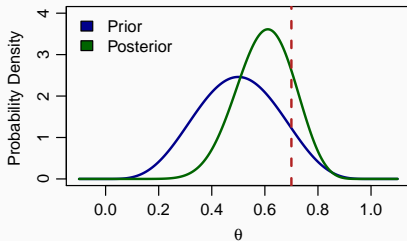
# Matching priors to goals of analysis

- There are priors that are most suitable for estimation.
- And there are priors most suitable for model comparison.
- And there are priors that are pretty good for both.
- Oh, and not everyone agrees on this classifications (or what “good means”).



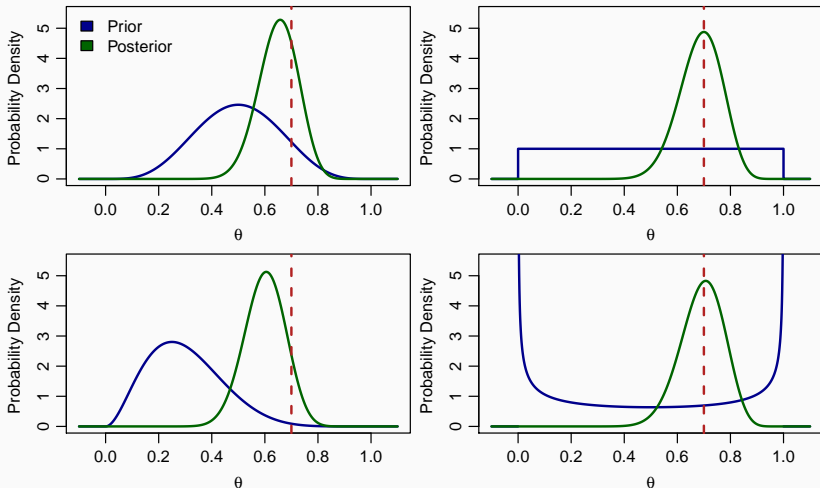
# Why Priors for Estimation Don't Matter That Much

Frank eats his food 7 out of 10 times.



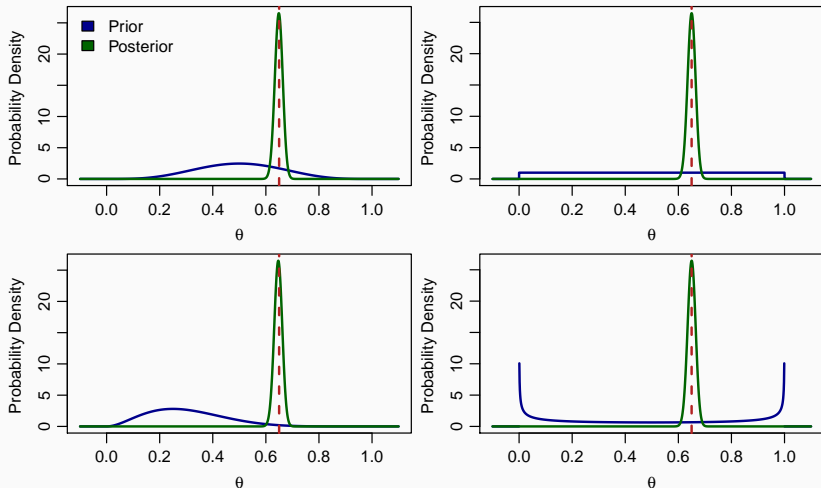
# Why Priors for Estimation Don't Matter That Much

Frank eats his food 21 out of 30 times.



# Why Priors for Estimation Don't Matter That Much

Frank eats his food 650 out of 1000 times.



# Questions?

