Cognitive Modeling Part I

Bayesian Modeling in brms

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A brief introduction to cognitive modeling

1. What kind of models are we talking about?

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2. Signal detection

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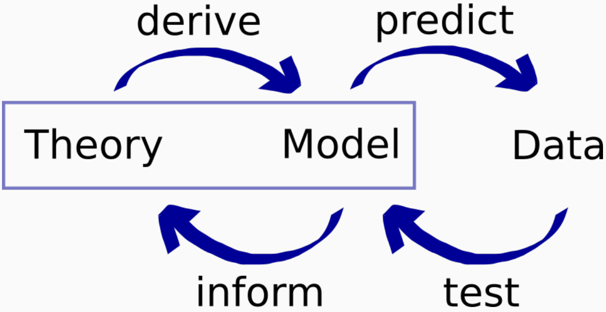
2. Signal detection
3. Application to perceptual decision making experiment

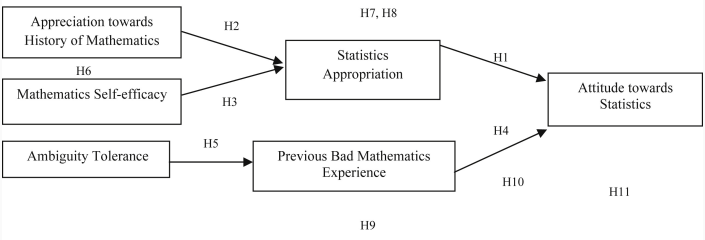
Theory, models, and data



Verbal vs. mathematical models

There are many things that people call models.

E.g. Prayoga, T., & Abraham, J. (2017). A psychological model explaining why we love or hate statistics.



Verbal vs. mathematical models

"A mathematical model or theory is a set of mathematical structures, including a set of *linkage statements*" van Zandt & Townsend (2012).

- Behavioral variables are related to components of psychological processes using equations.

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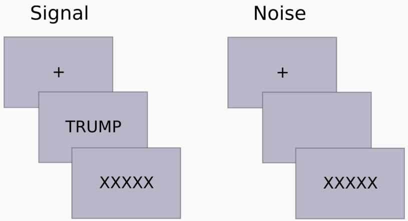
- Behavioral variables are related to components of psychological processes using equations.
- Psychological processes are expressed as parameters and functions.
- Behavior needs to be quantifiable (e.g. accuracy, response time).

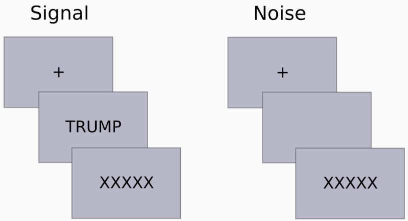
Signal Detection Models

Signal detection experiment



Stimulus	Present response	Absent Response	Total
Signal	75	25	100
Noise	30	20	50
Total	105	45	

Signal detection experiment



Stimulus	Present response	Absent Response	Total
Signal	75 (Hits)	25 (Misses)	100
Noise	30 (False Alarms)	20 (Correct Rejections)	50
Total	105	45	

Signal Detection Models

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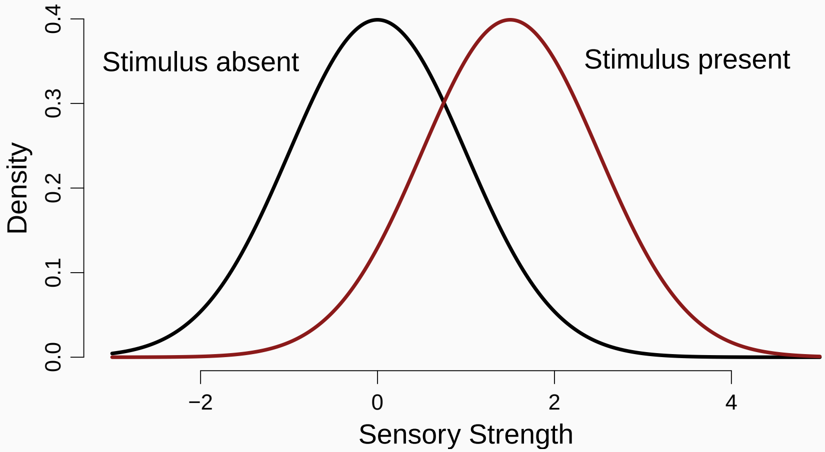
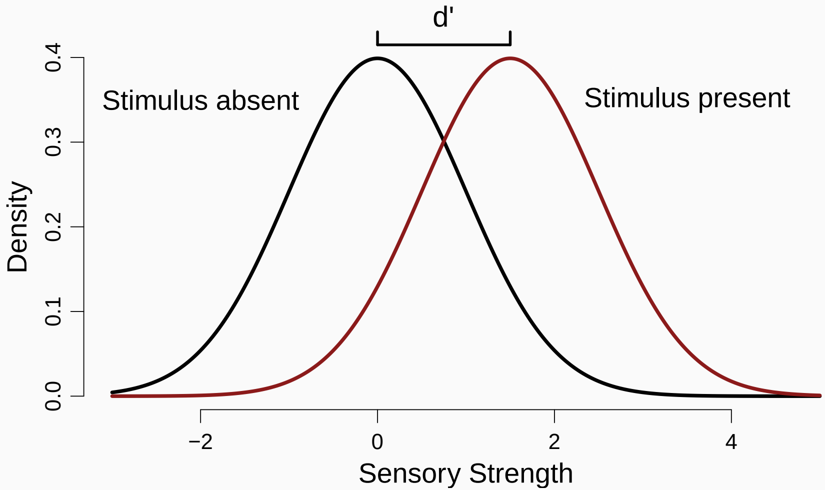
- $$S \sim \begin{cases} \text{Normal}(\mu = d', \sigma^2 = 1), & \text{for signal-present trials,} \\ \text{Normal}(\mu = 0, \sigma^2 = 1), & \text{for signal-absent trials.} \end{cases}$$

Signal Detection Model

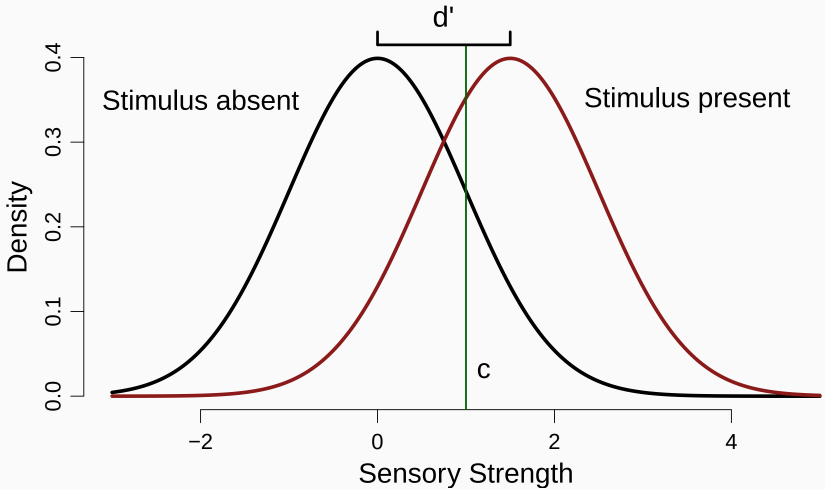


SDT model



$d' =$ Sensitivity.

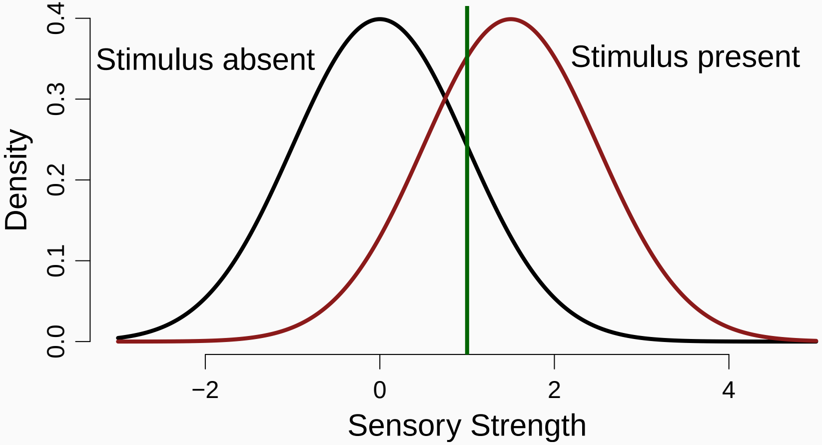
SDT model



c = Criterion, determines the response made.

SDT model predictions for H, M, F, C

What corresponds to the probability of hit?



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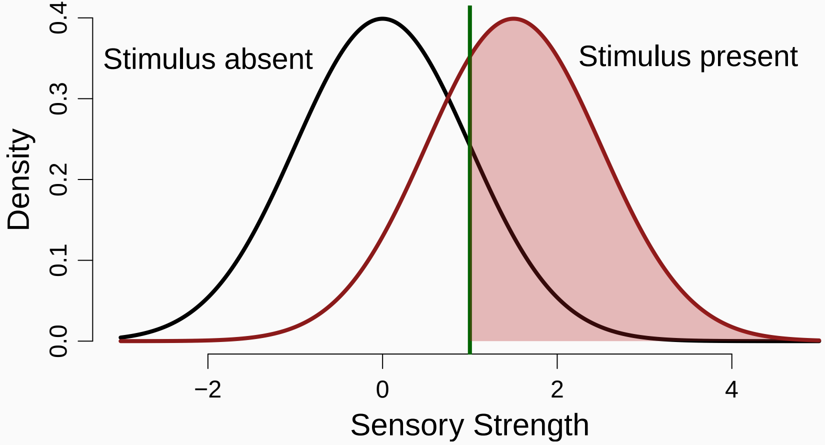
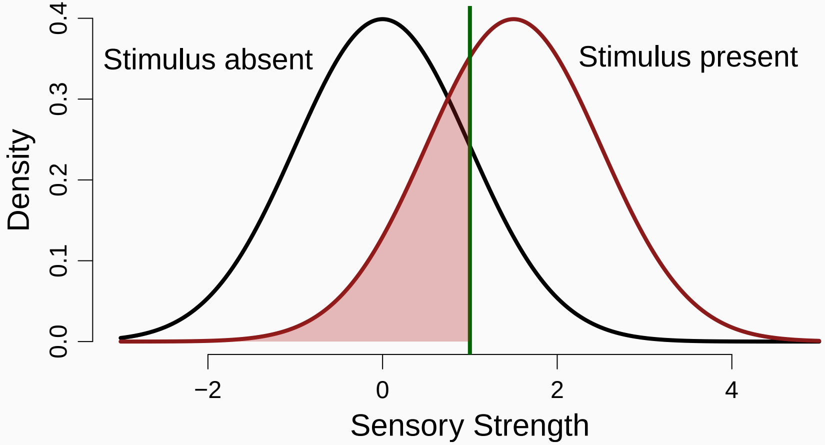


Area under the curve!

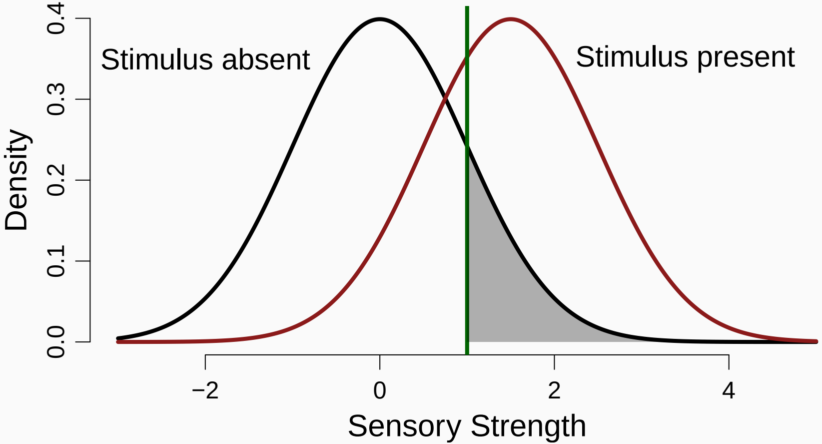
SDT model predictions for H, M, F, C

What corresponds to the probability of miss?



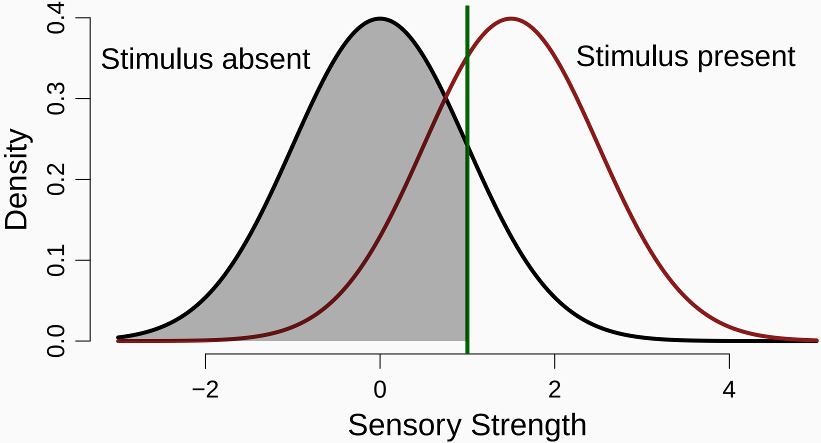
SDT model predictions for H, M, F, C

What corresponds to the probability of false alarm?



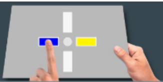
SDT model predictions for H, M, F, C

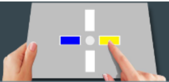
What corresponds to the probability of correct rejection?



Application to perceptual decision making experiment

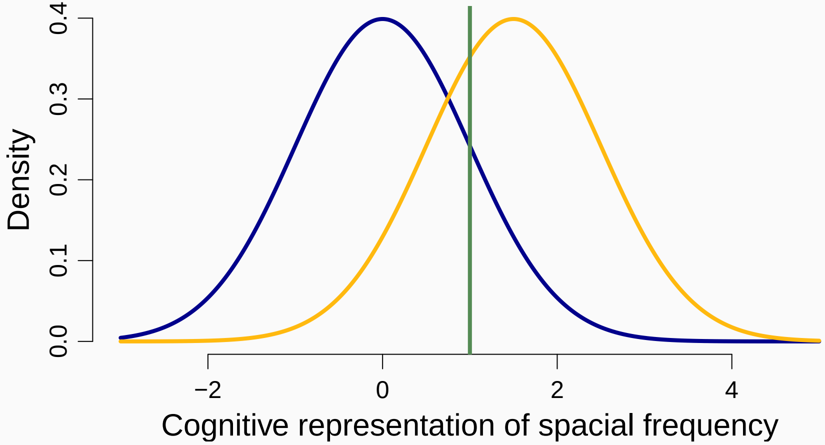
Collapsing across participants



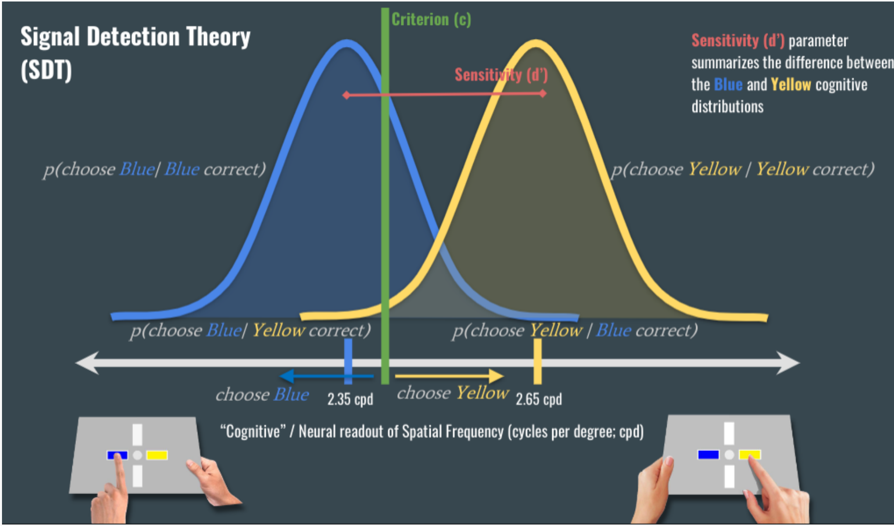


The Confusion Matrix	Chose Blue	Chose Yellow
Blue (2.35 cpd) was correct	81.9%	18.1%
Yellow (2.65 cpd) was correct	11.5%	88.5%

Application to perceptual decision making experiment

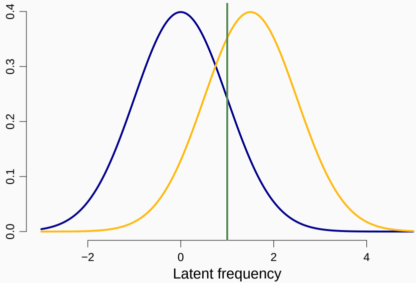


Application to perceptual decision making experiment



Signal Detection Model in brms

- Data are a coin flip and we model the probability: $Y_i \sim \text{Bernoulli}(p_i)$.



Signal Detection Model in brms

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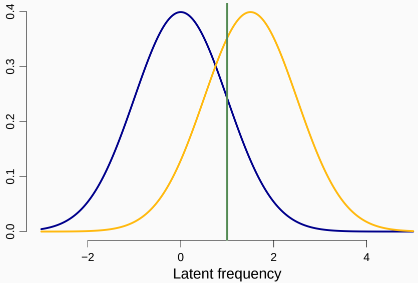
- Probabilities are transformed to the continuous latent space: $p_i = \Phi(\mu_i)$.



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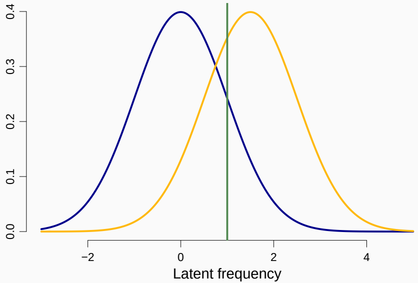
- In that space, we can use a linear model just as before:
  $\mu_i = \beta_0 + \beta_1 \text{spf}_i$,



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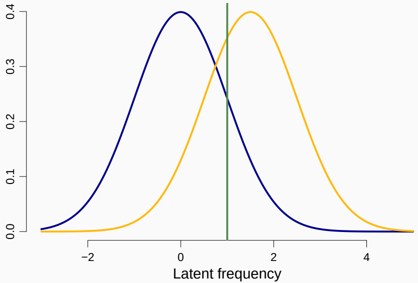

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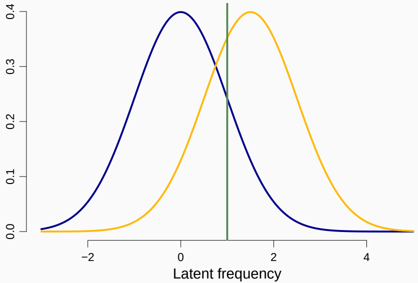
- and β_1, the slope, translates to d'.



Signal Detection Model in brms

$$Y_i \sim \text{Bernoulli}(p_i),$$

$$p_i = \Phi(\mu_i),$$

$$\mu_i = \beta_0 + \beta_1 \text{spf}_i.$$

```
fit1 <- brm(response ~ 1 + factor(spf),
            family = bernoulli(link="probit"),
            data = pdm[pdm$subject==1,])
```

Signal Detection Model in brms

Computing *responses* using accuracy and presented spacial frequency:

```
pdm$spfn <- 1 - (as.numeric(as.factor(pdm$spf)) - 1)
pdm$response <- ifelse(pdm$accuracy==1
                              , pdm$spfn
                              , 1 - pdm$spfn)
```

Signal Detection Model in brms

```
summary(fit1)
```

```
##  Family: bernoulli
##   Links: mu = probit
## Formula: response ~ 1 + factor(spf)
##    Data: pdm[pdm$subject == 1, ] (Number of observations: 562)
##   Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##          total post-warmup draws = 4000
##
## Population-Level Effects:
##              Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept       -0.11      0.08    -0.27     0.05 1.00     3757     2501
## factorspflow    -1.14      0.13    -1.37    -0.89 1.00     2495     2218
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Questions?