Assignment 5 - Solutions

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Exercise 1

```r
library(tidyverse)
library(brms)
```

```r
load("df_pupil_complete.rda")
df_pupil_complete <- df_pupil_complete %>%
  mutate(c_load = load - mean(load))
```

Fit a "maximal" model (correlated varying intercept and slopes for subjects) assuming a normal likelihood.

```r
fit_pupil <- brm(p_size ~ 1 + c_load + (c_load | subj),
  data = df_pupil_complete,
  family = gaussian(),
  prior = c(
    prior(normal(1000, 500), class = Intercept),
    prior(normal(0, 1000), class = sigma),
    prior(normal(0, 100), class = b, coef = c_load),
    prior(normal(0, 1000), class = sd),
    prior(lkj(2), class = cor)),
  control=list(adapt_delta=0.99, max_treedepth=15))
```

(a) Examine the effect of load on pupil size, and the average pupil size.

```r
fit_pupil
```

```
##  Family: gaussian
##   Links: mu = identity; sigma = identity
## Formula: p_size ~ 1 + c_load + (c_load | subj)
##    Data: df_pupil_complete (Number of observations: 2228)
##   Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##          total post-warmup draws = 4000
##
## Multilevel Hyperparameters:
## ~subj (Number of levels: 20)
##                       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS
## sd(Intercept)          3300.74    430.45  2521.74  4213.11 1.01      715
## sd(c_load)               71.95     15.88    47.61   108.87 1.00     1120
## cor(Intercept,c_load)     0.31      0.24    -0.20     0.71 1.00     1183
##                       Tail_ESS
## sd(Intercept)              967
## sd(c_load)                1702
## cor(Intercept,c_load)     1741
##
```

```
## Regression Coefficients:
##           Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept  2452.86    493.23  1425.88  3396.37 1.01      655     1177
## c_load       38.89     25.55   -12.19    87.01 1.00     1012     1513
##
## Further Distributional Parameters:
##       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma   505.32      7.57   491.26   520.40 1.00     4433     3085
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

If we want to just report the change in pupil size as a function of one unit increase in (centered) load:

```r
posterior_summary(fit_pupil, variable = "b_c_load")
```

```
##          Estimate Est.Error      Q2.5    Q97.5
## b_c_load 38.88977  25.54953 -12.18553 87.01479
```

There isn't a clear effect of load across all the subject.

But the intercept seems to be quite large,

```r
posterior_summary(fit_pupil, variable = "b_Intercept")
```

```
##             Estimate Est.Error     Q2.5    Q97.5
## b_Intercept 2452.864  493.2283 1425.885 3396.366
```

and we assumed that it shouldn't be:

```r
brms::prior_summary(fit_pupil)
```

```
##                 prior     class      coef group resp dpar nlpar lb ub
##                (flat)         b
##        normal(0, 100)         b    c_load
##     normal(1000, 500) Intercept
##  lkj_corr_cholesky(2)         L
##  lkj_corr_cholesky(2)         L             subj
##       normal(0, 1000)        sd                                  0
##       normal(0, 1000)        sd             subj                 0
##       normal(0, 1000)        sd    c_load   subj                 0
##       normal(0, 1000)        sd Intercept   subj                 0
##       normal(0, 1000)     sigma                                  0
##        source
##       default
##          user
##          user
##          user
##  (vectorized)
##          user
##  (vectorized)
##  (vectorized)
##  (vectorized)
##          user
```

See the row with class Intercept.

So maybe our prior for the intercept was overly informative.

(b) Do a sensitivity analysis for the prior on the intercept (α). What is the estimate of the effect (β) under different priors?

We'll try a wider prior for α. A more complete sensitivity analysis would investigate several possible priors

$$\alpha \sim \mathit{Normal}(4000, 2000)$$

```r
fit_pupil_2 <- brm(p_size ~ 1 + c_load + (c_load | subj),
  data = df_pupil_complete,
  family = gaussian(),
  prior = c(
    prior(normal(4000, 2000), class = Intercept),
    prior(normal(0, 1000), class = sigma),
    prior(normal(0, 100), class = b, coef = c_load),
    prior(normal(0, 2000), class = sd),
  prior(lkj(2), class = cor))
)
```

```r
fit_pupil_2
```

```
##  Family: gaussian
##   Links: mu = identity; sigma = identity
## Formula: p_size ~ 1 + c_load + (c_load | subj)
##    Data: df_pupil_complete (Number of observations: 2228)
##   Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##          total post-warmup draws = 4000
##
## Multilevel Hyperparameters:
## ~subj (Number of levels: 20)
##                      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS
## sd(Intercept)         2555.57    410.15  1882.41  3503.52 1.00      739
## sd(c_load)              69.13     14.19    46.43   100.51 1.01     1128
## cor(Intercept,c_load)    0.27      0.21    -0.17     0.64 1.00     1822
##                      Tail_ESS
## sd(Intercept)            1369
## sd(c_load)               1898
## cor(Intercept,c_load)    2364
##
## Regression Coefficients:
##           Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept  5467.07    550.72  4339.09  6505.09 1.01      414      745
## c_load       59.07     17.05    24.44    91.84 1.00     1279     1982
##
## Further Distributional Parameters:
##       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma   505.53      7.80   490.60   520.87 1.00     6281     2679
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Now there seems to be a clear effect! Our bad prior for the intercept was messing up our inferences!

(c) Is the effect of load consistent across subjects?

```r
## For the hierarchical model, this is more complicated,
# because we want the effect (beta) + adjustment:
# we extract the overall group level effect:
beta <- c(as_draws_df(fit_pupil_2)$b_c_load)
# We extract the individual adjustments
ind_effects_v <- paste0("r_subj[", unique(df_pupil_complete$subj), ",c_load]")
adjustment <- as.matrix(as_draws_df(fit_pupil)[ind_effects_v])
# We get the by subject effects in a data frame where each adjustment
# is in each column.
by_subj_effect <- as_tibble(beta + adjustment)
# We summarize them by getting a table with the mean and the
# quantiles for each column and then binding them.
par_h <- lapply(by_subj_effect, function(x) {
  tibble(
    Estimate = mean(x),
    Q2.5 = quantile(x, .025),
    Q97.5 = quantile(x, .975)
  )
}) %>%
  bind_rows() %>%
  # We add a column to identify that the model,
  # and one with the subject labels:
  mutate(
    subj = unique(df_pupil_complete$subj))%>%
  arrange(Estimate) %>%
  mutate(subj = factor(subj, levels = subj))


ggplot(
  par_h,
  aes(
    ymin = Q2.5, ymax = Q97.5, x = subj, y = Estimate
  )
) +
  geom_errorbar() +
  geom_point() +
  # We'll also add the mean and 95% CrI of the overall difference
  # to the plot:
  geom_hline(
    yintercept =
      posterior_summary(fit_pupil_2)["b_c_load", "Estimate"]
  ) +
  geom_hline(
    yintercept =
      posterior_summary(fit_pupil_2)["b_c_load", "Q2.5"],
    linetype = "dotted", linewidth = 0.5
  ) +
  geom_hline(
    yintercept =
      posterior_summary(fit_pupil_2)["b_c_load", "Q97.5"],
    linetype = "dotted", linewidth = 0.5
  ) +
```

```
xlab("Change in pupil size") +
coord_flip()
```