Mathematical Modelling

Signal Detection Model

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February/March 2021

Feedback

Let us know what went well and what could be improved!

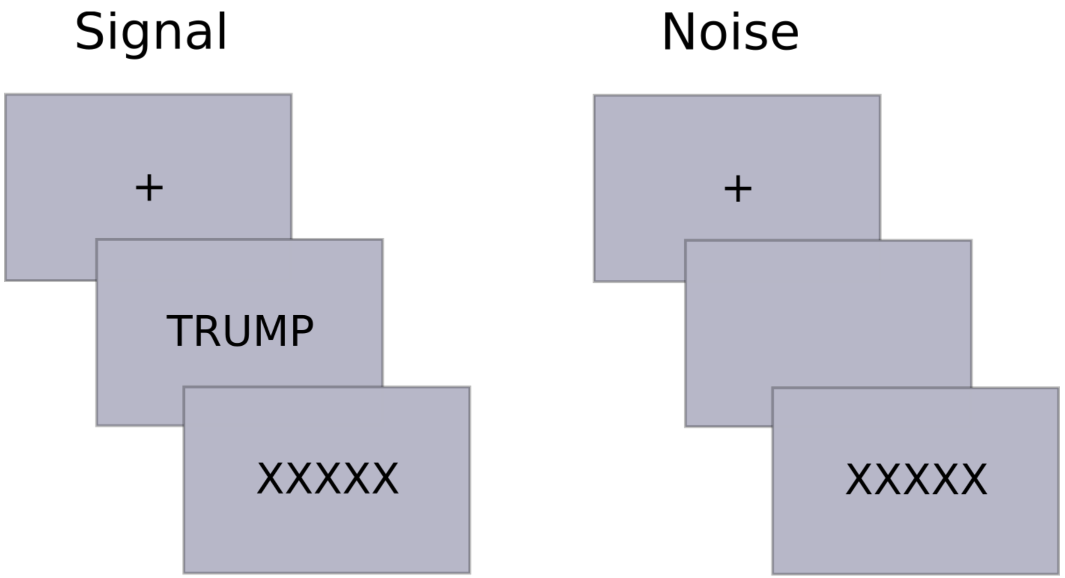
Go to feedback survey

Agenda

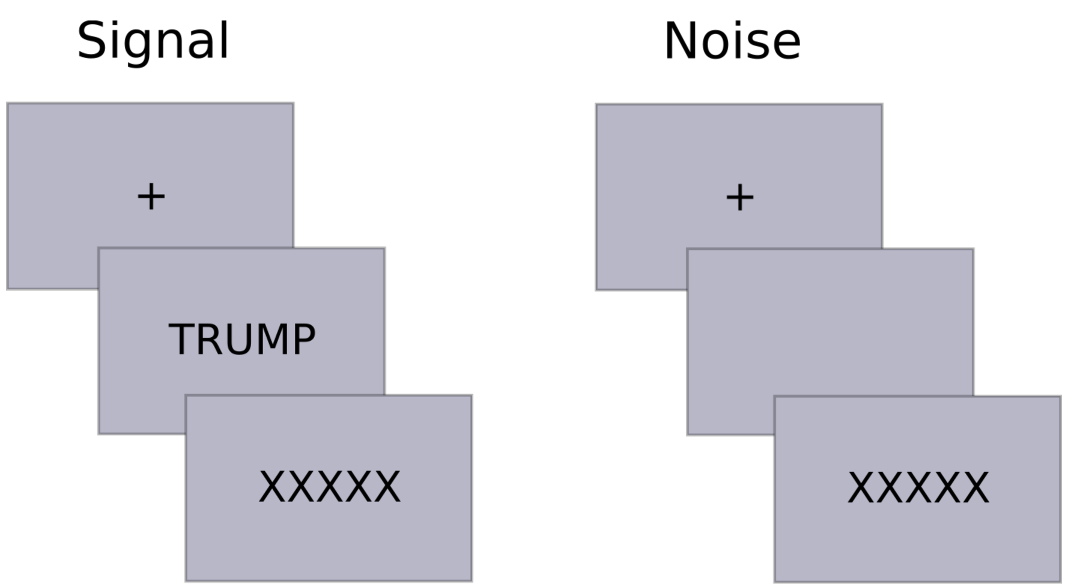
Signal detection models

1. Signal detection experiments
2. A bit more probability theory
3. Signal detection models
    - Receiving Operator Characteristics (ROC)
    - SDT in `R`
    - SDT in practice

Signal detection experiment

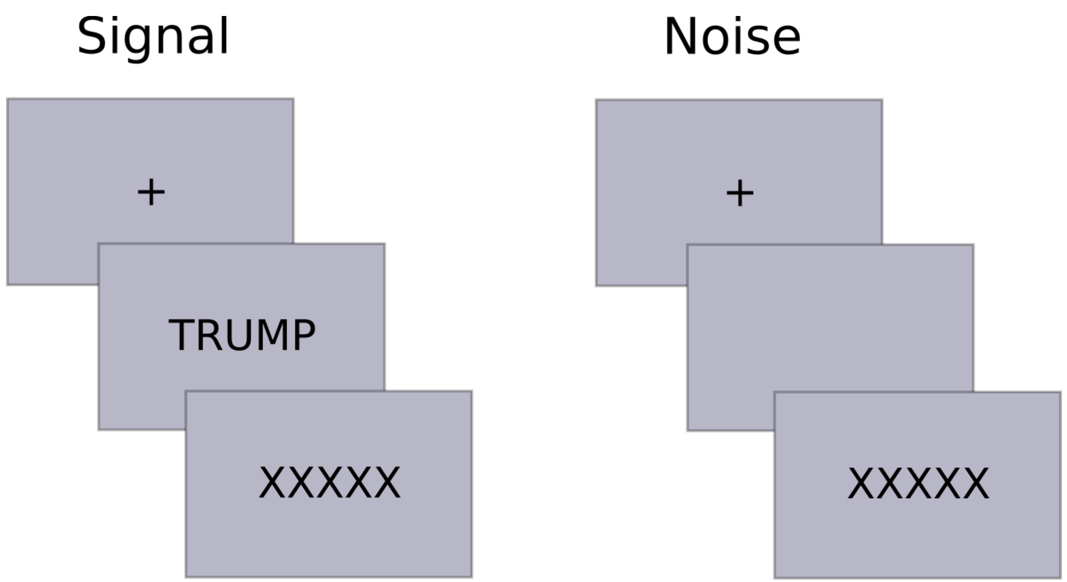


Signal detection experiment



Stimulus	Present response	Absent Response	Total
Signal	75	25	100
Noise	30	20	50
Total	105	45	

Signal detection experiment



Stimulus	Present response	Absent Response	Total
Signal	75 (Hits)	25 (Misses)	100
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Signal Detection Theory vs. Signal detection experiment

- Signal Detection Theory (SDT): Model-based method of assessing performance

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- Originally developed for signal detection experiments
- Now used for perception, memory, decision theory, attitude research, ...

Green & Swets, 1966

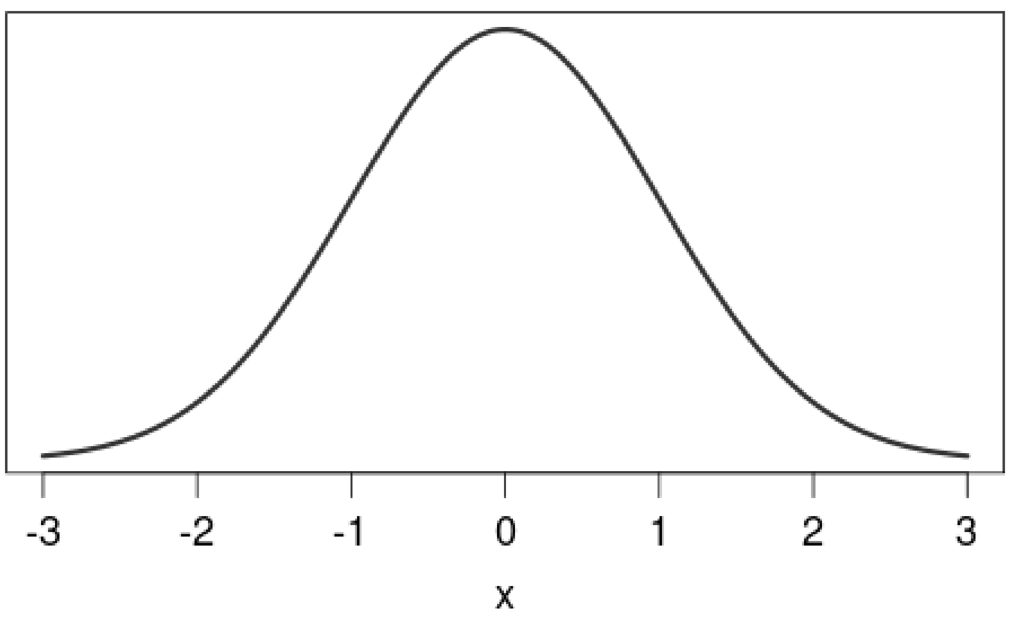
Signal Detection Theory vs. Signal detection experiment

- Signal Detection Theory (SDT): Model-based method of assessing performance
- Originally developed for signal detection experiments
- Now used for perception, memory, decision theory, attitude research, ...
- Originally for two-choice tasks
- Now extended to any form of forced-choice task including rating scales, continuous measures, ...

Green & Swets, 1966

Signal Detection Theory

Models are a bit more complicated and rely on understanding of continuous random variables.



A bit more probability theory

Density function

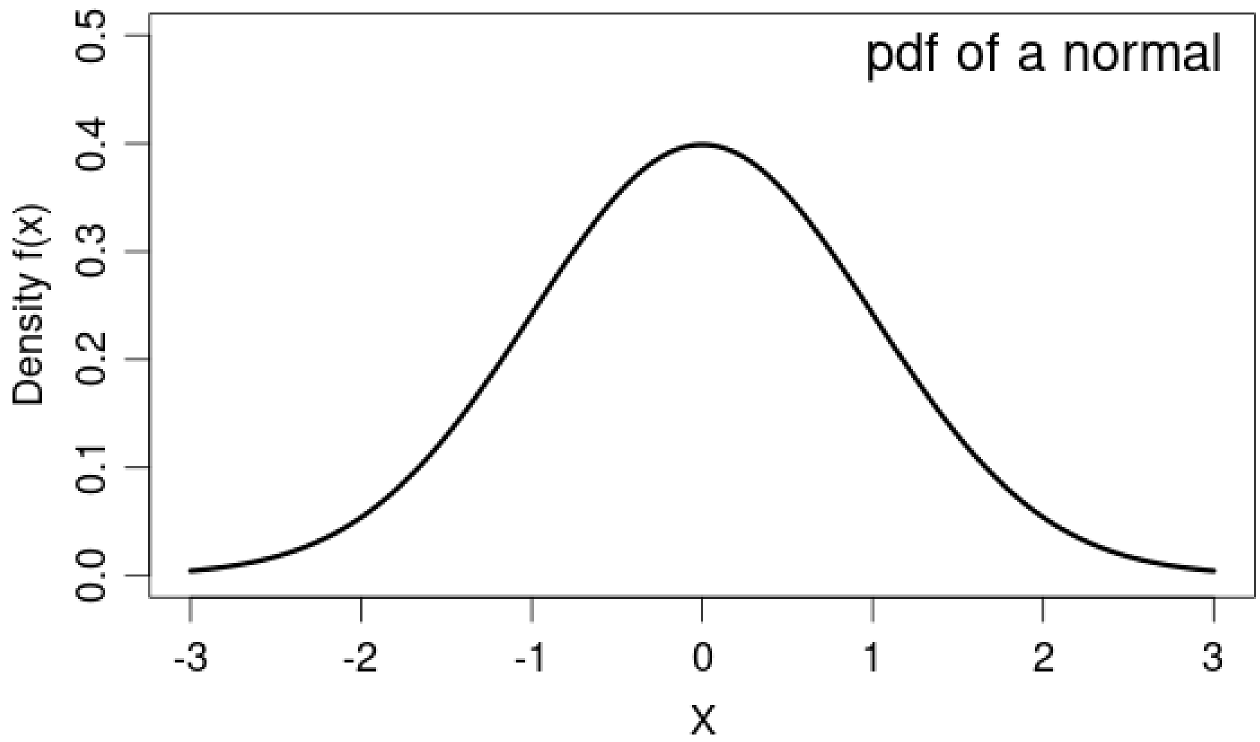
Definition *Density function:* The density function (pdf) of a continuous random variable *X*, *f(x)*, is a function such that the area under the function between *a* and *b* corresponds to the probability

$$Pr(a < x \leq b).$$

The value of *f(x)* is the probability density.

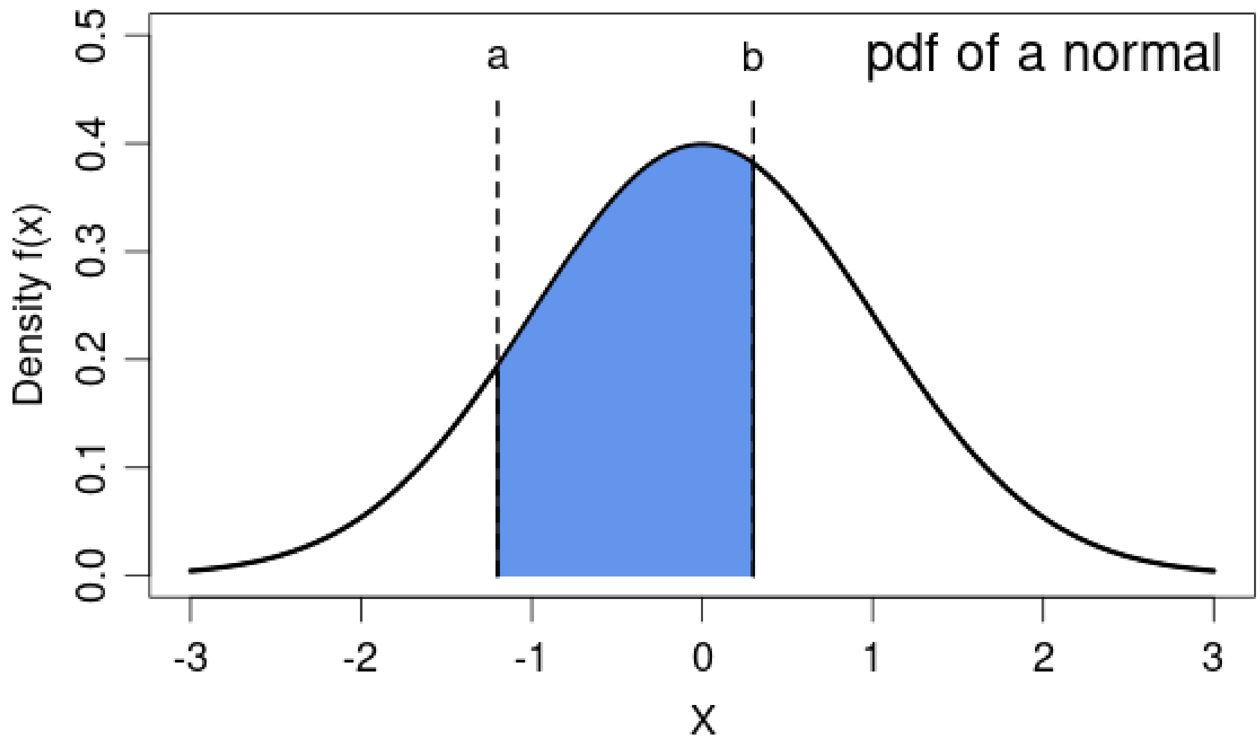
Density function

Example



Density function

Example



Density function in R

```r
dnorm(0, mean = 0, sd = 1)
```

```
## [1] 0.3989423
```

Density function in R

```r
x <- seq(-3, 3, .01)

head(x)
```

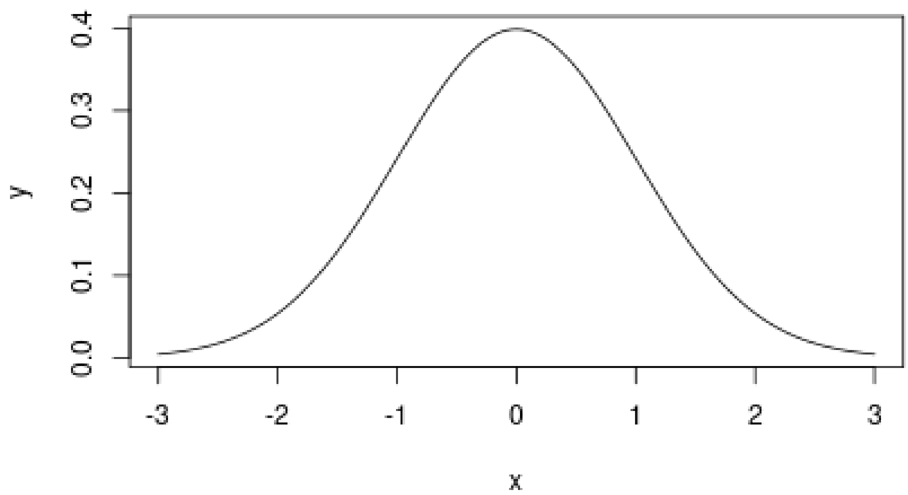
```
## [1] -3.00 -2.99 -2.98 -2.97 -2.96 -2.95
```

```r
y <- dnorm(x = x, mean = 0, sd = 1)

head(y)
```

```
## [1] 0.004431848 0.004566590 0.004704958 0.004847033 0.004992899 0.00514264
```

Density function in R

```r
x <- seq(-3, 3, .01)
y <- dnorm(x = x, mean = 0, sd = 1)

plot(x, y, type = "l")
```

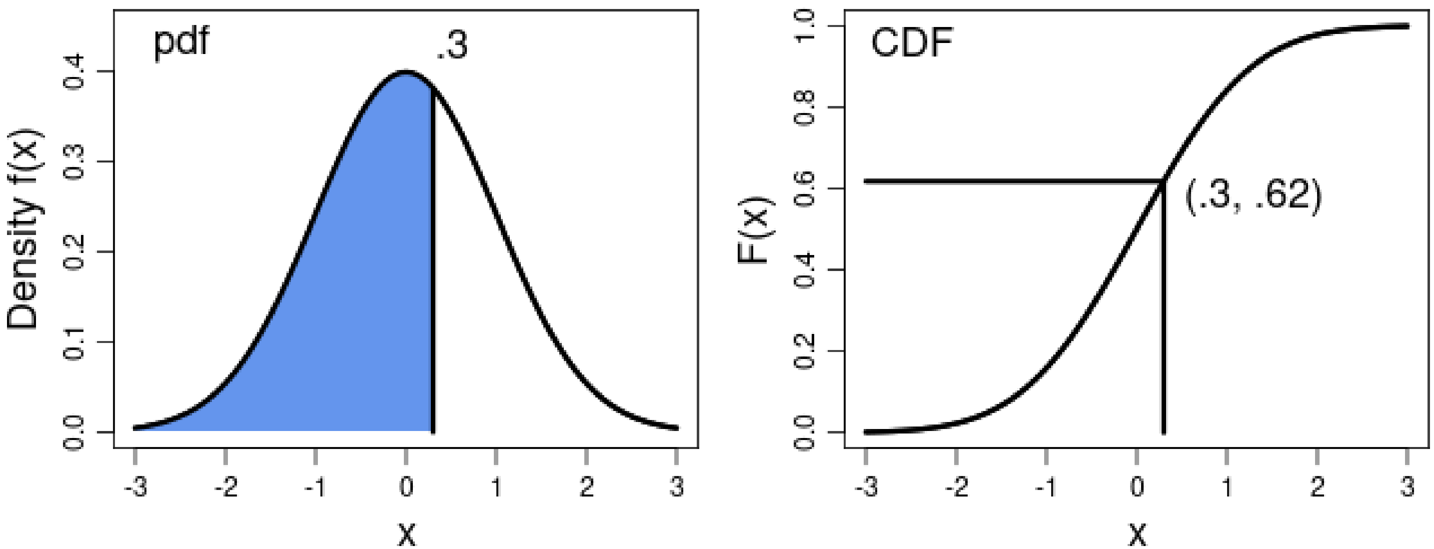


Cumulative distribution function

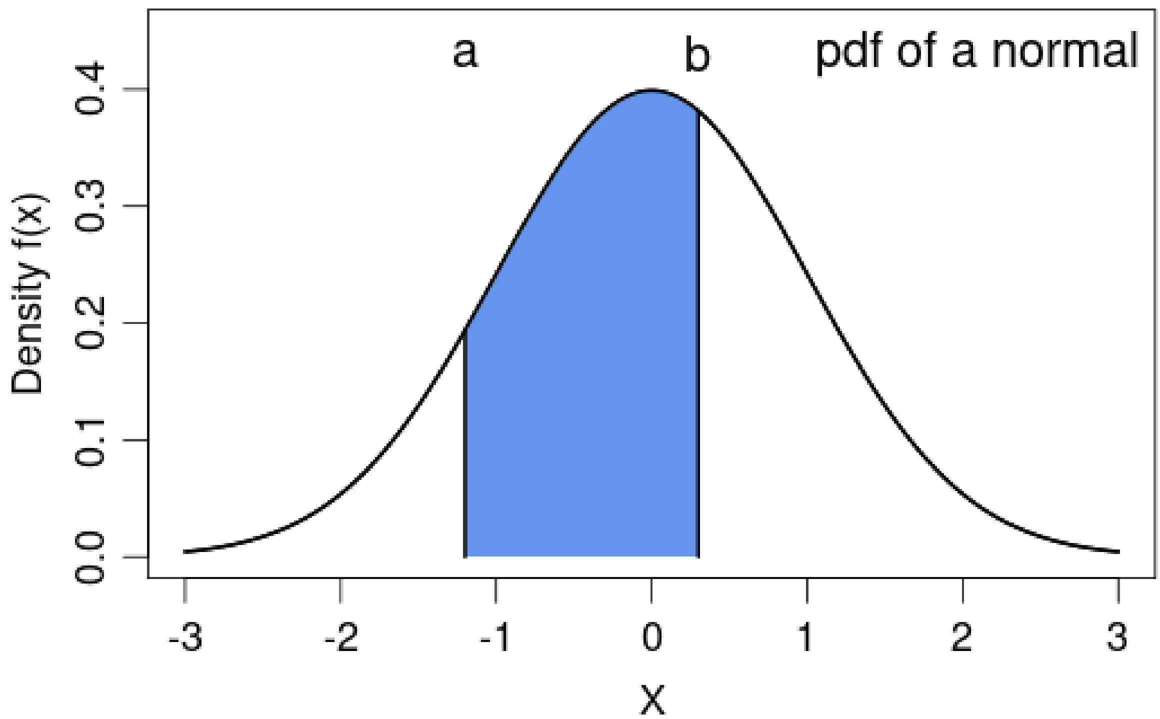
Definition *Cumulative distribution function:* Let F denote the cumulative distribution function (CDF) of random variable X. Then,

$$F(x) = Pr(X \leq x).$$

Example

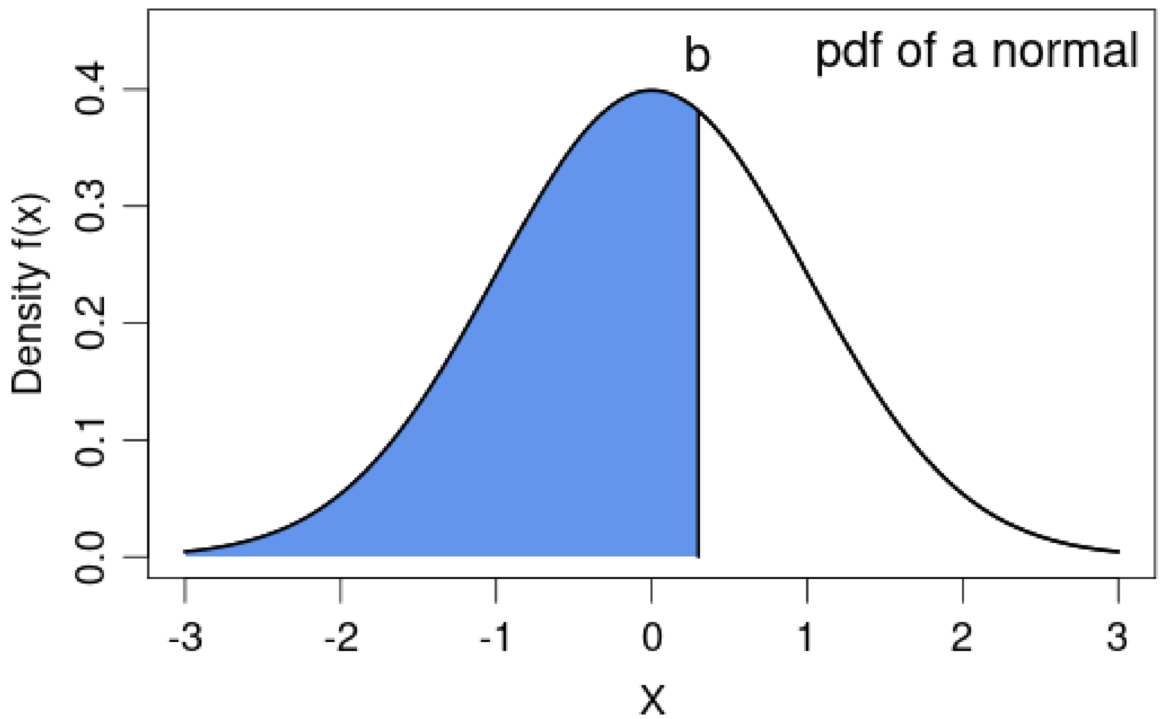


Using CDF and pdf



What is $Pr(a < x \leq b)$

Using CDF and pdf



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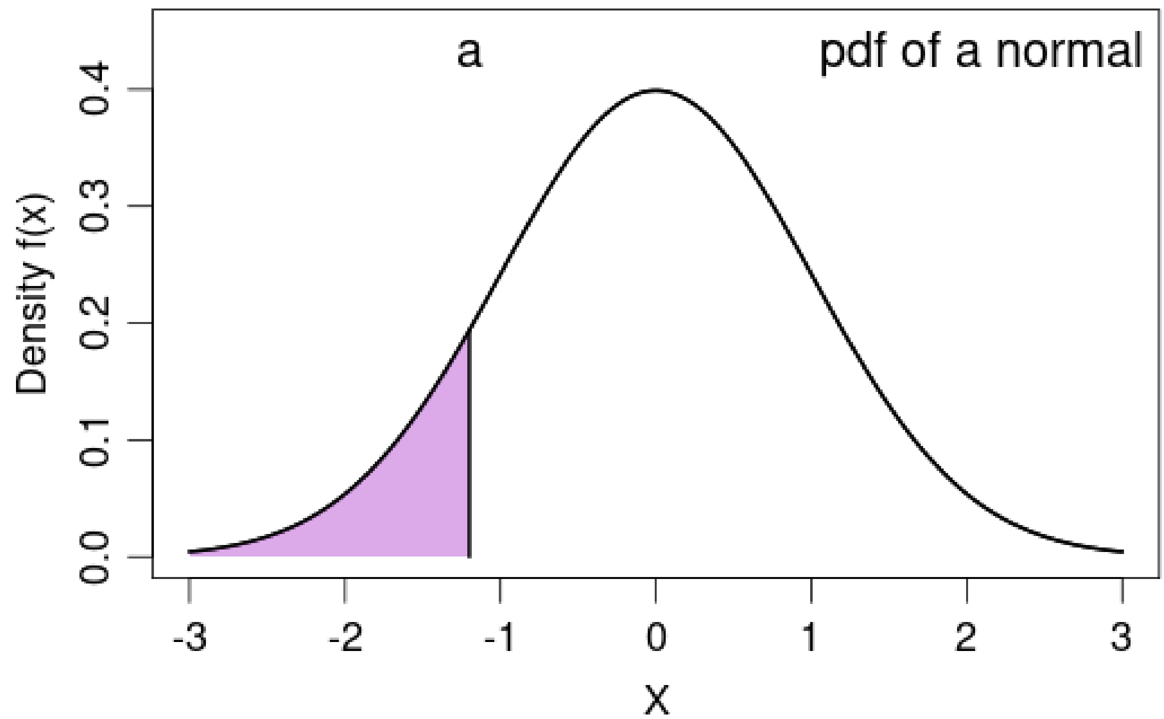

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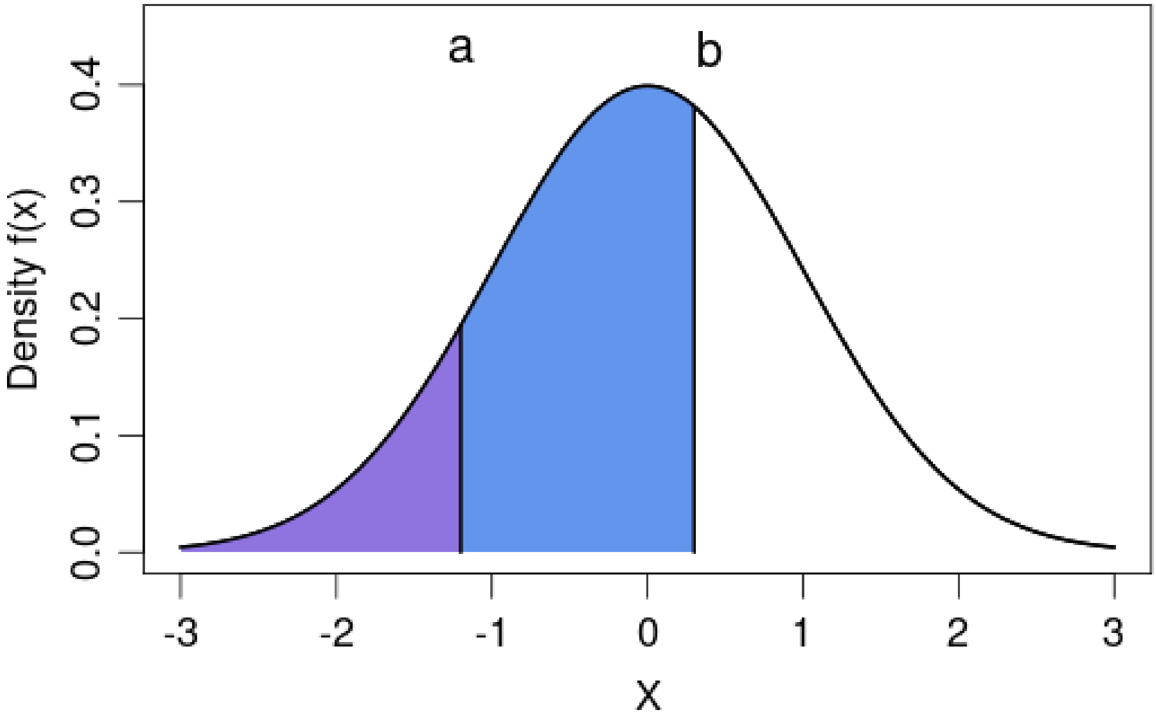
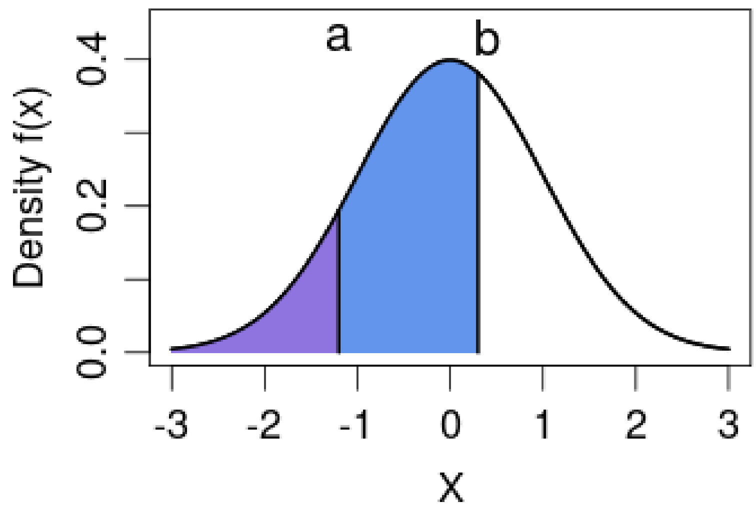


$$Pr(a < x \leq b) = F(b) - F(a)$$

Using CDF and pdf



$$Pr(a < x \leq b) = F(b) - F(a)$$

```
pnorm(0.3, 0, 1) - pnorm(-1.2, 0, 1)
```

```
## [1] 0.5028418
```

Quantile function

Definition *Quantile function*: The pth quantile of a distribution is the value q_p such that

$$Pr(X \leq q_p) = p.$$

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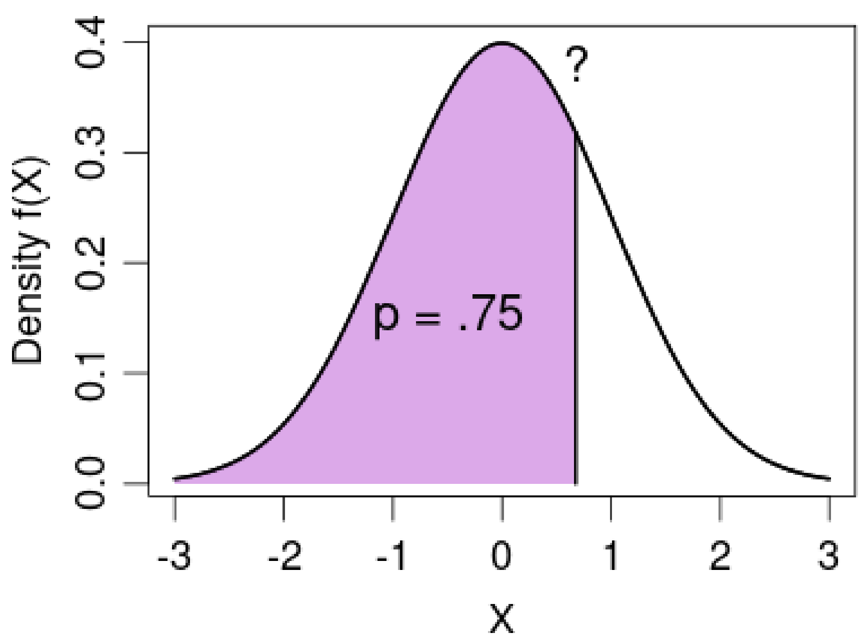
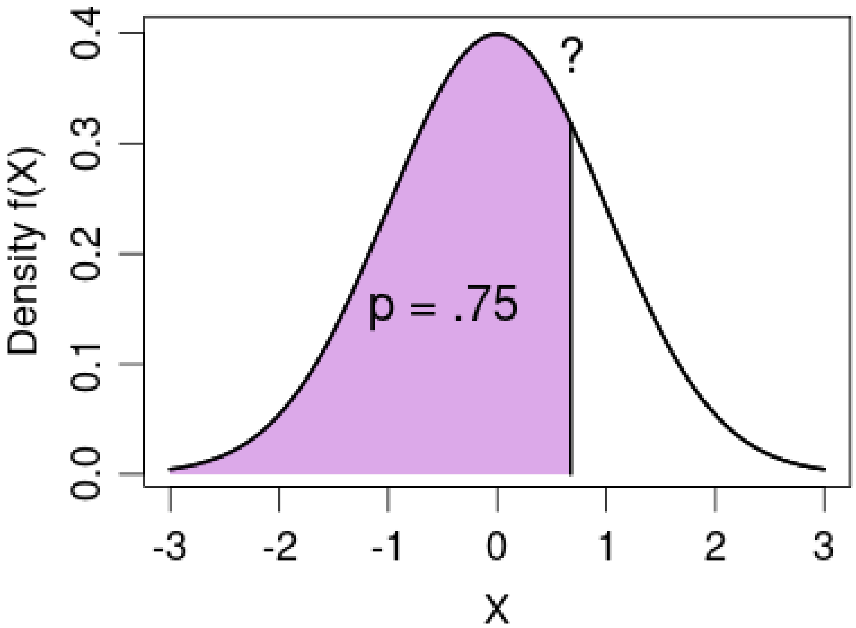
Reverse of the CDF



Quantile function




```
qnorm(.75, mean = 0, sd = 1)
```

```
## [1] 0.6744898
```

Signal Detection Theory model

SDT model for SD experiment

- General idea: Perception strength S varies gradually.
- On average, perceptual strength is higher when the stimulus is present compared to absent.

Signal Detection Theory model

SDT model for SD experiment

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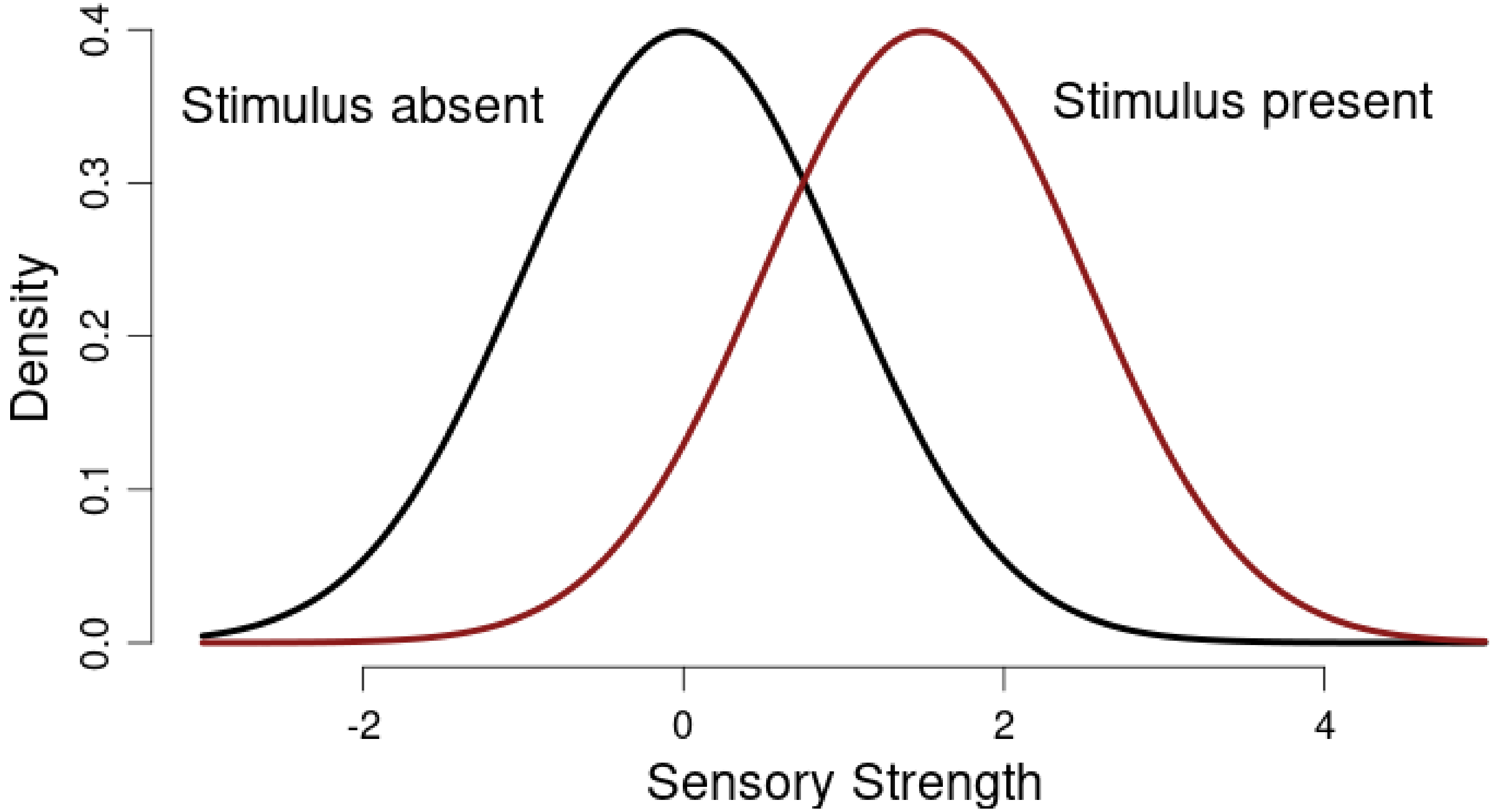
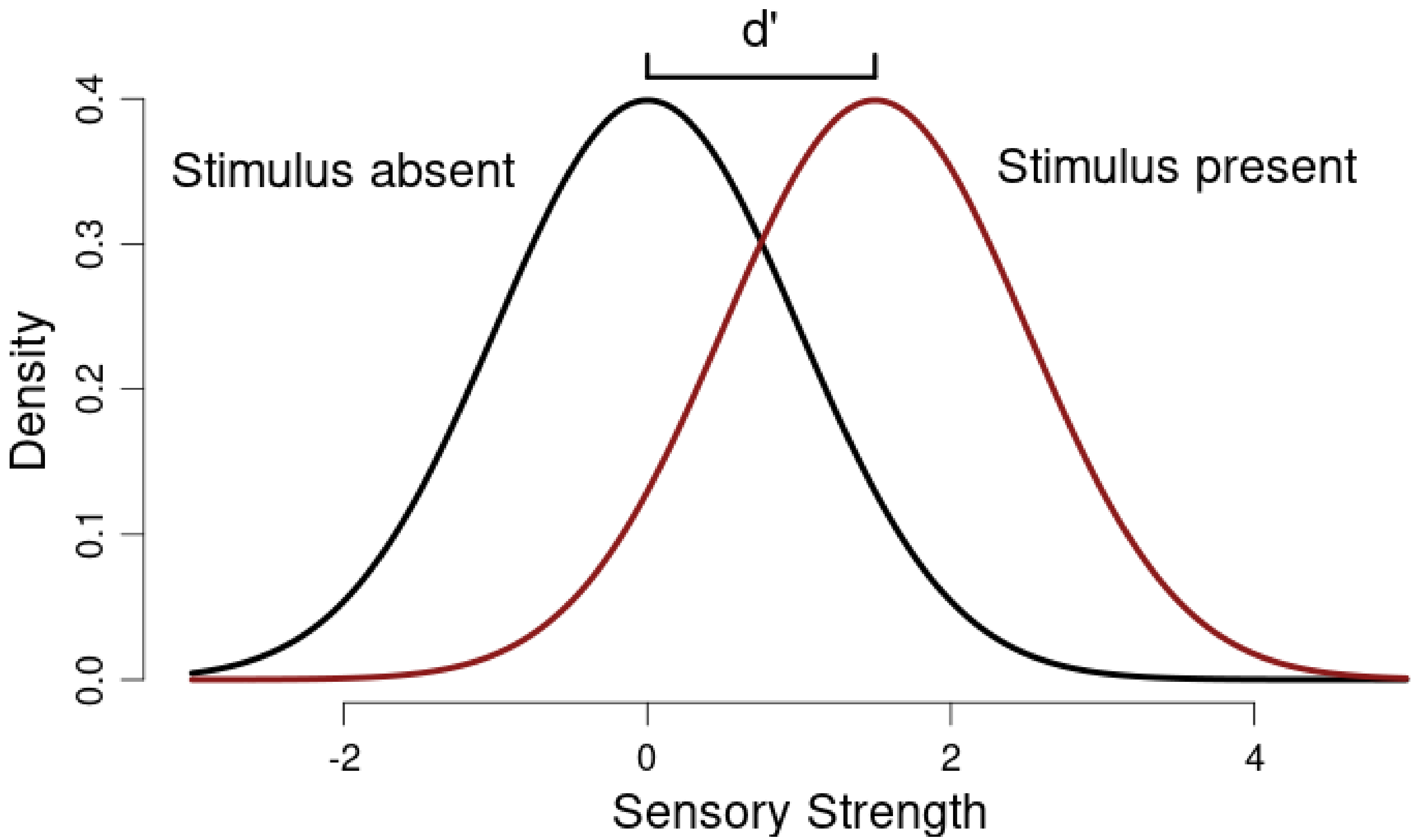
$$S \sim \begin{cases} \text{Normal}(\mu = d', \sigma^2 = 1), & \text{for signal-present trials,} \\ \text{Normal}(\mu = 0, \sigma^2 = 1), & \text{for signal-absent trials.} \end{cases}$$

SDT model



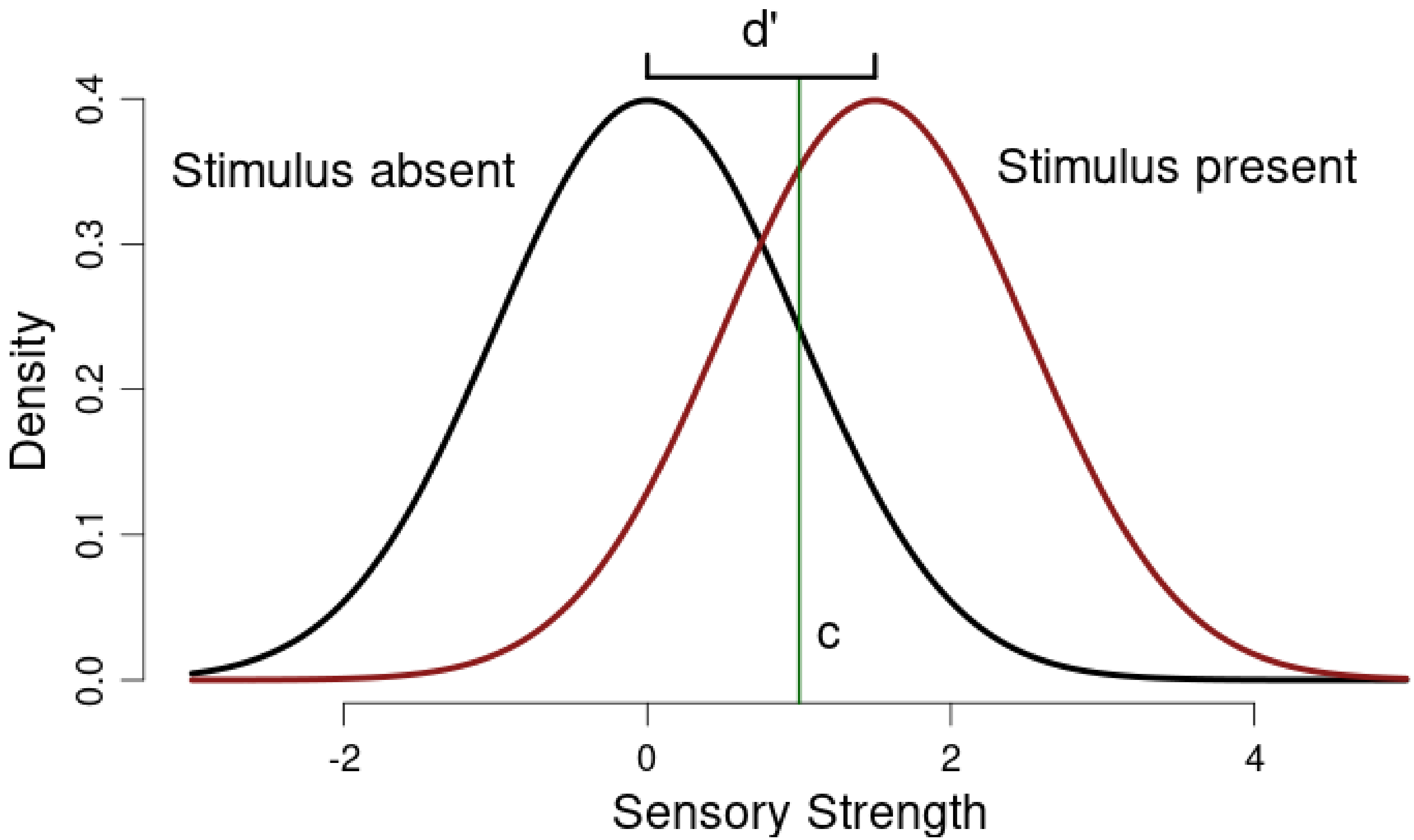
Stimulus absent
Stimulus present
Density
0.0
0.1
0.2
0.3
0.4
-2
0
2
4
Sensory Strength

SDT model



d' = Sensitivity.

SDT model



$c =$ Criterion, determines the response made.

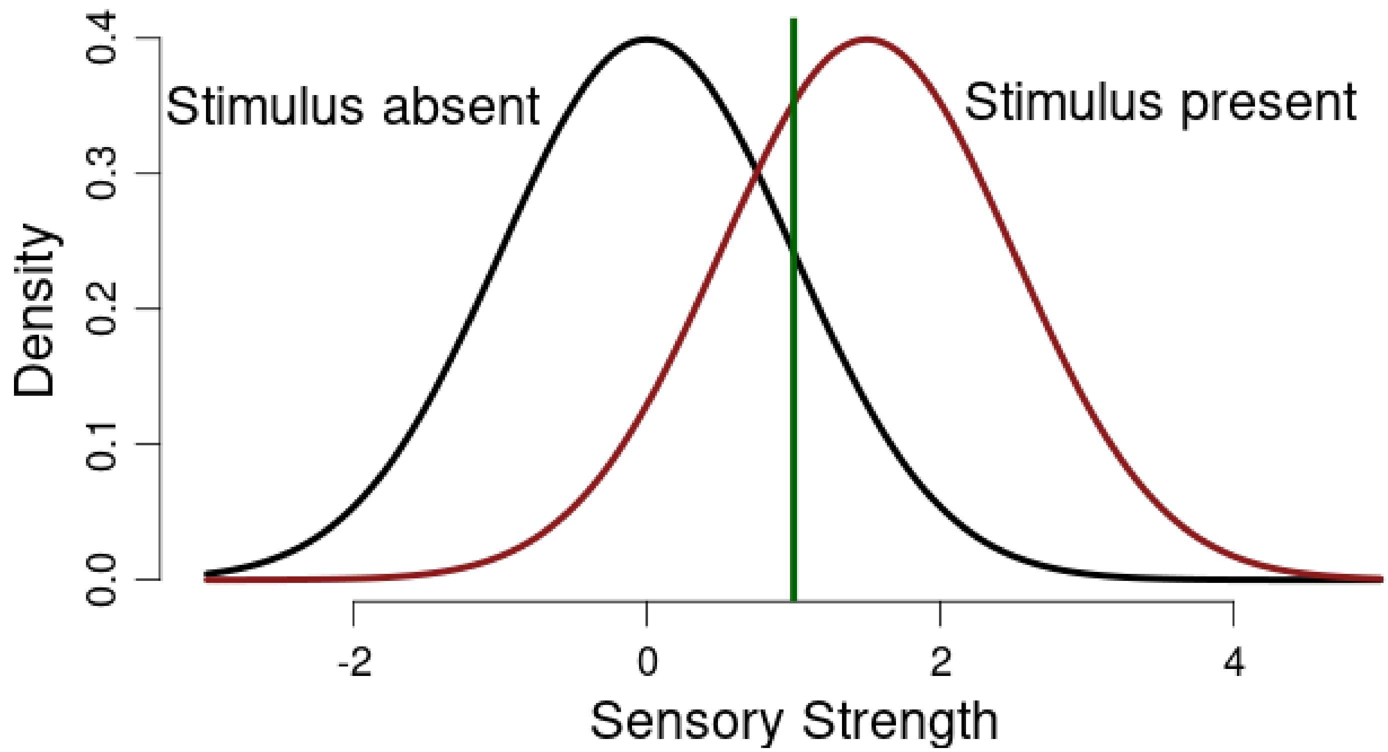
SDT model compared to HTM

One parameter for perception ability, one for response bias:

- d' corresponds to d.
- c corresponds to g.

SDT model predictions for H, M, F, C

What corresponds to the probability of hit?



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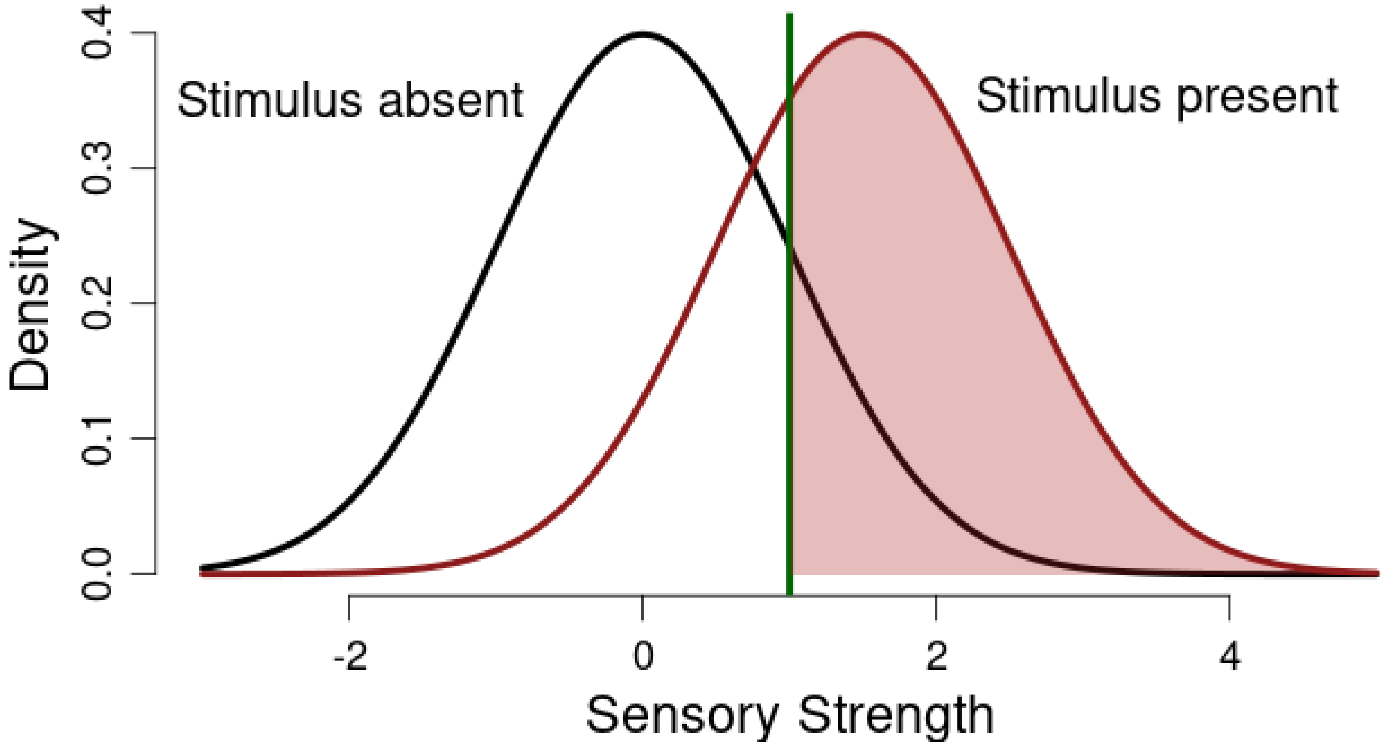
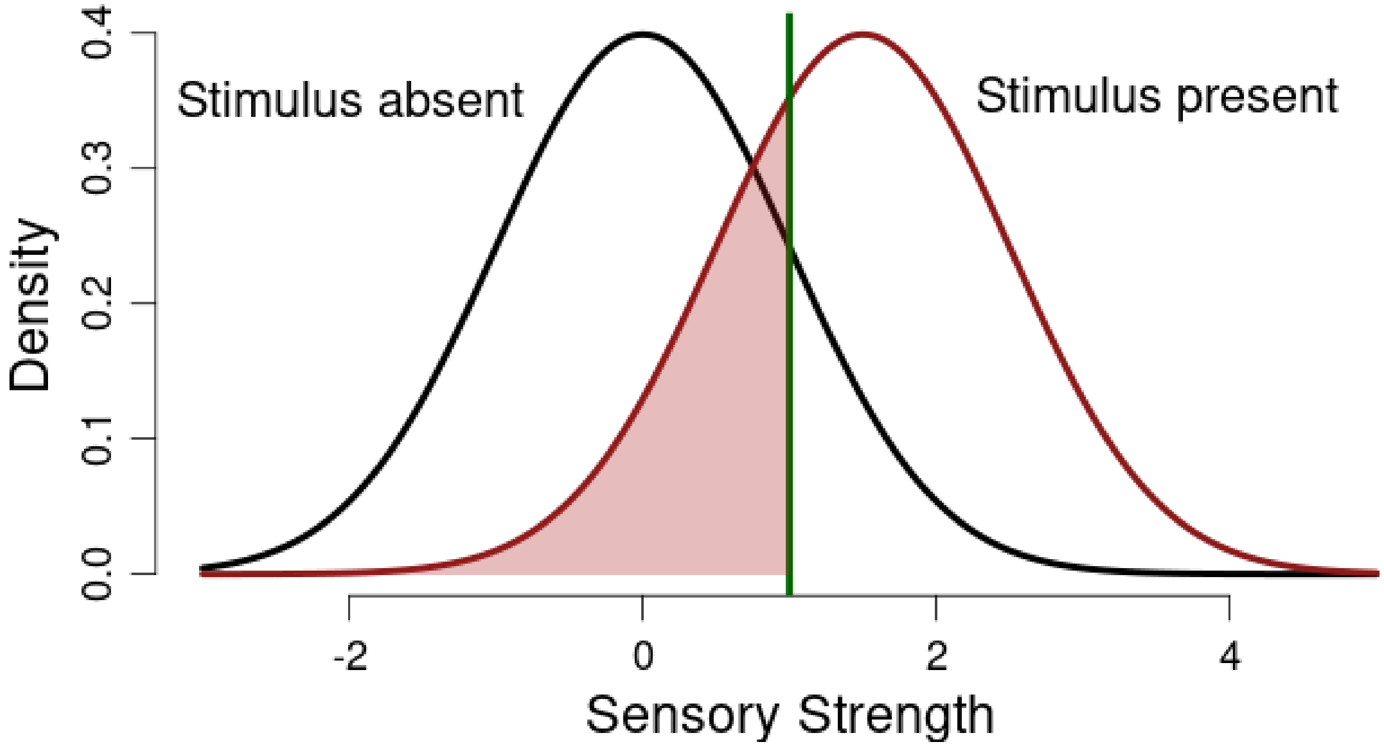


Area under the curve!

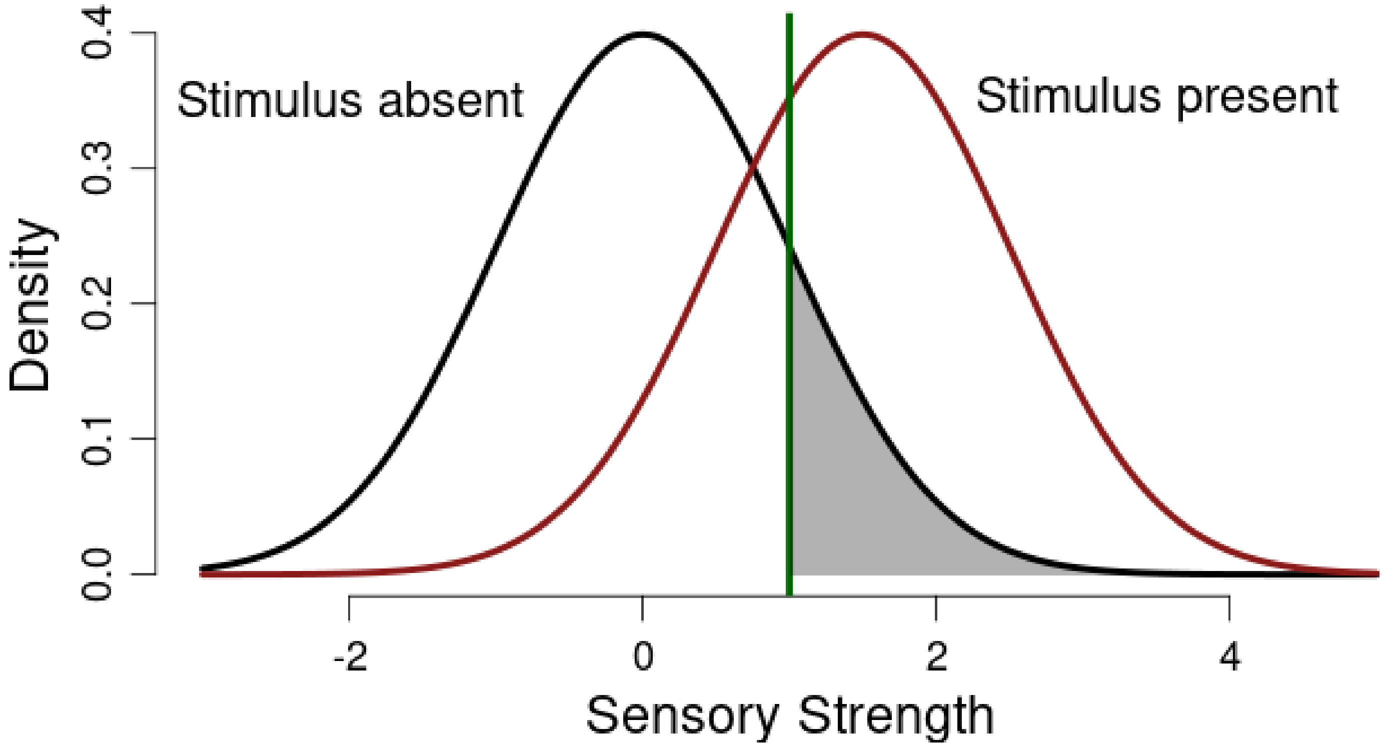
SDT model predictions for H, M, F, C

What corresponds to the probability of miss?



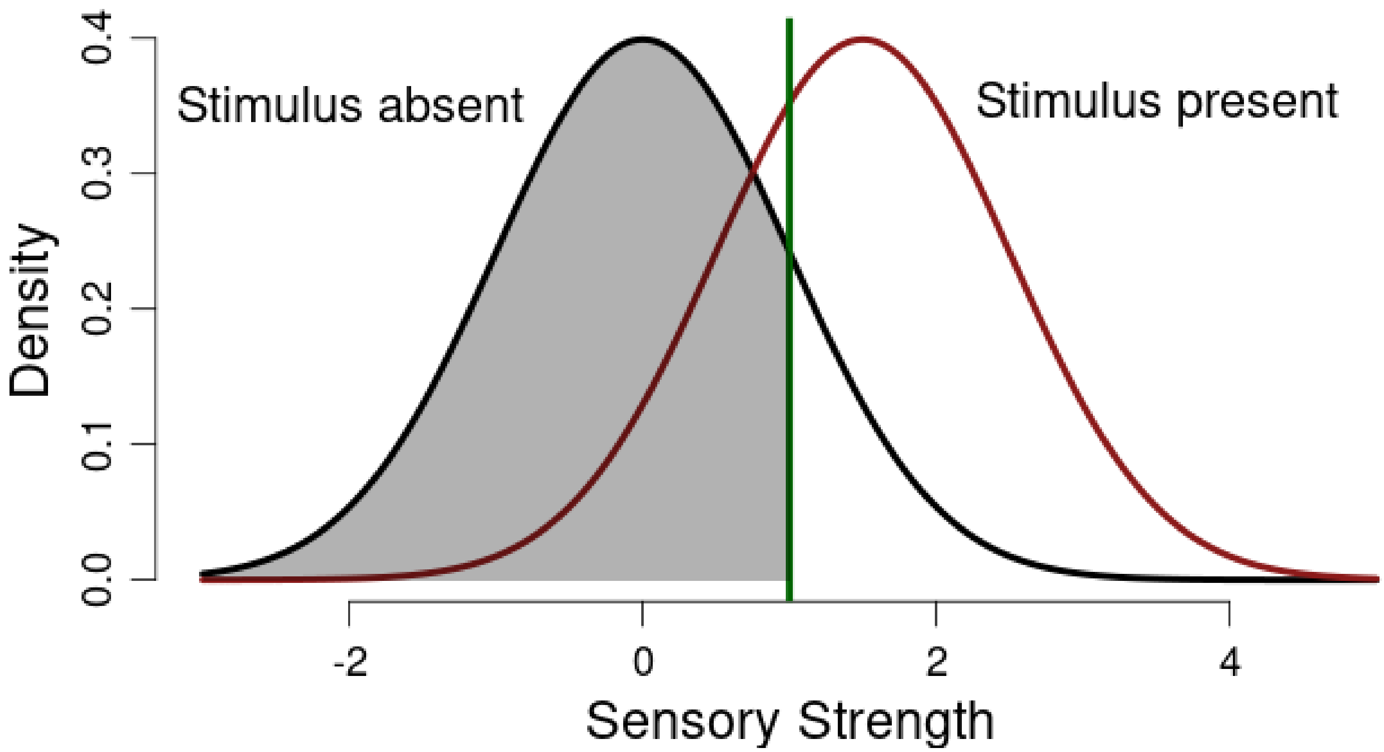
SDT model predictions for H, M, F, C

What corresponds to the probability of false alarm?

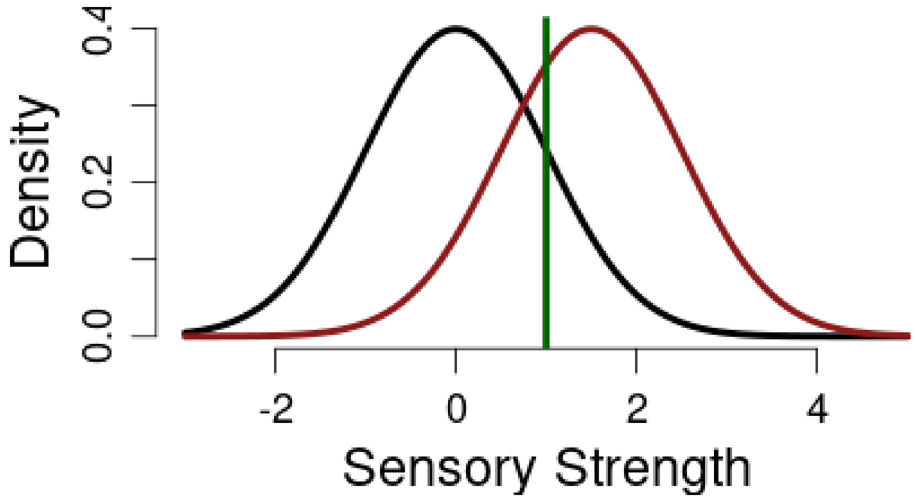


SDT model predictions for H, M, F, C

What corresponds to the probability of correct rejection?



SDT model equations



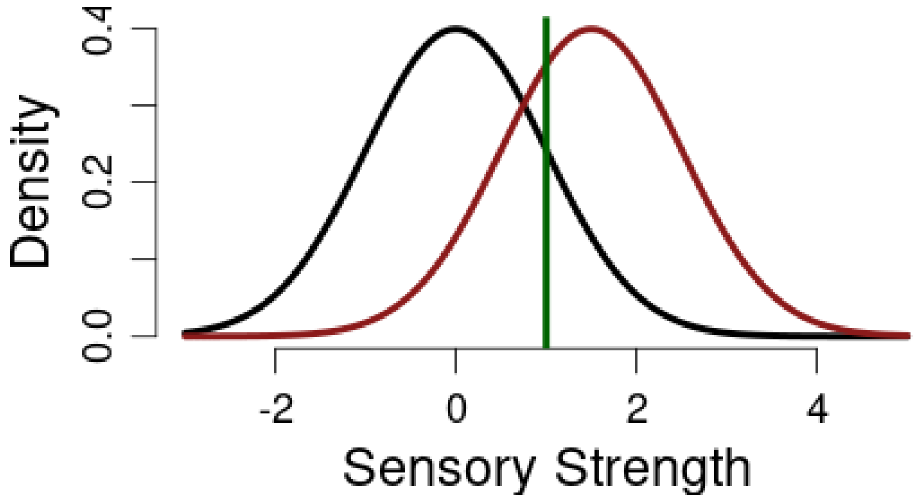

```
c <- 0.5; dprime = 1

pnorm(c, mean = dprime, sd = 1)
```

```
## [1] 0.3085375
```

```
pnorm(c - dprime, mean = 0, sd = 1)
```

```
## [1] 0.3085375
```

SDT model equations



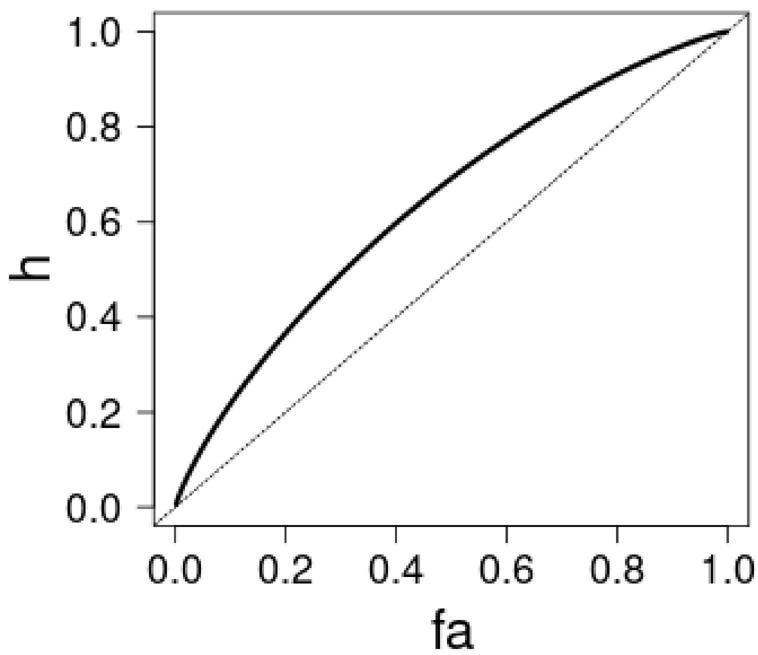
Equations for hits, p_1, and false alarms, p_2:

$$p_1 = p(\text{"Signal"} \mid \text{Signal}) = 1 - \Phi(c - d'),$$
$$p_2 = p(\text{"Signal"} \mid \text{Noise}) = 1 - \Phi(c)$$

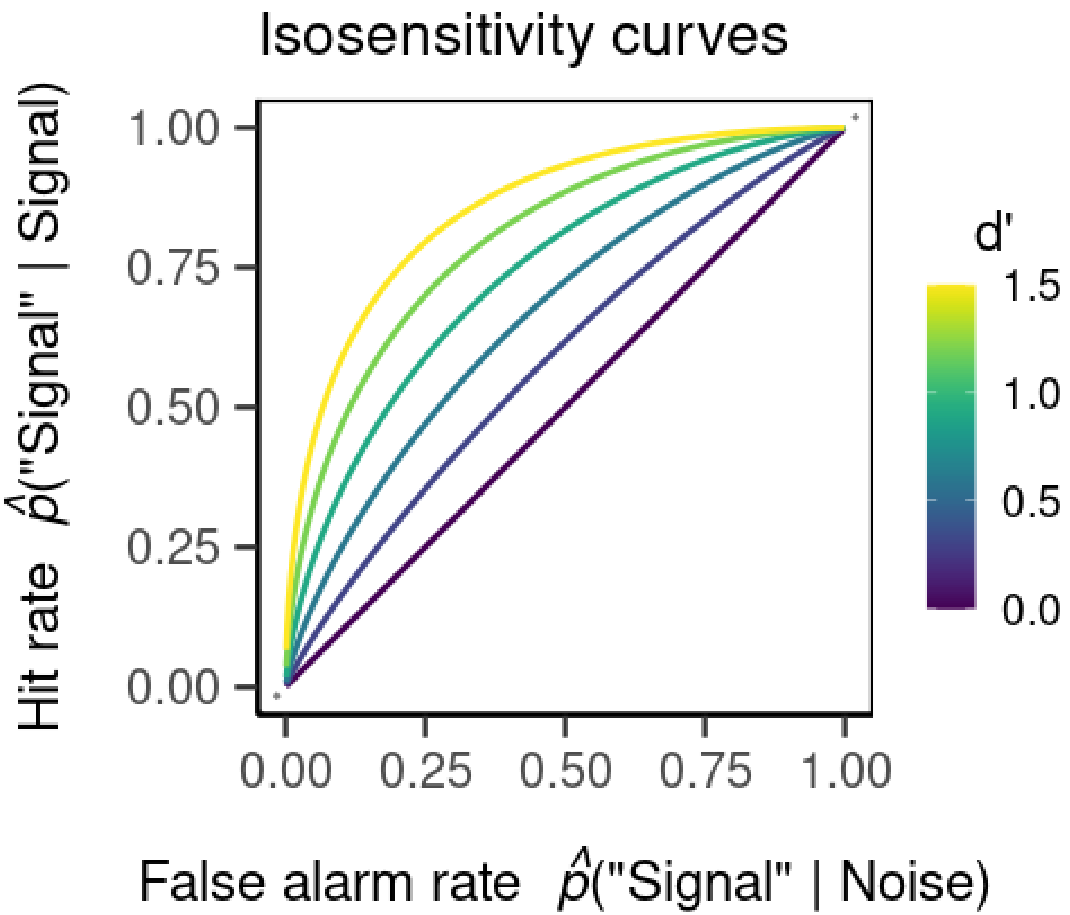
Φ = CDF of a standard normal.

SDT model predictions

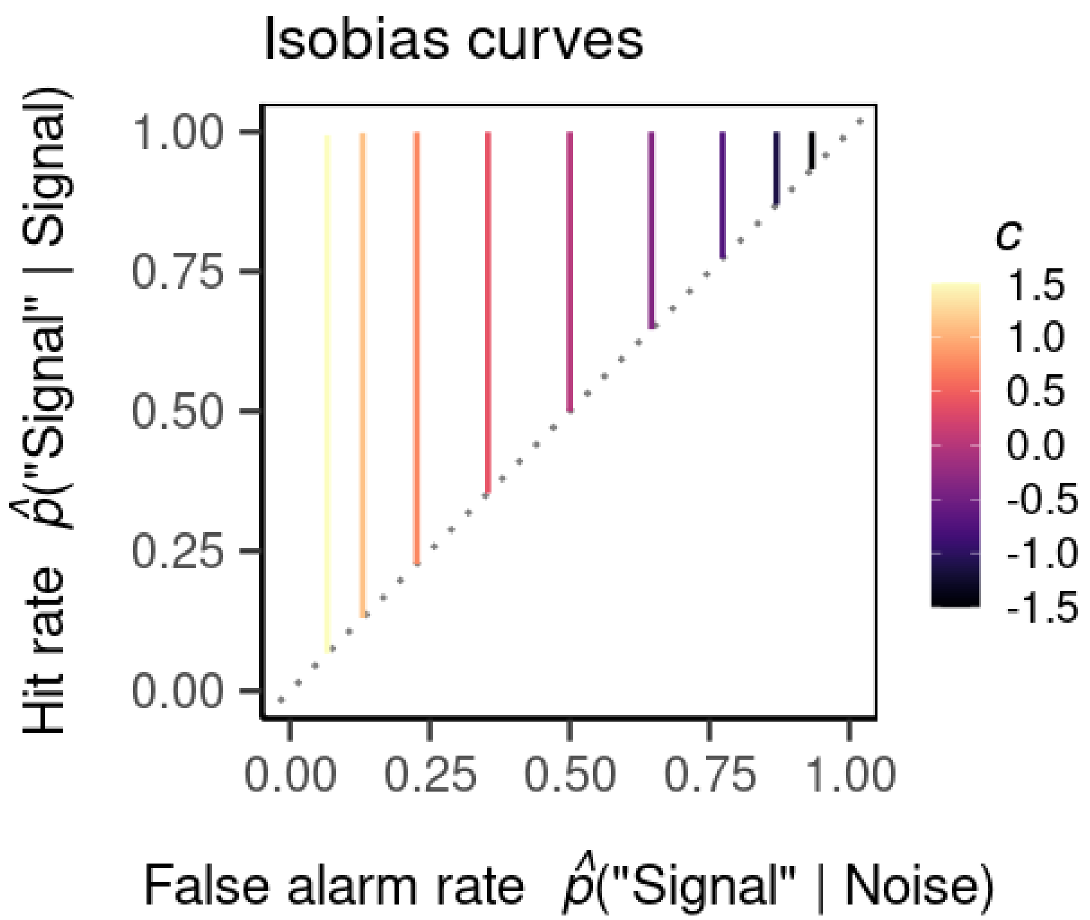
```r
# Simulate predictions
dprime <- 0.5
crit <- seq(-3, 3, .01)
fa <- 1 - pnorm(crit)
h <- 1 - pnorm(crit - dprime)

# Plot predictions
par(mar = c(5, 4.5, 4, 2) + 0.1
plot(
  fa, h, type = "l", las = 1,
  ylim = c(0, 1), lwd = 3,
  cex.lab = 2, cex.axis = 1.5
)
abline(0, 1, lty = "22")
```



Receiving Operator Characteristics (ROC) for SDT



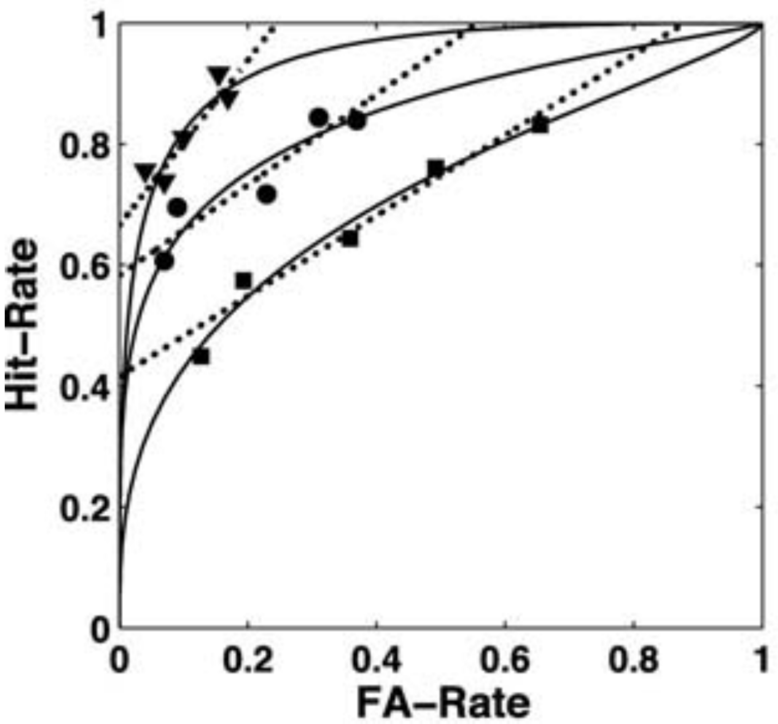
ROC for SDT



Isobias curves
Hit rate $\hat{p}$("Signal" | Signal)
1.00
0.75
0.50
0.25
0.00
0.00
0.25
0.50
0.75
1.00
False alarm rate $\hat{p}$("Signal" | Noise)
c
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5

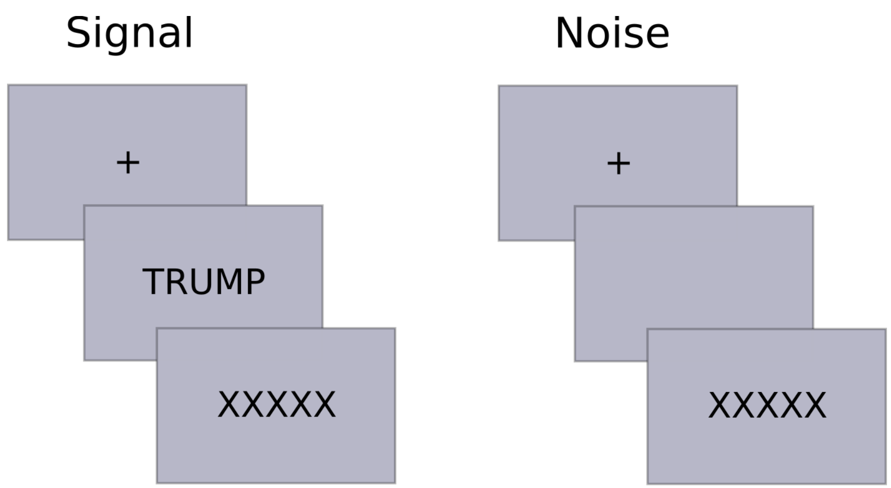
Predictions from SDT model

- Oftentimes good fit to data.
- SDT model is very popular because it is both flexible (ROC curves) and simple (Number of parameters).



(Bröder & Schütz, 2009)

Signal Detection model in R



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Noise	30 (False Alarms)	20 (Correct Rejections)	50
Total	105	45	

```r
y <- c("hit" = 75, "fa" = 30)
n <- c("signal" = 100, "noise" = 50)
```

Signal Detection model in R

```
#Log likelihood for signal detection
sdtm <- function(par, y, n) { #par = c(d', c) y = c(hit, fa)
   p1 <- 1 - pnorm(par["c"] - par["d"])
   p2 <- 1 - pnorm(par["c"])
  -2 * (dbinom(y["hit"], n["signal"], p1, log = TRUE) +
    dbinom(y["fa"], n["noise"], p2, log = TRUE))
}
# Model fitting
out <- optim(par = c(d = 0, c = 0), fn = sdtm, y = y, n = n)
out$par
```

```
##          d          c
##  0.4211693 -0.2533812
```

Signal Detection model for >1 conditions

- As with high-threshold models we often want to estimate the model for data with several conditions.
- For example, in our signal detection experiment we might have noise trials, trials where the signal is presented for 12ms and trials where the signal is presented for 20ms.

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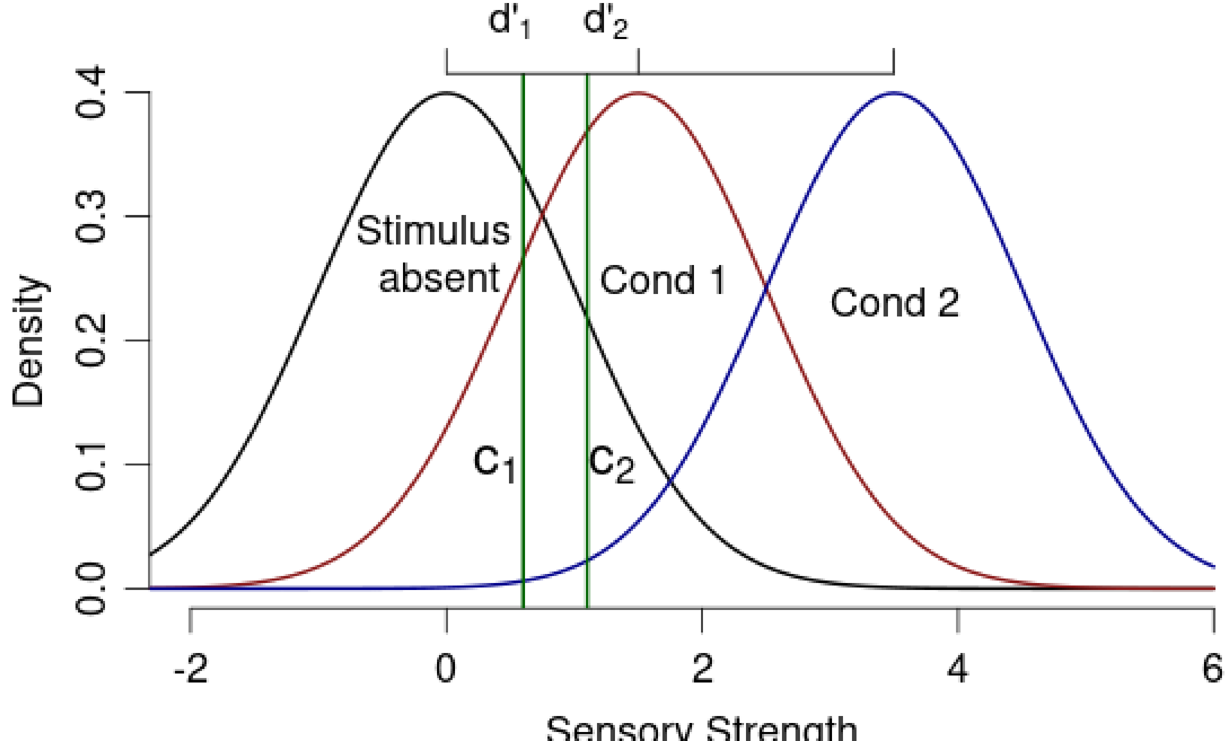

Null counts

- Null counts in the data are a big problem for SDT models.
- When there are no misses, the hit-rate is $\hat{p}_h = 1.0$.
- When $\hat{p}_h = 1.0$ then $\Phi^-1(\hat{p}_h) = \infty$ leading to an estimate of $d' = \infty$.

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Correction for null counts

$$\hat{p}_h = \frac{y_h + 0.5}{N_s + 1}$$

$$\hat{p}_f = \frac{y_f + 0.5}{N_n + 1}$$

Snodgrass and Corwin (1988)

Wrap up

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Signal Detection models imply that

- perceptual strength (or memory strength) is graded/continuous.
- perception is not all-or-none.

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Signal Detection models imply that

- perceptual strength (or memory strength) is graded/continuous.
- perception is not all-or-none.

Signal Detection models

- are flexible and simple.
- can be extended to be even more flexible.

:)