Trabajo_Practico_3

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1 Trabajo Práctico 3

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8. Encontrar un perceptrón multicapa que resuelva una XOR de 2 entradas mediante simulated annealing. Graficar el error a lo largo del proceso de aprendizaje.

[4]:
```python
import numpy as np
import matplotlib.pyplot as plt


from IPython.core.magic import register_cell_magic

@register_cell_magic
def skip(line, cell):
    return
```

Para este ejercicio, se utiliza Simulated Annealing para resolver la XOR de 2 entradas, un problema no lineal, que no puede ser resuelto por un perceptrón simple. El aprendizaje a través de simulated annealing es un método de optimización que simula el proceso de enfriamiento de un material en un sistema físico. En este caso, se utiliza para encontrar los pesos que minimizan el error cuadrático medio de la red neuronal. Lo que sucede es que cuando se disminuye la temperatura determina la probabilidad de aceptar una solución peor que la actual a través de una distribución Boltzmann, lo que permite explorar el espacio de soluciones de manera más eficiente. En teoría, el algoritmo converge a la solución óptima, pero en la práctica, se puede quedar atascado en un mínimo local.

[183]:
```python
class MLP:
    def __init__(self, input_size, hidden_size, output_size):
        # Initialize weights and biases
        self.weights_input_hidden = np.random.randn(input_size, hidden_size)
        self.weights_hidden_output = np.random.randn(hidden_size, output_size)
        self.bias_hidden = np.zeros(hidden_size)
        self.bias_output = np.zeros(output_size)

    def forward(self, X):
```

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        hidden = np.dot(X, self.weights_input_hidden) + self.bias_hidden
        hidden_activation = self.activate(hidden)
        output = np.dot(hidden_activation, self.weights_hidden_output) + self.
↪bias_output
        return self.activate(output)

    def activate(self, x):
        return 1 / (1 + np.exp(-x))

    def compute_loss(self, y_true, y_pred):
        return np.mean((y_true - y_pred) ** 2)

    def simulated_annealing(self, X, y, initial_temp=20000, final_temp=0.1,␣
↪alpha=0.995, max_iter=1000, best=True):
        current_temp = initial_temp
        current_weights_input_hidden = self.weights_input_hidden.copy()
        current_weights_hidden_output = self.weights_hidden_output.copy()
        current_bias_hidden = self.bias_hidden.copy()
        current_bias_output = self.bias_output.copy()
        best_loss = self.compute_loss(y, self.forward(X))
        best_weights_input_hidden = current_weights_input_hidden.copy()
        best_weights_hidden_output = current_weights_hidden_output.copy()
        best_bias_hidden = current_bias_hidden.copy()
        best_bias_output = current_bias_output.copy()
        errors = []
        best_errors = []

        iteration = 0
        while current_temp > final_temp:
            iteration += 1
            for _ in range(max_iter):
                step_size = 1e-1
                new_weights_input_hidden = current_weights_input_hidden + np.
↪random.normal(0, step_size, current_weights_input_hidden.shape)
                new_weights_hidden_output = current_weights_hidden_output + np.
↪random.normal(0, step_size, current_weights_hidden_output.shape)
                new_bias_hidden = current_bias_hidden + np.random.normal(0,␣
↪step_size, current_bias_hidden.shape)
                new_bias_output = current_bias_output + np.random.normal(0,␣
↪step_size, current_bias_output.shape)

                self.weights_input_hidden = new_weights_input_hidden
                self.weights_hidden_output = new_weights_hidden_output
                self.bias_hidden = new_bias_hidden
                self.bias_output = new_bias_output
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            y_pred = self.forward(X)
            new_loss = self.compute_loss(y, y_pred)
            errors.append(new_loss)

            delta_loss = new_loss - best_loss
            acceptance_probability = np.exp(delta_loss / current_temp) if␣
↪delta_loss > 0 else 1

            if acceptance_probability > np.random.rand():
                current_weights_input_hidden = new_weights_input_hidden
                current_weights_hidden_output = new_weights_hidden_output
                current_bias_hidden = new_bias_hidden
                current_bias_output = new_bias_output
                if new_loss < best_loss:
                    best_loss = new_loss
                    best_weights_input_hidden = new_weights_input_hidden.
↪copy()
                    best_weights_hidden_output = new_weights_hidden_output.
↪copy()
                    best_bias_hidden = new_bias_hidden.copy()
                    best_bias_output = new_bias_output.copy()

            best_errors.append(best_loss)
        current_temp *= alpha

        if iteration % 10 == 0:
            print(f"Iteration {iteration}, Temperature {current_temp:.2f},␣
↪Best Loss {best_loss:.6f}, Current Loss {new_loss:.6f}")

    if best:
        self.weights_input_hidden = best_weights_input_hidden
        self.weights_hidden_output = best_weights_hidden_output
        self.bias_hidden = best_bias_hidden
        self.bias_output = best_bias_output
    else:
        self.weights_input_hidden = current_weights_input_hidden
        self.weights_hidden_output = current_weights_hidden_output
        self.bias_hidden = current_bias_hidden
        self.bias_output = current_bias_output

    return best_errors, errors
```

Se propone un algoritmo de simulated annealing para resolver la XOR de 2 entradas. Se utiliza una red neuronal con 2 entradas, 2 neuronas en la capa oculta y 1 neurona en la capa de salida. Se inicializan los pesos aleatoriamente y se calcula el error cuadrático medio. Luego, se actualizan los pesos de manera aleatoria y se calcula el nuevo error cuadrático medio. Si el nuevo error es menor que el anterior, se acepta la solución. Si el nuevo error es mayor que el anterior, se acepta la solución con una probabilidad determinada por la temperatura y la diferencia de error. Se repite este proceso hasta que se alcance un número máximo de iteraciones o se alcance un error mínimo. Además se ha propuesto un criterio de conservar los pesos sinápticos más óptimos de la corrida tal que se pueda utilizar la configuración más óptima obtenida.

[ ]:
```
X = np.array([
    [0, 0],
    [0, 1],
    [1, 0],
    [1, 1]
])
y = np.array([
    [0],
    [1],
    [1],
    [0]
])
mlp = MLP(input_size=2, hidden_size=6, output_size=1)

best_errors, errors = mlp.simulated_annealing(
    X, y,
    initial_temp=200,
    final_temp=1,
    alpha=0.999,
    best = True
)
plt.plot(errors)
plt.plot(best_errors)
plt.xlabel('Iteración')
plt.ylabel('Error')
plt.legend(['Error', 'Mejor Error'])
plt.title('Error vs Iteración')
plt.show()

outputs = mlp.forward(X)
print("Predicciones:")
for input_data, output, target in zip(X, outputs, y):
    print(f"Input: {input_data}, Predecido: {output[0]:.4f}, Esperado:␣
 ↪{target[0]}")
```

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Iteration 10, Temperature 198.01, Best Loss 0.068077, Current Loss 0.435235
Iteration 20, Temperature 196.04, Best Loss 0.068077, Current Loss 0.499555
Iteration 30, Temperature 194.09, Best Loss 0.068077, Current Loss 0.499954
Iteration 40, Temperature 192.15, Best Loss 0.068077, Current Loss 0.494845
Iteration 50, Temperature 190.24, Best Loss 0.068077, Current Loss 0.500000
Iteration 60, Temperature 188.35, Best Loss 0.068077, Current Loss 0.253198
Iteration 70, Temperature 186.47, Best Loss 0.068077, Current Loss 0.499878
Iteration 80, Temperature 184.62, Best Loss 0.068077, Current Loss 0.256191
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Iteration 90, Temperature 182.78, Best Loss 0.068077, Current Loss 0.250000
Iteration 100, Temperature 180.96, Best Loss 0.068077, Current Loss 0.367783
Iteration 110, Temperature 179.16, Best Loss 0.038360, Current Loss 0.050587
Iteration 120, Temperature 177.37, Best Loss 0.001438, Current Loss 0.245910
Iteration 130, Temperature 175.61, Best Loss 0.000005, Current Loss 0.322447
Iteration 140, Temperature 173.86, Best Loss 0.000005, Current Loss 0.500000
Iteration 150, Temperature 172.13, Best Loss 0.000005, Current Loss 0.500000
Iteration 160, Temperature 170.42, Best Loss 0.000005, Current Loss 0.500000
Iteration 170, Temperature 168.72, Best Loss 0.000005, Current Loss 0.500000
Iteration 180, Temperature 167.04, Best Loss 0.000005, Current Loss 0.500000
Iteration 190, Temperature 165.38, Best Loss 0.000005, Current Loss 0.499999
Iteration 200, Temperature 163.73, Best Loss 0.000005, Current Loss 0.500000
Iteration 210, Temperature 162.10, Best Loss 0.000005, Current Loss 0.500000
Iteration 220, Temperature 160.49, Best Loss 0.000005, Current Loss 0.500000
Iteration 230, Temperature 158.89, Best Loss 0.000005, Current Loss 0.500000
Iteration 240, Temperature 157.31, Best Loss 0.000005, Current Loss 0.500000
Iteration 250, Temperature 155.74, Best Loss 0.000005, Current Loss 0.500000
Iteration 260, Temperature 154.19, Best Loss 0.000005, Current Loss 0.500000
Iteration 270, Temperature 152.66, Best Loss 0.000005, Current Loss 0.500000
Iteration 280, Temperature 151.14, Best Loss 0.000005, Current Loss 0.500000
Iteration 290, Temperature 149.63, Best Loss 0.000005, Current Loss 0.500000
Iteration 300, Temperature 148.14, Best Loss 0.000005, Current Loss 0.500000
Iteration 310, Temperature 146.67, Best Loss 0.000005, Current Loss 0.500000
Iteration 320, Temperature 145.21, Best Loss 0.000005, Current Loss 0.500000
Iteration 330, Temperature 143.76, Best Loss 0.000005, Current Loss 0.500000
Iteration 340, Temperature 142.33, Best Loss 0.000005, Current Loss 0.500000
Iteration 350, Temperature 140.91, Best Loss 0.000005, Current Loss 0.500000
Iteration 360, Temperature 139.51, Best Loss 0.000005, Current Loss 0.500000
Iteration 370, Temperature 138.12, Best Loss 0.000005, Current Loss 0.500000
Iteration 380, Temperature 136.75, Best Loss 0.000005, Current Loss 0.500000
Iteration 390, Temperature 135.38, Best Loss 0.000005, Current Loss 0.500000
Iteration 400, Temperature 134.04, Best Loss 0.000005, Current Loss 0.500000
Iteration 410, Temperature 132.70, Best Loss 0.000005, Current Loss 0.500000
Iteration 420, Temperature 131.38, Best Loss 0.000005, Current Loss 0.500000
Iteration 430, Temperature 130.07, Best Loss 0.000005, Current Loss 0.500000
Iteration 440, Temperature 128.78, Best Loss 0.000005, Current Loss 0.500000
Iteration 450, Temperature 127.50, Best Loss 0.000005, Current Loss 0.500000
Iteration 460, Temperature 126.23, Best Loss 0.000005, Current Loss 0.500000
Iteration 470, Temperature 124.97, Best Loss 0.000005, Current Loss 0.500000
Iteration 480, Temperature 123.73, Best Loss 0.000005, Current Loss 0.500000
Iteration 490, Temperature 122.50, Best Loss 0.000005, Current Loss 0.500000
Iteration 500, Temperature 121.28, Best Loss 0.000005, Current Loss 0.500000
Iteration 510, Temperature 120.07, Best Loss 0.000005, Current Loss 0.500000
Iteration 520, Temperature 118.87, Best Loss 0.000005, Current Loss 0.500000
Iteration 530, Temperature 117.69, Best Loss 0.000005, Current Loss 0.500000
Iteration 540, Temperature 116.52, Best Loss 0.000005, Current Loss 0.500000
Iteration 550, Temperature 115.36, Best Loss 0.000005, Current Loss 0.500000
Iteration 560, Temperature 114.21, Best Loss 0.000005, Current Loss 0.500000
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Iteration 570, Temperature 113.07, Best Loss 0.000005, Current Loss 0.500000
Iteration 580, Temperature 111.95, Best Loss 0.000005, Current Loss 0.500000
Iteration 590, Temperature 110.83, Best Loss 0.000005, Current Loss 0.500000
Iteration 600, Temperature 109.73, Best Loss 0.000005, Current Loss 0.500000
Iteration 610, Temperature 108.64, Best Loss 0.000005, Current Loss 0.500000
Iteration 620, Temperature 107.56, Best Loss 0.000005, Current Loss 0.500000
Iteration 630, Temperature 106.48, Best Loss 0.000005, Current Loss 0.500000
Iteration 640, Temperature 105.42, Best Loss 0.000005, Current Loss 0.500000
Iteration 650, Temperature 104.38, Best Loss 0.000005, Current Loss 0.500000
Iteration 660, Temperature 103.34, Best Loss 0.000005, Current Loss 0.500000
Iteration 670, Temperature 102.31, Best Loss 0.000005, Current Loss 0.500000
Iteration 680, Temperature 101.29, Best Loss 0.000005, Current Loss 0.500000
Iteration 690, Temperature 100.28, Best Loss 0.000005, Current Loss 0.500000
Iteration 700, Temperature 99.28, Best Loss 0.000005, Current Loss 0.500000
Iteration 710, Temperature 98.29, Best Loss 0.000005, Current Loss 0.500000
Iteration 720, Temperature 97.32, Best Loss 0.000005, Current Loss 0.500000
Iteration 730, Temperature 96.35, Best Loss 0.000005, Current Loss 0.500000
Iteration 740, Temperature 95.39, Best Loss 0.000005, Current Loss 0.500000
Iteration 750, Temperature 94.44, Best Loss 0.000005, Current Loss 0.500000
Iteration 760, Temperature 93.50, Best Loss 0.000005, Current Loss 0.500000
Iteration 770, Temperature 92.57, Best Loss 0.000005, Current Loss 0.500000
Iteration 780, Temperature 91.65, Best Loss 0.000005, Current Loss 0.500000
Iteration 790, Temperature 90.73, Best Loss 0.000005, Current Loss 0.500000
Iteration 800, Temperature 89.83, Best Loss 0.000005, Current Loss 0.500000
Iteration 810, Temperature 88.94, Best Loss 0.000005, Current Loss 0.500000
Iteration 820, Temperature 88.05, Best Loss 0.000005, Current Loss 0.500000
Iteration 830, Temperature 87.17, Best Loss 0.000005, Current Loss 0.500000
Iteration 840, Temperature 86.31, Best Loss 0.000005, Current Loss 0.500000
Iteration 850, Temperature 85.45, Best Loss 0.000005, Current Loss 0.499929
Iteration 860, Temperature 84.60, Best Loss 0.000005, Current Loss 0.250204
Iteration 870, Temperature 83.75, Best Loss 0.000005, Current Loss 0.499989
Iteration 880, Temperature 82.92, Best Loss 0.000005, Current Loss 0.500000
Iteration 890, Temperature 82.09, Best Loss 0.000005, Current Loss 0.500000
Iteration 900, Temperature 81.28, Best Loss 0.000005, Current Loss 0.471134
Iteration 910, Temperature 80.47, Best Loss 0.000005, Current Loss 0.434991
Iteration 920, Temperature 79.67, Best Loss 0.000005, Current Loss 0.499996
Iteration 930, Temperature 78.87, Best Loss 0.000005, Current Loss 0.250000
Iteration 940, Temperature 78.09, Best Loss 0.000005, Current Loss 0.500000
Iteration 950, Temperature 77.31, Best Loss 0.000005, Current Loss 0.500000
Iteration 960, Temperature 76.54, Best Loss 0.000005, Current Loss 0.500000
Iteration 970, Temperature 75.78, Best Loss 0.000005, Current Loss 0.500000
Iteration 980, Temperature 75.03, Best Loss 0.000005, Current Loss 0.500000
Iteration 990, Temperature 74.28, Best Loss 0.000005, Current Loss 0.500000
Iteration 1000, Temperature 73.54, Best Loss 0.000005, Current Loss 0.500000
Iteration 1010, Temperature 72.81, Best Loss 0.000005, Current Loss 0.500000
Iteration 1020, Temperature 72.08, Best Loss 0.000005, Current Loss 0.500000
Iteration 1030, Temperature 71.36, Best Loss 0.000005, Current Loss 0.500000
Iteration 1040, Temperature 70.65, Best Loss 0.000005, Current Loss 0.500000
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Iteration 1050, Temperature 69.95, Best Loss 0.000005, Current Loss 0.500000
Iteration 1060, Temperature 69.25, Best Loss 0.000005, Current Loss 0.500000
Iteration 1070, Temperature 68.56, Best Loss 0.000005, Current Loss 0.500000
Iteration 1080, Temperature 67.88, Best Loss 0.000005, Current Loss 0.500000
Iteration 1090, Temperature 67.21, Best Loss 0.000005, Current Loss 0.500000
Iteration 1100, Temperature 66.54, Best Loss 0.000005, Current Loss 0.500000
Iteration 1110, Temperature 65.88, Best Loss 0.000005, Current Loss 0.500000
Iteration 1120, Temperature 65.22, Best Loss 0.000005, Current Loss 0.500000
Iteration 1130, Temperature 64.57, Best Loss 0.000005, Current Loss 0.500000
Iteration 1140, Temperature 63.93, Best Loss 0.000005, Current Loss 0.500000
Iteration 1150, Temperature 63.29, Best Loss 0.000005, Current Loss 0.500000
Iteration 1160, Temperature 62.66, Best Loss 0.000005, Current Loss 0.500000
Iteration 1170, Temperature 62.04, Best Loss 0.000005, Current Loss 0.500000
Iteration 1180, Temperature 61.42, Best Loss 0.000005, Current Loss 0.500000
Iteration 1190, Temperature 60.81, Best Loss 0.000005, Current Loss 0.750000
Iteration 1200, Temperature 60.20, Best Loss 0.000005, Current Loss 0.500000
Iteration 1210, Temperature 59.60, Best Loss 0.000005, Current Loss 0.453425
Iteration 1220, Temperature 59.01, Best Loss 0.000005, Current Loss 0.500000
Iteration 1230, Temperature 58.42, Best Loss 0.000005, Current Loss 0.500000
Iteration 1240, Temperature 57.84, Best Loss 0.000005, Current Loss 0.500000
Iteration 1250, Temperature 57.27, Best Loss 0.000005, Current Loss 0.500000
Iteration 1260, Temperature 56.70, Best Loss 0.000005, Current Loss 0.500000
Iteration 1270, Temperature 56.13, Best Loss 0.000005, Current Loss 0.500000
Iteration 1280, Temperature 55.57, Best Loss 0.000005, Current Loss 0.500000
Iteration 1290, Temperature 55.02, Best Loss 0.000005, Current Loss 0.500000
Iteration 1300, Temperature 54.47, Best Loss 0.000005, Current Loss 0.500000
Iteration 1310, Temperature 53.93, Best Loss 0.000005, Current Loss 0.500000
Iteration 1320, Temperature 53.39, Best Loss 0.000005, Current Loss 0.500000
Iteration 1330, Temperature 52.86, Best Loss 0.000005, Current Loss 0.500000
Iteration 1340, Temperature 52.33, Best Loss 0.000005, Current Loss 0.750000
Iteration 1350, Temperature 51.81, Best Loss 0.000005, Current Loss 0.750000
Iteration 1360, Temperature 51.30, Best Loss 0.000005, Current Loss 0.750000
Iteration 1370, Temperature 50.79, Best Loss 0.000005, Current Loss 0.749936
Iteration 1380, Temperature 50.28, Best Loss 0.000005, Current Loss 0.750000
Iteration 1390, Temperature 49.78, Best Loss 0.000005, Current Loss 0.500000
Iteration 1400, Temperature 49.28, Best Loss 0.000005, Current Loss 0.250034
Iteration 1410, Temperature 48.79, Best Loss 0.000005, Current Loss 0.500000
Iteration 1420, Temperature 48.31, Best Loss 0.000005, Current Loss 0.495477
Iteration 1430, Temperature 47.83, Best Loss 0.000005, Current Loss 0.500000
Iteration 1440, Temperature 47.35, Best Loss 0.000005, Current Loss 0.331553
Iteration 1450, Temperature 46.88, Best Loss 0.000005, Current Loss 0.599679
Iteration 1460, Temperature 46.41, Best Loss 0.000005, Current Loss 0.750000
Iteration 1470, Temperature 45.95, Best Loss 0.000005, Current Loss 0.750000
Iteration 1480, Temperature 45.49, Best Loss 0.000005, Current Loss 0.750000
Iteration 1490, Temperature 45.04, Best Loss 0.000005, Current Loss 0.749999
Iteration 1500, Temperature 44.59, Best Loss 0.000005, Current Loss 0.749978
Iteration 1510, Temperature 44.15, Best Loss 0.000005, Current Loss 0.750000
Iteration 1520, Temperature 43.71, Best Loss 0.000005, Current Loss 0.750000
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Iteration 1530, Temperature 43.27, Best Loss 0.000005, Current Loss 0.750000
Iteration 1540, Temperature 42.84, Best Loss 0.000005, Current Loss 0.213142
Iteration 1550, Temperature 42.42, Best Loss 0.000005, Current Loss 0.494735
Iteration 1560, Temperature 41.99, Best Loss 0.000005, Current Loss 0.499999
Iteration 1570, Temperature 41.58, Best Loss 0.000005, Current Loss 0.500000
Iteration 1580, Temperature 41.16, Best Loss 0.000005, Current Loss 0.500000
Iteration 1590, Temperature 40.75, Best Loss 0.000005, Current Loss 0.498880
Iteration 1600, Temperature 40.35, Best Loss 0.000005, Current Loss 0.727721
Iteration 1610, Temperature 39.95, Best Loss 0.000005, Current Loss 0.750000
Iteration 1620, Temperature 39.55, Best Loss 0.000005, Current Loss 0.500000
Iteration 1630, Temperature 39.15, Best Loss 0.000005, Current Loss 0.500000
Iteration 1640, Temperature 38.76, Best Loss 0.000005, Current Loss 0.500000
Iteration 1650, Temperature 38.38, Best Loss 0.000005, Current Loss 0.500000
Iteration 1660, Temperature 38.00, Best Loss 0.000005, Current Loss 0.500000
Iteration 1670, Temperature 37.62, Best Loss 0.000005, Current Loss 0.500000
Iteration 1680, Temperature 37.24, Best Loss 0.000005, Current Loss 0.500000
Iteration 1690, Temperature 36.87, Best Loss 0.000005, Current Loss 0.749946
Iteration 1700, Temperature 36.51, Best Loss 0.000005, Current Loss 0.500000
Iteration 1710, Temperature 36.14, Best Loss 0.000005, Current Loss 0.500000
Iteration 1720, Temperature 35.78, Best Loss 0.000005, Current Loss 0.500000
Iteration 1730, Temperature 35.43, Best Loss 0.000005, Current Loss 0.500000
Iteration 1740, Temperature 35.07, Best Loss 0.000005, Current Loss 0.750000
Iteration 1750, Temperature 34.72, Best Loss 0.000005, Current Loss 0.500000
Iteration 1760, Temperature 34.38, Best Loss 0.000005, Current Loss 0.499996
Iteration 1770, Temperature 34.04, Best Loss 0.000005, Current Loss 0.250000
Iteration 1780, Temperature 33.70, Best Loss 0.000005, Current Loss 0.734786
Iteration 1790, Temperature 33.36, Best Loss 0.000005, Current Loss 0.502963
Iteration 1800, Temperature 33.03, Best Loss 0.000005, Current Loss 0.500000
Iteration 1810, Temperature 32.70, Best Loss 0.000005, Current Loss 0.250000
Iteration 1820, Temperature 32.38, Best Loss 0.000005, Current Loss 0.250000
Iteration 1830, Temperature 32.05, Best Loss 0.000005, Current Loss 0.250003
Iteration 1840, Temperature 31.73, Best Loss 0.000005, Current Loss 0.456882
Iteration 1850, Temperature 31.42, Best Loss 0.000005, Current Loss 0.749745
Iteration 1860, Temperature 31.11, Best Loss 0.000005, Current Loss 0.250000
Iteration 1870, Temperature 30.80, Best Loss 0.000005, Current Loss 0.499986
Iteration 1880, Temperature 30.49, Best Loss 0.000005, Current Loss 0.500000
Iteration 1890, Temperature 30.19, Best Loss 0.000005, Current Loss 0.500000
Iteration 1900, Temperature 29.89, Best Loss 0.000005, Current Loss 0.500000
Iteration 1910, Temperature 29.59, Best Loss 0.000005, Current Loss 0.500000
Iteration 1920, Temperature 29.29, Best Loss 0.000005, Current Loss 0.250000
Iteration 1930, Temperature 29.00, Best Loss 0.000005, Current Loss 0.500000
Iteration 1940, Temperature 28.71, Best Loss 0.000005, Current Loss 0.500000
Iteration 1950, Temperature 28.43, Best Loss 0.000005, Current Loss 0.500000
Iteration 1960, Temperature 28.14, Best Loss 0.000005, Current Loss 0.500000
Iteration 1970, Temperature 27.86, Best Loss 0.000005, Current Loss 0.500000
Iteration 1980, Temperature 27.59, Best Loss 0.000005, Current Loss 0.500000
Iteration 1990, Temperature 27.31, Best Loss 0.000005, Current Loss 0.500000
Iteration 2000, Temperature 27.04, Best Loss 0.000005, Current Loss 0.500000
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Iteration 2010, Temperature 26.77, Best Loss 0.000005, Current Loss 0.500000
Iteration 2020, Temperature 26.50, Best Loss 0.000005, Current Loss 0.499992
Iteration 2030, Temperature 26.24, Best Loss 0.000005, Current Loss 0.495500
Iteration 2040, Temperature 25.98, Best Loss 0.000005, Current Loss 0.500000
Iteration 2050, Temperature 25.72, Best Loss 0.000005, Current Loss 0.500000
Iteration 2060, Temperature 25.46, Best Loss 0.000005, Current Loss 0.500000
Iteration 2070, Temperature 25.21, Best Loss 0.000005, Current Loss 0.499999
Iteration 2080, Temperature 24.96, Best Loss 0.000005, Current Loss 0.499997
Iteration 2090, Temperature 24.71, Best Loss 0.000005, Current Loss 0.500000
Iteration 2100, Temperature 24.47, Best Loss 0.000005, Current Loss 0.500000
Iteration 2110, Temperature 24.22, Best Loss 0.000005, Current Loss 0.500000
Iteration 2120, Temperature 23.98, Best Loss 0.000005, Current Loss 0.500000
Iteration 2130, Temperature 23.74, Best Loss 0.000005, Current Loss 0.500000
Iteration 2140, Temperature 23.51, Best Loss 0.000005, Current Loss 0.500000
Iteration 2150, Temperature 23.27, Best Loss 0.000005, Current Loss 0.500000
Iteration 2160, Temperature 23.04, Best Loss 0.000005, Current Loss 0.500000
Iteration 2170, Temperature 22.81, Best Loss 0.000005, Current Loss 0.500000
Iteration 2180, Temperature 22.58, Best Loss 0.000005, Current Loss 0.500000
Iteration 2190, Temperature 22.36, Best Loss 0.000005, Current Loss 0.500000
Iteration 2200, Temperature 22.14, Best Loss 0.000005, Current Loss 0.500000
Iteration 2210, Temperature 21.92, Best Loss 0.000005, Current Loss 0.500000
Iteration 2220, Temperature 21.70, Best Loss 0.000005, Current Loss 0.371632
Iteration 2230, Temperature 21.48, Best Loss 0.000005, Current Loss 0.153116
Iteration 2240, Temperature 21.27, Best Loss 0.000005, Current Loss 0.499393
Iteration 2250, Temperature 21.06, Best Loss 0.000005, Current Loss 0.387535
Iteration 2260, Temperature 20.85, Best Loss 0.000005, Current Loss 0.499968
Iteration 2270, Temperature 20.64, Best Loss 0.000005, Current Loss 0.395525
Iteration 2280, Temperature 20.43, Best Loss 0.000005, Current Loss 0.499205
Iteration 2290, Temperature 20.23, Best Loss 0.000005, Current Loss 0.500000
Iteration 2300, Temperature 20.03, Best Loss 0.000005, Current Loss 0.499993
Iteration 2310, Temperature 19.83, Best Loss 0.000005, Current Loss 0.401039
Iteration 2320, Temperature 19.63, Best Loss 0.000005, Current Loss 0.499956
Iteration 2330, Temperature 19.44, Best Loss 0.000005, Current Loss 0.249977
Iteration 2340, Temperature 19.24, Best Loss 0.000005, Current Loss 0.249999
Iteration 2350, Temperature 19.05, Best Loss 0.000005, Current Loss 0.500000
Iteration 2360, Temperature 18.86, Best Loss 0.000005, Current Loss 0.249996
Iteration 2370, Temperature 18.67, Best Loss 0.000005, Current Loss 0.499996
Iteration 2380, Temperature 18.49, Best Loss 0.000005, Current Loss 0.499928
Iteration 2390, Temperature 18.30, Best Loss 0.000005, Current Loss 0.424919
Iteration 2400, Temperature 18.12, Best Loss 0.000005, Current Loss 0.249999
Iteration 2410, Temperature 17.94, Best Loss 0.000005, Current Loss 0.498674
Iteration 2420, Temperature 17.76, Best Loss 0.000005, Current Loss 0.499991
Iteration 2430, Temperature 17.59, Best Loss 0.000005, Current Loss 0.148846
Iteration 2440, Temperature 17.41, Best Loss 0.000005, Current Loss 0.195660
Iteration 2450, Temperature 17.24, Best Loss 0.000005, Current Loss 0.438199
Iteration 2460, Temperature 17.07, Best Loss 0.000005, Current Loss 0.499992
Iteration 2470, Temperature 16.90, Best Loss 0.000005, Current Loss 0.498410
Iteration 2480, Temperature 16.73, Best Loss 0.000005, Current Loss 0.498236
```

```
Iteration 2490, Temperature 16.56, Best Loss 0.000005, Current Loss 0.500000
Iteration 2500, Temperature 16.40, Best Loss 0.000005, Current Loss 0.500000
Iteration 2510, Temperature 16.23, Best Loss 0.000005, Current Loss 0.500000
Iteration 2520, Temperature 16.07, Best Loss 0.000005, Current Loss 0.500000
Iteration 2530, Temperature 15.91, Best Loss 0.000005, Current Loss 0.500000
Iteration 2540, Temperature 15.75, Best Loss 0.000005, Current Loss 0.500000
Iteration 2550, Temperature 15.60, Best Loss 0.000005, Current Loss 0.500000
Iteration 2560, Temperature 15.44, Best Loss 0.000005, Current Loss 0.500000
Iteration 2570, Temperature 15.29, Best Loss 0.000005, Current Loss 0.500000
Iteration 2580, Temperature 15.14, Best Loss 0.000005, Current Loss 0.500000
Iteration 2590, Temperature 14.98, Best Loss 0.000005, Current Loss 0.500000
Iteration 2600, Temperature 14.84, Best Loss 0.000005, Current Loss 0.500004
Iteration 2610, Temperature 14.69, Best Loss 0.000005, Current Loss 0.500000
Iteration 2620, Temperature 14.54, Best Loss 0.000005, Current Loss 0.500000
Iteration 2630, Temperature 14.40, Best Loss 0.000005, Current Loss 0.500000
Iteration 2640, Temperature 14.25, Best Loss 0.000005, Current Loss 0.500000
Iteration 2650, Temperature 14.11, Best Loss 0.000005, Current Loss 0.500000
Iteration 2660, Temperature 13.97, Best Loss 0.000005, Current Loss 0.500000
Iteration 2670, Temperature 13.83, Best Loss 0.000005, Current Loss 0.500000
Iteration 2680, Temperature 13.69, Best Loss 0.000005, Current Loss 0.500000
Iteration 2690, Temperature 13.56, Best Loss 0.000005, Current Loss 0.500000
Iteration 2700, Temperature 13.42, Best Loss 0.000005, Current Loss 0.500000
Iteration 2710, Temperature 13.29, Best Loss 0.000005, Current Loss 0.500000
Iteration 2720, Temperature 13.16, Best Loss 0.000005, Current Loss 0.500000
Iteration 2730, Temperature 13.03, Best Loss 0.000005, Current Loss 0.500000
Iteration 2740, Temperature 12.90, Best Loss 0.000005, Current Loss 0.500000
Iteration 2750, Temperature 12.77, Best Loss 0.000005, Current Loss 0.500000
Iteration 2760, Temperature 12.64, Best Loss 0.000005, Current Loss 0.500000
Iteration 2770, Temperature 12.52, Best Loss 0.000005, Current Loss 0.500000
Iteration 2780, Temperature 12.39, Best Loss 0.000005, Current Loss 0.500000
Iteration 2790, Temperature 12.27, Best Loss 0.000005, Current Loss 0.500000
Iteration 2800, Temperature 12.14, Best Loss 0.000005, Current Loss 0.500000
Iteration 2810, Temperature 12.02, Best Loss 0.000005, Current Loss 0.500000
Iteration 2820, Temperature 11.90, Best Loss 0.000005, Current Loss 0.500000
Iteration 2830, Temperature 11.79, Best Loss 0.000005, Current Loss 0.499997
Iteration 2840, Temperature 11.67, Best Loss 0.000005, Current Loss 0.500000
Iteration 2850, Temperature 11.55, Best Loss 0.000005, Current Loss 0.500000
Iteration 2860, Temperature 11.44, Best Loss 0.000005, Current Loss 0.500000
Iteration 2870, Temperature 11.32, Best Loss 0.000005, Current Loss 0.500000
Iteration 2880, Temperature 11.21, Best Loss 0.000005, Current Loss 0.500000
Iteration 2890, Temperature 11.10, Best Loss 0.000005, Current Loss 0.500000
Iteration 2900, Temperature 10.99, Best Loss 0.000005, Current Loss 0.500000
Iteration 2910, Temperature 10.88, Best Loss 0.000005, Current Loss 0.500000
Iteration 2920, Temperature 10.77, Best Loss 0.000005, Current Loss 0.500000
Iteration 2930, Temperature 10.66, Best Loss 0.000005, Current Loss 0.500000
Iteration 2940, Temperature 10.56, Best Loss 0.000005, Current Loss 0.500000
Iteration 2950, Temperature 10.45, Best Loss 0.000005, Current Loss 0.500000
Iteration 2960, Temperature 10.35, Best Loss 0.000005, Current Loss 0.500000
```

```
Iteration 2970, Temperature 10.25, Best Loss 0.000005, Current Loss 0.500000
Iteration 2980, Temperature 10.14, Best Loss 0.000005, Current Loss 0.500000
Iteration 2990, Temperature 10.04, Best Loss 0.000005, Current Loss 0.500000
Iteration 3000, Temperature 9.94, Best Loss 0.000005, Current Loss 0.500000
Iteration 3010, Temperature 9.84, Best Loss 0.000005, Current Loss 0.500000
Iteration 3020, Temperature 9.75, Best Loss 0.000005, Current Loss 0.500000
Iteration 3030, Temperature 9.65, Best Loss 0.000005, Current Loss 0.500000
Iteration 3040, Temperature 9.55, Best Loss 0.000005, Current Loss 0.500000
Iteration 3050, Temperature 9.46, Best Loss 0.000005, Current Loss 0.500000
Iteration 3060, Temperature 9.36, Best Loss 0.000005, Current Loss 0.500000
Iteration 3070, Temperature 9.27, Best Loss 0.000005, Current Loss 0.500000
Iteration 3080, Temperature 9.18, Best Loss 0.000005, Current Loss 0.500000
Iteration 3090, Temperature 9.09, Best Loss 0.000005, Current Loss 0.500000
Iteration 3100, Temperature 9.00, Best Loss 0.000005, Current Loss 0.500000
Iteration 3110, Temperature 8.91, Best Loss 0.000005, Current Loss 0.500000
Iteration 3120, Temperature 8.82, Best Loss 0.000005, Current Loss 0.500000
Iteration 3130, Temperature 8.73, Best Loss 0.000005, Current Loss 0.500000
Iteration 3140, Temperature 8.64, Best Loss 0.000005, Current Loss 0.500000
Iteration 3150, Temperature 8.56, Best Loss 0.000005, Current Loss 0.500000
Iteration 3160, Temperature 8.47, Best Loss 0.000005, Current Loss 0.500000
Iteration 3170, Temperature 8.39, Best Loss 0.000005, Current Loss 0.500000
Iteration 3180, Temperature 8.30, Best Loss 0.000005, Current Loss 0.500000
Iteration 3190, Temperature 8.22, Best Loss 0.000005, Current Loss 0.500000
Iteration 3200, Temperature 8.14, Best Loss 0.000005, Current Loss 0.500000
Iteration 3210, Temperature 8.06, Best Loss 0.000005, Current Loss 0.500000
Iteration 3220, Temperature 7.98, Best Loss 0.000005, Current Loss 0.500000
Iteration 3230, Temperature 7.90, Best Loss 0.000005, Current Loss 0.500000
Iteration 3240, Temperature 7.82, Best Loss 0.000005, Current Loss 0.500000
Iteration 3250, Temperature 7.74, Best Loss 0.000005, Current Loss 0.500000
Iteration 3260, Temperature 7.67, Best Loss 0.000005, Current Loss 0.500000
Iteration 3270, Temperature 7.59, Best Loss 0.000005, Current Loss 0.500000
Iteration 3280, Temperature 7.51, Best Loss 0.000005, Current Loss 0.500000
Iteration 3290, Temperature 7.44, Best Loss 0.000005, Current Loss 0.500000
Iteration 3300, Temperature 7.36, Best Loss 0.000005, Current Loss 0.500000
Iteration 3310, Temperature 7.29, Best Loss 0.000005, Current Loss 0.500000
Iteration 3320, Temperature 7.22, Best Loss 0.000005, Current Loss 0.500000
Iteration 3330, Temperature 7.15, Best Loss 0.000005, Current Loss 0.500000
Iteration 3340, Temperature 7.08, Best Loss 0.000005, Current Loss 0.500000
Iteration 3350, Temperature 7.01, Best Loss 0.000005, Current Loss 0.500000
Iteration 3360, Temperature 6.94, Best Loss 0.000005, Current Loss 0.500000
Iteration 3370, Temperature 6.87, Best Loss 0.000005, Current Loss 0.500000
Iteration 3380, Temperature 6.80, Best Loss 0.000005, Current Loss 0.500000
Iteration 3390, Temperature 6.73, Best Loss 0.000005, Current Loss 0.500000
Iteration 3400, Temperature 6.66, Best Loss 0.000005, Current Loss 0.500000
Iteration 3410, Temperature 6.60, Best Loss 0.000005, Current Loss 0.500000
Iteration 3420, Temperature 6.53, Best Loss 0.000005, Current Loss 0.500000
Iteration 3430, Temperature 6.47, Best Loss 0.000005, Current Loss 0.500000
Iteration 3440, Temperature 6.40, Best Loss 0.000005, Current Loss 0.500000
```

```
Iteration 3450, Temperature 6.34, Best Loss 0.000005, Current Loss 0.500000
Iteration 3460, Temperature 6.28, Best Loss 0.000005, Current Loss 0.500000
Iteration 3470, Temperature 6.21, Best Loss 0.000005, Current Loss 0.500000
Iteration 3480, Temperature 6.15, Best Loss 0.000005, Current Loss 0.500000
Iteration 3490, Temperature 6.09, Best Loss 0.000005, Current Loss 0.500000
Iteration 3500, Temperature 6.03, Best Loss 0.000005, Current Loss 0.500000
Iteration 3510, Temperature 5.97, Best Loss 0.000005, Current Loss 0.528083
Iteration 3520, Temperature 5.91, Best Loss 0.000005, Current Loss 0.750000
Iteration 3530, Temperature 5.85, Best Loss 0.000005, Current Loss 0.750000
Iteration 3540, Temperature 5.79, Best Loss 0.000005, Current Loss 0.750000
Iteration 3550, Temperature 5.73, Best Loss 0.000005, Current Loss 0.500000
Iteration 3560, Temperature 5.68, Best Loss 0.000005, Current Loss 0.500000
Iteration 3570, Temperature 5.62, Best Loss 0.000005, Current Loss 0.500014
Iteration 3580, Temperature 5.57, Best Loss 0.000005, Current Loss 0.500000
Iteration 3590, Temperature 5.51, Best Loss 0.000005, Current Loss 0.500000
Iteration 3600, Temperature 5.45, Best Loss 0.000005, Current Loss 0.500000
Iteration 3610, Temperature 5.40, Best Loss 0.000005, Current Loss 0.500000
Iteration 3620, Temperature 5.35, Best Loss 0.000005, Current Loss 0.500000
Iteration 3630, Temperature 5.29, Best Loss 0.000005, Current Loss 0.500000
Iteration 3640, Temperature 5.24, Best Loss 0.000005, Current Loss 0.500000
Iteration 3650, Temperature 5.19, Best Loss 0.000005, Current Loss 0.500000
Iteration 3660, Temperature 5.14, Best Loss 0.000005, Current Loss 0.500000
Iteration 3670, Temperature 5.09, Best Loss 0.000005, Current Loss 0.500000
Iteration 3680, Temperature 5.04, Best Loss 0.000005, Current Loss 0.500000
Iteration 3690, Temperature 4.99, Best Loss 0.000005, Current Loss 0.500000
Iteration 3700, Temperature 4.94, Best Loss 0.000005, Current Loss 0.500000
Iteration 3710, Temperature 4.89, Best Loss 0.000005, Current Loss 0.500000
Iteration 3720, Temperature 4.84, Best Loss 0.000005, Current Loss 0.500000
Iteration 3730, Temperature 4.79, Best Loss 0.000005, Current Loss 0.500000
Iteration 3740, Temperature 4.74, Best Loss 0.000005, Current Loss 0.500000
Iteration 3750, Temperature 4.69, Best Loss 0.000005, Current Loss 0.500000
Iteration 3760, Temperature 4.65, Best Loss 0.000005, Current Loss 0.500000
Iteration 3770, Temperature 4.60, Best Loss 0.000005, Current Loss 0.500000
Iteration 3780, Temperature 4.56, Best Loss 0.000005, Current Loss 0.500000
Iteration 3790, Temperature 4.51, Best Loss 0.000005, Current Loss 0.500000
Iteration 3800, Temperature 4.47, Best Loss 0.000005, Current Loss 0.500000
Iteration 3810, Temperature 4.42, Best Loss 0.000005, Current Loss 0.500000
Iteration 3820, Temperature 4.38, Best Loss 0.000005, Current Loss 0.500000
Iteration 3830, Temperature 4.33, Best Loss 0.000005, Current Loss 0.500000
Iteration 3840, Temperature 4.29, Best Loss 0.000005, Current Loss 0.500000
Iteration 3850, Temperature 4.25, Best Loss 0.000005, Current Loss 0.500000
Iteration 3860, Temperature 4.21, Best Loss 0.000005, Current Loss 0.500000
Iteration 3870, Temperature 4.16, Best Loss 0.000005, Current Loss 0.500000
Iteration 3880, Temperature 4.12, Best Loss 0.000005, Current Loss 0.500000
Iteration 3890, Temperature 4.08, Best Loss 0.000005, Current Loss 0.500000
Iteration 3900, Temperature 4.04, Best Loss 0.000005, Current Loss 0.500000
Iteration 3910, Temperature 4.00, Best Loss 0.000005, Current Loss 0.500000
Iteration 3920, Temperature 3.96, Best Loss 0.000005, Current Loss 0.500000
```

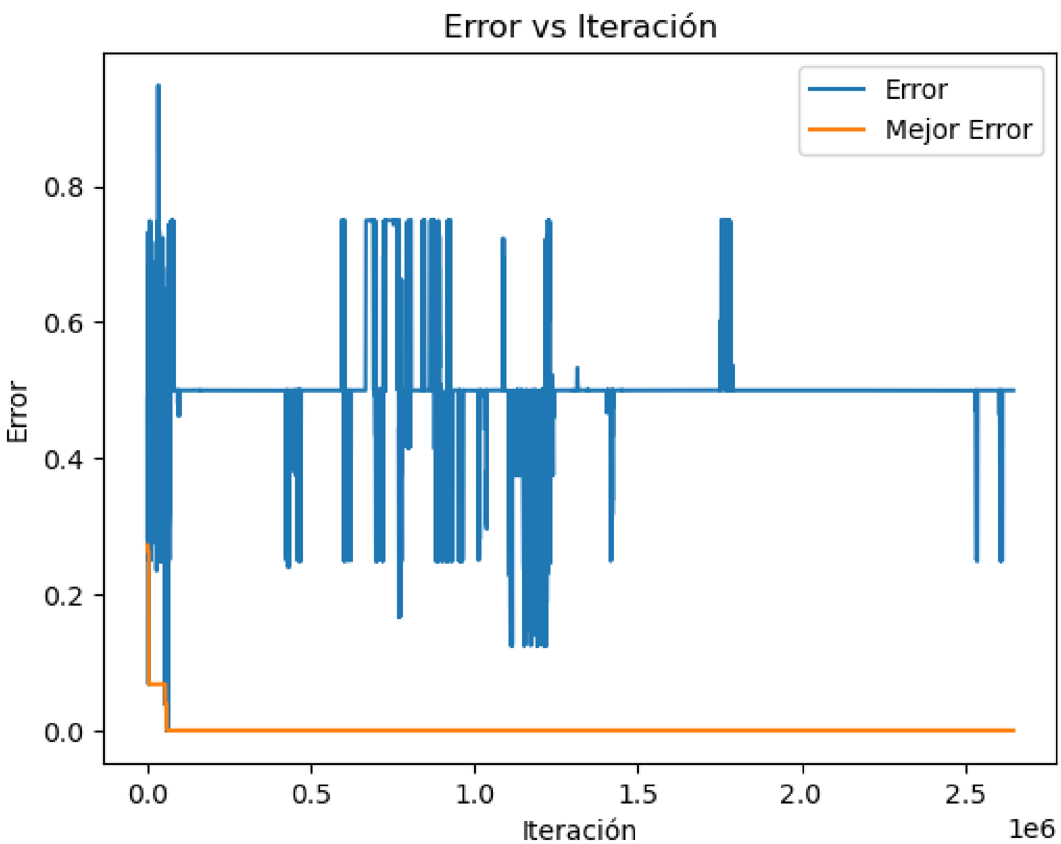
```
Iteration 3930, Temperature 3.92, Best Loss 0.000005, Current Loss 0.500000
Iteration 3940, Temperature 3.88, Best Loss 0.000005, Current Loss 0.500000
Iteration 3950, Temperature 3.84, Best Loss 0.000005, Current Loss 0.500000
Iteration 3960, Temperature 3.81, Best Loss 0.000005, Current Loss 0.500000
Iteration 3970, Temperature 3.77, Best Loss 0.000005, Current Loss 0.500000
Iteration 3980, Temperature 3.73, Best Loss 0.000005, Current Loss 0.500000
Iteration 3990, Temperature 3.69, Best Loss 0.000005, Current Loss 0.500000
Iteration 4000, Temperature 3.66, Best Loss 0.000005, Current Loss 0.500000
Iteration 4010, Temperature 3.62, Best Loss 0.000005, Current Loss 0.500000
Iteration 4020, Temperature 3.58, Best Loss 0.000005, Current Loss 0.500000
Iteration 4030, Temperature 3.55, Best Loss 0.000005, Current Loss 0.500000
Iteration 4040, Temperature 3.51, Best Loss 0.000005, Current Loss 0.500000
Iteration 4050, Temperature 3.48, Best Loss 0.000005, Current Loss 0.500000
Iteration 4060, Temperature 3.44, Best Loss 0.000005, Current Loss 0.500000
Iteration 4070, Temperature 3.41, Best Loss 0.000005, Current Loss 0.500000
Iteration 4080, Temperature 3.37, Best Loss 0.000005, Current Loss 0.500000
Iteration 4090, Temperature 3.34, Best Loss 0.000005, Current Loss 0.500000
Iteration 4100, Temperature 3.31, Best Loss 0.000005, Current Loss 0.500000
Iteration 4110, Temperature 3.27, Best Loss 0.000005, Current Loss 0.500000
Iteration 4120, Temperature 3.24, Best Loss 0.000005, Current Loss 0.500000
Iteration 4130, Temperature 3.21, Best Loss 0.000005, Current Loss 0.500000
Iteration 4140, Temperature 3.18, Best Loss 0.000005, Current Loss 0.500000
Iteration 4150, Temperature 3.15, Best Loss 0.000005, Current Loss 0.500000
Iteration 4160, Temperature 3.12, Best Loss 0.000005, Current Loss 0.500000
Iteration 4170, Temperature 3.08, Best Loss 0.000005, Current Loss 0.500000
Iteration 4180, Temperature 3.05, Best Loss 0.000005, Current Loss 0.500000
Iteration 4190, Temperature 3.02, Best Loss 0.000005, Current Loss 0.500000
Iteration 4200, Temperature 2.99, Best Loss 0.000005, Current Loss 0.500000
Iteration 4210, Temperature 2.96, Best Loss 0.000005, Current Loss 0.500000
Iteration 4220, Temperature 2.93, Best Loss 0.000005, Current Loss 0.500000
Iteration 4230, Temperature 2.90, Best Loss 0.000005, Current Loss 0.500000
```

```
/tmp/ipykernel_318702/4153003468.py:16: RuntimeWarning: overflow encountered in
exp
  return 1 / (1 + np.exp(-x))
```

```
Iteration 4240, Temperature 2.88, Best Loss 0.000005, Current Loss 0.500000
Iteration 4250, Temperature 2.85, Best Loss 0.000005, Current Loss 0.500000
Iteration 4260, Temperature 2.82, Best Loss 0.000005, Current Loss 0.500000
Iteration 4270, Temperature 2.79, Best Loss 0.000005, Current Loss 0.500000
Iteration 4280, Temperature 2.76, Best Loss 0.000005, Current Loss 0.500000
Iteration 4290, Temperature 2.74, Best Loss 0.000005, Current Loss 0.500000
Iteration 4300, Temperature 2.71, Best Loss 0.000005, Current Loss 0.500000
Iteration 4310, Temperature 2.68, Best Loss 0.000005, Current Loss 0.500000
Iteration 4320, Temperature 2.65, Best Loss 0.000005, Current Loss 0.500000
Iteration 4330, Temperature 2.63, Best Loss 0.000005, Current Loss 0.500000
Iteration 4340, Temperature 2.60, Best Loss 0.000005, Current Loss 0.500000
Iteration 4350, Temperature 2.58, Best Loss 0.000005, Current Loss 0.500000
Iteration 4360, Temperature 2.55, Best Loss 0.000005, Current Loss 0.500000
```

```
Iteration 4370, Temperature 2.52, Best Loss 0.000005, Current Loss 0.500000
Iteration 4380, Temperature 2.50, Best Loss 0.000005, Current Loss 0.500000
Iteration 4390, Temperature 2.47, Best Loss 0.000005, Current Loss 0.500000
Iteration 4400, Temperature 2.45, Best Loss 0.000005, Current Loss 0.500000
Iteration 4410, Temperature 2.43, Best Loss 0.000005, Current Loss 0.500000
Iteration 4420, Temperature 2.40, Best Loss 0.000005, Current Loss 0.500000
Iteration 4430, Temperature 2.38, Best Loss 0.000005, Current Loss 0.500000
Iteration 4440, Temperature 2.35, Best Loss 0.000005, Current Loss 0.500000
Iteration 4450, Temperature 2.33, Best Loss 0.000005, Current Loss 0.500000
Iteration 4460, Temperature 2.31, Best Loss 0.000005, Current Loss 0.500000
Iteration 4470, Temperature 2.28, Best Loss 0.000005, Current Loss 0.500000
Iteration 4480, Temperature 2.26, Best Loss 0.000005, Current Loss 0.500000
Iteration 4490, Temperature 2.24, Best Loss 0.000005, Current Loss 0.500000
Iteration 4500, Temperature 2.22, Best Loss 0.000005, Current Loss 0.500000
Iteration 4510, Temperature 2.19, Best Loss 0.000005, Current Loss 0.500000
Iteration 4520, Temperature 2.17, Best Loss 0.000005, Current Loss 0.500000
Iteration 4530, Temperature 2.15, Best Loss 0.000005, Current Loss 0.500000
Iteration 4540, Temperature 2.13, Best Loss 0.000005, Current Loss 0.500000
Iteration 4550, Temperature 2.11, Best Loss 0.000005, Current Loss 0.500000
Iteration 4560, Temperature 2.09, Best Loss 0.000005, Current Loss 0.500000
Iteration 4570, Temperature 2.07, Best Loss 0.000005, Current Loss 0.500000
Iteration 4580, Temperature 2.05, Best Loss 0.000005, Current Loss 0.500000
Iteration 4590, Temperature 2.03, Best Loss 0.000005, Current Loss 0.500000
Iteration 4600, Temperature 2.01, Best Loss 0.000005, Current Loss 0.500000
Iteration 4610, Temperature 1.99, Best Loss 0.000005, Current Loss 0.500000
Iteration 4620, Temperature 1.97, Best Loss 0.000005, Current Loss 0.500000
Iteration 4630, Temperature 1.95, Best Loss 0.000005, Current Loss 0.500000
Iteration 4640, Temperature 1.93, Best Loss 0.000005, Current Loss 0.500000
Iteration 4650, Temperature 1.91, Best Loss 0.000005, Current Loss 0.500000
Iteration 4660, Temperature 1.89, Best Loss 0.000005, Current Loss 0.500000
Iteration 4670, Temperature 1.87, Best Loss 0.000005, Current Loss 0.500000
Iteration 4680, Temperature 1.85, Best Loss 0.000005, Current Loss 0.500000
Iteration 4690, Temperature 1.83, Best Loss 0.000005, Current Loss 0.500000
Iteration 4700, Temperature 1.81, Best Loss 0.000005, Current Loss 0.500000
Iteration 4710, Temperature 1.80, Best Loss 0.000005, Current Loss 0.500000
Iteration 4720, Temperature 1.78, Best Loss 0.000005, Current Loss 0.500000
Iteration 4730, Temperature 1.76, Best Loss 0.000005, Current Loss 0.500000
Iteration 4740, Temperature 1.74, Best Loss 0.000005, Current Loss 0.500000
Iteration 4750, Temperature 1.73, Best Loss 0.000005, Current Loss 0.500000
Iteration 4760, Temperature 1.71, Best Loss 0.000005, Current Loss 0.500000
Iteration 4770, Temperature 1.69, Best Loss 0.000005, Current Loss 0.500000
Iteration 4780, Temperature 1.68, Best Loss 0.000005, Current Loss 0.500000
Iteration 4790, Temperature 1.66, Best Loss 0.000005, Current Loss 0.500000
Iteration 4800, Temperature 1.64, Best Loss 0.000005, Current Loss 0.500000
Iteration 4810, Temperature 1.63, Best Loss 0.000005, Current Loss 0.500000
Iteration 4820, Temperature 1.61, Best Loss 0.000005, Current Loss 0.500000
Iteration 4830, Temperature 1.59, Best Loss 0.000005, Current Loss 0.500000
Iteration 4840, Temperature 1.58, Best Loss 0.000005, Current Loss 0.500000
```

```
Iteration 4850, Temperature 1.56, Best Loss 0.000005, Current Loss 0.500000
Iteration 4860, Temperature 1.55, Best Loss 0.000005, Current Loss 0.500000
Iteration 4870, Temperature 1.53, Best Loss 0.000005, Current Loss 0.500000
Iteration 4880, Temperature 1.52, Best Loss 0.000005, Current Loss 0.500000
Iteration 4890, Temperature 1.50, Best Loss 0.000005, Current Loss 0.500000
Iteration 4900, Temperature 1.49, Best Loss 0.000005, Current Loss 0.500000
Iteration 4910, Temperature 1.47, Best Loss 0.000005, Current Loss 0.500000
Iteration 4920, Temperature 1.46, Best Loss 0.000005, Current Loss 0.500000
Iteration 4930, Temperature 1.44, Best Loss 0.000005, Current Loss 0.500000
Iteration 4940, Temperature 1.43, Best Loss 0.000005, Current Loss 0.500000
Iteration 4950, Temperature 1.41, Best Loss 0.000005, Current Loss 0.500000
Iteration 4960, Temperature 1.40, Best Loss 0.000005, Current Loss 0.500000
Iteration 4970, Temperature 1.39, Best Loss 0.000005, Current Loss 0.500000
Iteration 4980, Temperature 1.37, Best Loss 0.000005, Current Loss 0.500000
Iteration 4990, Temperature 1.36, Best Loss 0.000005, Current Loss 0.500000
Iteration 5000, Temperature 1.34, Best Loss 0.000005, Current Loss 0.500000
Iteration 5010, Temperature 1.33, Best Loss 0.000005, Current Loss 0.500000
Iteration 5020, Temperature 1.32, Best Loss 0.000005, Current Loss 0.500000
Iteration 5030, Temperature 1.30, Best Loss 0.000005, Current Loss 0.500000
Iteration 5040, Temperature 1.29, Best Loss 0.000005, Current Loss 0.500000
Iteration 5050, Temperature 1.28, Best Loss 0.000005, Current Loss 0.500000
Iteration 5060, Temperature 1.27, Best Loss 0.000005, Current Loss 0.500000
Iteration 5070, Temperature 1.25, Best Loss 0.000005, Current Loss 0.500000
Iteration 5080, Temperature 1.24, Best Loss 0.000005, Current Loss 0.500000
Iteration 5090, Temperature 1.23, Best Loss 0.000005, Current Loss 0.500000
Iteration 5100, Temperature 1.22, Best Loss 0.000005, Current Loss 0.500000
Iteration 5110, Temperature 1.20, Best Loss 0.000005, Current Loss 0.500000
Iteration 5120, Temperature 1.19, Best Loss 0.000005, Current Loss 0.500000
Iteration 5130, Temperature 1.18, Best Loss 0.000005, Current Loss 0.500000
Iteration 5140, Temperature 1.17, Best Loss 0.000005, Current Loss 0.500000
Iteration 5150, Temperature 1.16, Best Loss 0.000005, Current Loss 0.500000
Iteration 5160, Temperature 1.15, Best Loss 0.000005, Current Loss 0.500000
Iteration 5170, Temperature 1.13, Best Loss 0.000005, Current Loss 0.500000
Iteration 5180, Temperature 1.12, Best Loss 0.000005, Current Loss 0.500000
Iteration 5190, Temperature 1.11, Best Loss 0.000005, Current Loss 0.500000
Iteration 5200, Temperature 1.10, Best Loss 0.000005, Current Loss 0.500000
Iteration 5210, Temperature 1.09, Best Loss 0.000005, Current Loss 0.500000
Iteration 5220, Temperature 1.08, Best Loss 0.000005, Current Loss 0.500000
Iteration 5230, Temperature 1.07, Best Loss 0.000005, Current Loss 0.500000
Iteration 5240, Temperature 1.06, Best Loss 0.000005, Current Loss 0.500000
Iteration 5250, Temperature 1.05, Best Loss 0.000005, Current Loss 0.500000
Iteration 5260, Temperature 1.04, Best Loss 0.000005, Current Loss 0.500000
Iteration 5270, Temperature 1.03, Best Loss 0.000005, Current Loss 0.500000
Iteration 5280, Temperature 1.02, Best Loss 0.000005, Current Loss 0.500000
Iteration 5290, Temperature 1.01, Best Loss 0.000005, Current Loss 0.500000
```




```
Predicciones:
Input: [0 0], Predecido: 0.0030, Esperado: 0
Input: [0 1], Predecido: 1.0000, Esperado: 1
Input: [1 0], Predecido: 1.0000, Esperado: 1
Input: [1 1], Predecido: 0.0034, Esperado: 0
```

Se puede ver como el error real tiene saltos bruscos al principio y luego se estabiliza. Esto se debe a que el algoritmo de simulated annealing explora el espacio de soluciones de manera aleatoria y no determinista, lo que puede llevar a soluciones subóptimas. Sin embargo, en este caso, el algoritmo converge a una solución que no es la más óptima, pero como se guarda la mejor configuración de pesos, se puede utilizar para predecir la salida de la red neuronal con una precisión aceptable.

1. Construya una red de Kohonen de 2 entradas que aprenda una distribución uniforme dentro del círculo unitario. Mostrar el mapa de preservación de topología. Probar con distribuciones uniformes dentro de otras figuras geométricas.

Para construir la red Kohonen, o el mapa auto-organizado, se declara la clase SOM que contiene los métodos necesarios para entrenar la red y visualizar los resultados. Se inicializan los pesos de manera aleatoria y se actualizan de acuerdo a la distancia euclidiana o distancia circular entre el vector de entrada y el vector de peso. Se utiliza un factor de aprendizaje que disminuye con el tiempo y un radio que se reduce con el tiempo. Se entrena la red con una distribución uniforme dentro del círculo unitario y se visualiza el mapa de preservación de topología. Luego, se prueba con distribuciones uniformes dentro de otras figuras geométricas, como el cuadrado y el triángulo, y se visualizan los resultados.

[1]:
```
import math
import numpy as np
import matplotlib.pyplot as plt

class SOM:
    def __init__(self, input_dimension, num_neurons, initial_radius,␣
 ↪learning_rate, decay_rate, min_radius):
        self.input_dimension = input_dimension
        self.num_neurons = num_neurons
        self.radius = initial_radius
        self.learning_rate = learning_rate
        self.decay_rate = decay_rate
        self.min_radius = min_radius
        self.weights = np.random.uniform(-0.01, 0.01, (num_neurons,␣
 ↪input_dimension))
        self.grid_size = int(math.sqrt(num_neurons))  # Tamaño de la cuadrícula

    def train(self, training_data, circular=False):

        while self.radius > self.min_radius:
            np.random.shuffle(training_data)

            for input_vector in training_data:
                distances = np.array([np.sqrt(np.sum((input_vector - w)**2))␣
 ↪for w in self.weights])
                winner_idx = np.argmin(distances)

                if circular:
                    # Topología circular
                    neighborhood = self.
 ↪_calculate_circular_neighborhood(winner_idx)
                else:
                    # Topología de cuadriculada
                    neighborhood = self._calculate_grid_neighborhood(winner_idx)

                self._update_weights(input_vector, neighborhood)

            self.radius -= self.radius * self.decay_rate

        return self.weights

    def _calculate_circular_neighborhood(self, winner_idx):
        angles = np.array([[math.sin(2*math.pi/self.num_neurons*i),
                            math.cos(2*math.pi/self.num_neurons*i)]
```

```python
                            for i in range(self.num_neurons)])
        winner_angle = angles[winner_idx]

        distances = np.sqrt(np.sum((angles - winner_angle)**2, axis=1))
        return np.exp(-distances**2 / (2 * self.radius**2))

    def _calculate_grid_neighborhood(self, winner_idx):
        winner_pos = np.array([winner_idx // self.grid_size, winner_idx % self.
↪grid_size])

        neighborhood = np.array([[np.exp(-np.sum((np.array([i, j]) -␣
↪winner_pos)**2) /
                                    (2 * self.radius**2))
                                  for j in range(self.grid_size)]
                                 for i in range(self.grid_size)])

        return neighborhood.flatten()

    def _update_weights(self, input_vector, neighborhood):
        for i in range(self.num_neurons):
            self.weights[i] += (self.learning_rate * neighborhood[i] *
                                (input_vector - self.weights[i]))
```

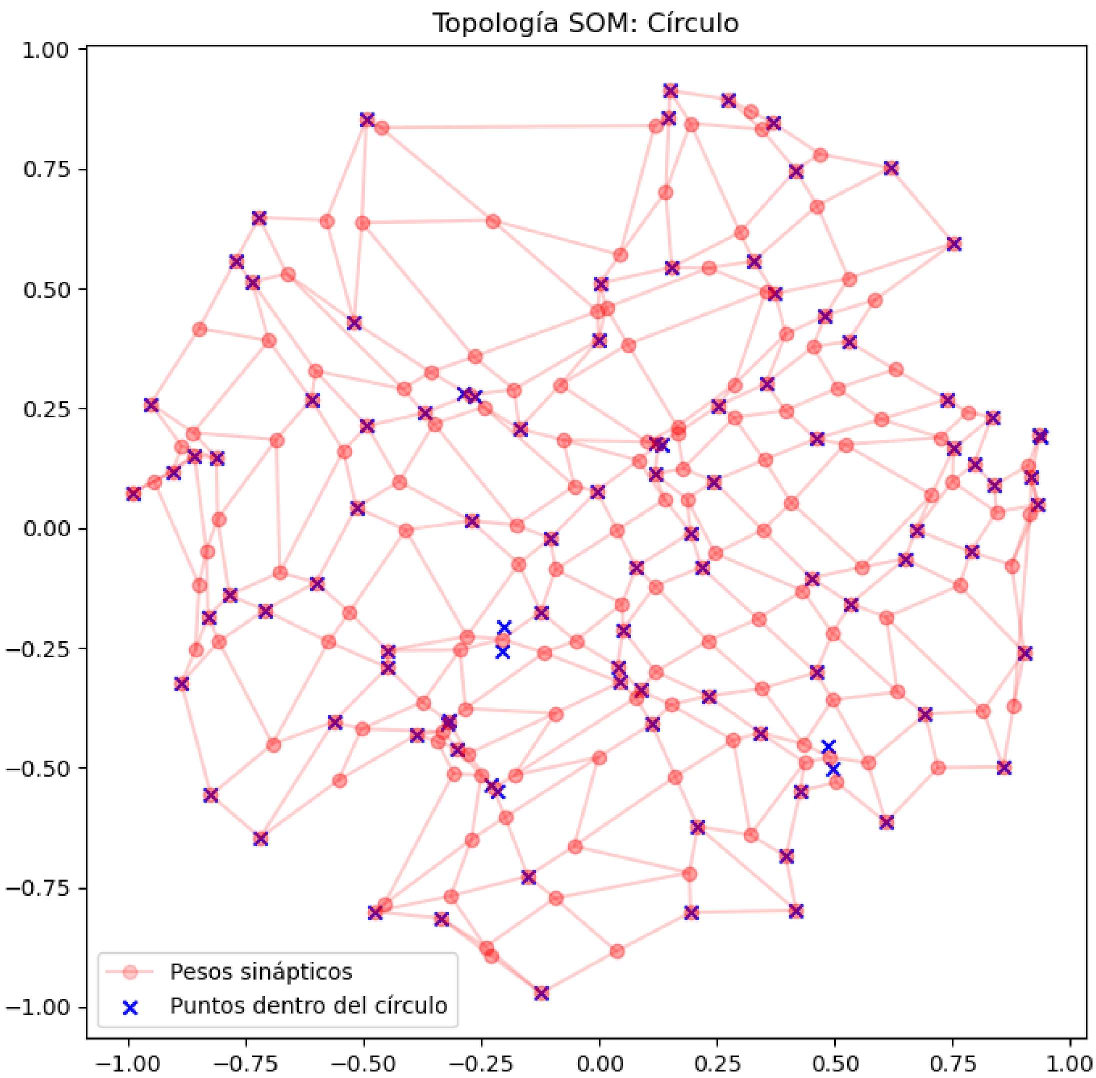
[44]:

```python
N = 100
M = int(math.sqrt(N))
x = np.random.uniform(-1, 1, N)
y = np.array([np.random.uniform(-math.sqrt(1 - v**2), math.sqrt(1 - v**2)) for␣
↪v in x ])
circle_arr = np.array([[x[i], y[i]] for i in range(N)])

som = SOM(2, 15*15, 10, 0.1, 0.01, 0.01)
W = som.train(circle_arr)
```

[203]:

```python
plt.figure(figsize=(8, 8))
plt.plot(W[:, 0].reshape(15, 15), W[:, 1].reshape(15, 15), c='red', marker='o',␣
↪alpha=0.2)
plt.plot(np.transpose(W[:, 0].reshape(15, 15)), np.transpose(W[:, 1].
↪reshape(15, 15)), c='red', marker='o', alpha=0.2)
plt.plot(W[0, 0], W[0, 1], c='red', marker='o', alpha=0.2, label = 'Pesos␣
↪sinápticos')
plt.scatter(circle_arr[:, 0], circle_arr[:, 1], c='blue', marker='x',␣
↪label='Puntos dentro del círculo')
plt.title("Topología SOM: Círculo")
plt.legend()
plt.show()
```

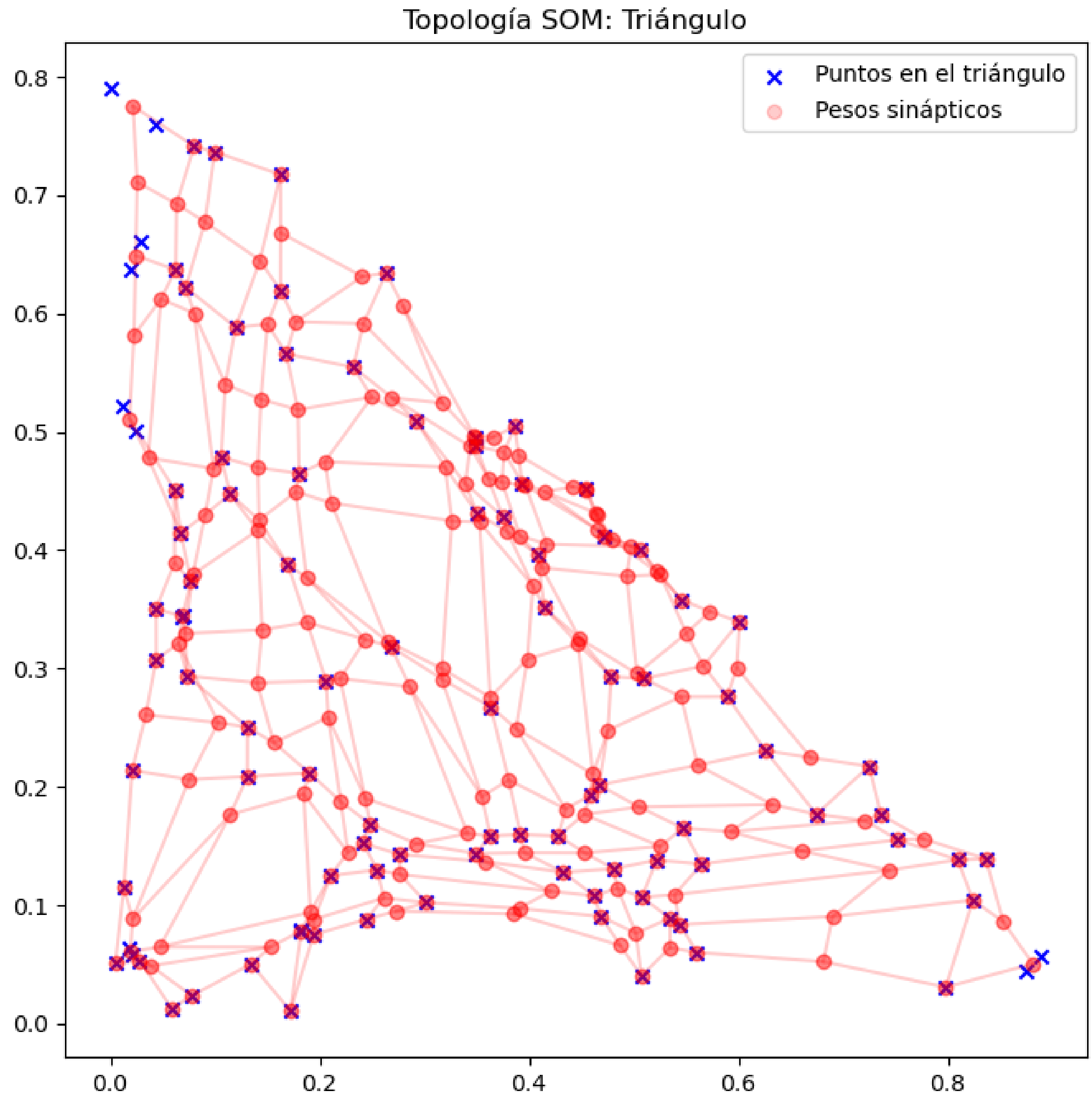


[ ]:
```
def generate_points_in_triangle(num_points):
    points = []
    for _ in range(num_points):
        r1, r2 = np.random.rand(2)
        if r1 + r2 > 1:
            r1, r2 = 1 - r1, 1 - r2
        x = r1
        y = r2 * (np.sqrt(3) / 2)
        points.append([x, y])
    return np.array(points)

num_points = 100
triangle_points = generate_points_in_triangle(num_points)
```

```
som2 = SOM(2, 15*15, 10, 0.1, 0.01, 0.01)
W2 = som2.train(triangle_points)
```

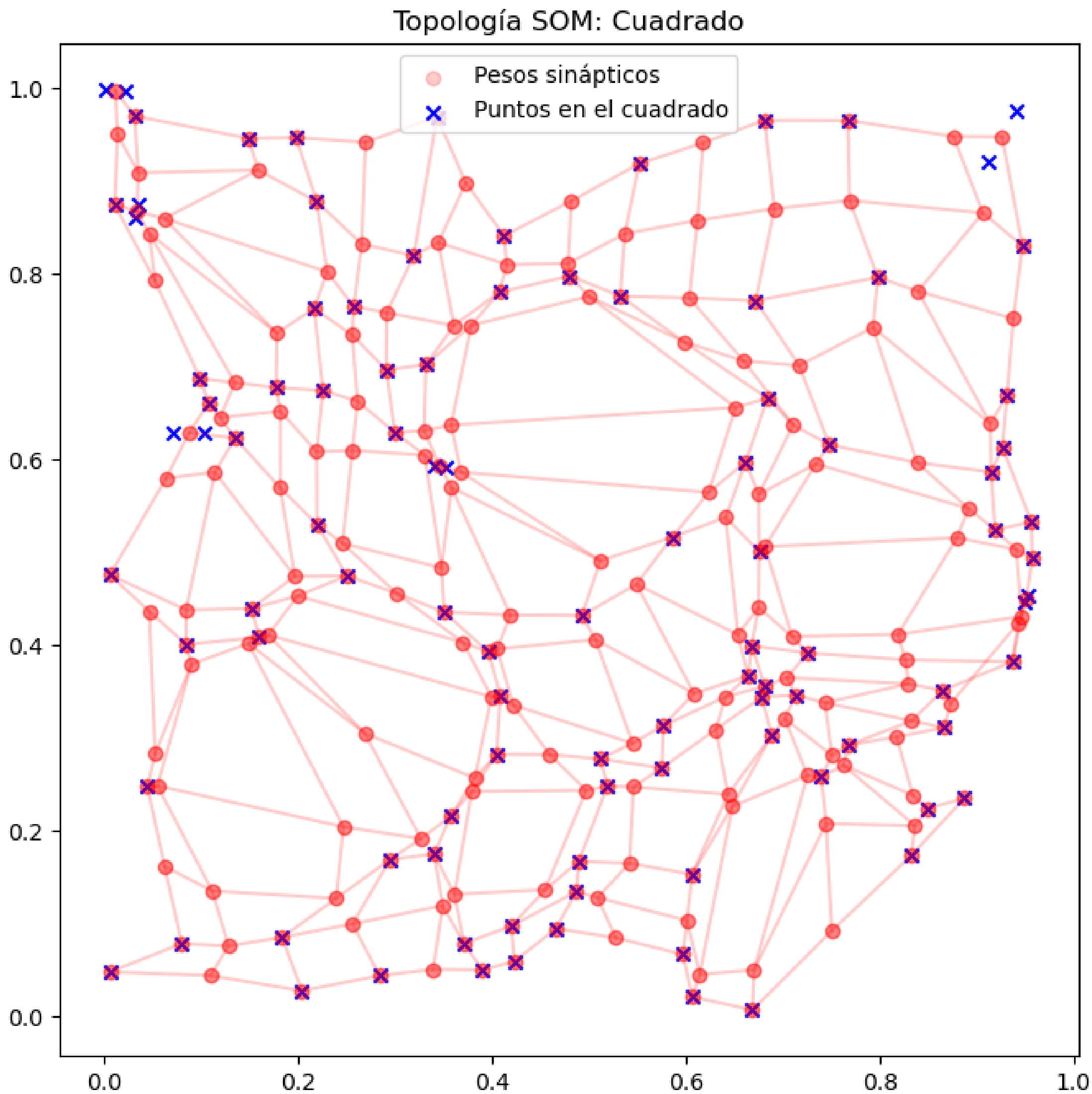
[207]:
```
plt.figure(figsize=(8, 8))
plt.scatter(triangle_points[:, 0], triangle_points[:, 1], c='blue', marker='x',␣
 ↪label='Puntos en el triángulo')
plt.plot(W2[:, 0].reshape(15, 15), W2[:, 1].reshape(15, 15), c='red',␣
 ↪marker='o', alpha=0.2)
plt.scatter(W2[:, 0].reshape(15, 15), W2[:, 1].reshape(15, 15), c='red',␣
 ↪marker='o', alpha=0.2, label = 'Pesos sinápticos')
plt.plot(np.transpose(W2[:, 0].reshape(15, 15)), np.transpose(W2[:, 1].
 ↪reshape(15, 15)), c='red', marker='o', alpha=0.2)
plt.title("Topología SOM: Triángulo")
plt.legend()
plt.show()
```



[194]:
```
num_points = 100
square_points = np.random.rand(num_points, 2)
som3 = SOM(2, 15*15, 10, 0.1, 0.01, 0.01)
W3 = som3.train(square_points)
```

[208]:
```
plt.figure(figsize=(8, 8))
plt.plot(W3[:, 0].reshape(15, 15), W3[:, 1].reshape(15, 15), c='red',␣
 ↪marker='o', alpha=0.2)
plt.plot(np.transpose(W3[:, 0].reshape(15, 15)), np.transpose(W3[:, 1].
 ↪reshape(15, 15)), c='red', marker='o', alpha=0.2)
plt.scatter(W3[:, 0].reshape(15, 15), W3[:, 1].reshape(15, 15), c='red',␣
 ↪marker='o', alpha=0.2, label = 'Pesos sinápticos')
```

```
plt.scatter(square_points[:, 0], square_points[:, 1], c='blue', marker='x',␣
 ↪label='Puntos en el cuadrado')
plt.title("Topología SOM: Cuadrado")
plt.legend()
plt.show()
```



Se puede ver que para estas redes, se acomodan los pesos sinápticos al rededor de los puntos de entrada. Además se puede ver como preserva la topología de los datos de entrada, es decir, los puntos que están cerca en el espacio de entrada, están cerca en el espacio de salida. Esto se debe a que la red Kohonen es una red auto-organizada que aprende la distribución de los datos de entrada y los agrupa en regiones similares.

2. Resuelva (aproximadamente) el "Traveling salesman problem" para 200 ciudades con una red de Kohonen

Para este ejercicio simplemente se genera una red de Kohonen de la que se entrenará con una función de vecindad circular para poder terminar e iniciar el recorrido en el mismo punto. Para 200 ciudades, se propone como criterio utilizar el doble de neuronas que la cantidad de puntos necesarios adecuarse simplemente para que cada ciudad o punto tuviese su propio peso sináptico asociado a sí mismo. Se puede ver como la red de Kohonen se acomoda a los puntos de entrada y se puede visualizar el recorrido óptimo a través de las ciudades. Sin embargo, se puede ver que la solución no es la más óptima, ya que la red Kohonen no es un algoritmo de optimización, sino de agrupamiento. Por lo tanto, se puede utilizar la red Kohonen para encontrar una solución aproximada al problema del vendedor viajero, pero no la solución óptima.

[5]:
```
num_cities = 200
cities = np.random.uniform(0, 1, size=(num_cities, 2))
som2 = SOM(2, 2*num_cities, 10, 0.1, 0.01, 0.01)
W_2 = som2.train(cities, circular=True)
```

[42]:
```
fig, axes = plt.subplots(1, 2, figsize=(16, 8))
axes[0].scatter(cities[:, 0], cities[:, 1], c='blue', marker='x',␣
 ↪label='Ciudades Aleatorias')
axes[0].legend()
axes[1].plot(W_2[:, 0], W_2[:, 1], c='red', marker='o', alpha=0.4, label='Ruta␣
 ↪Óptima')
axes[1].scatter(cities[:, 0], cities[:, 1], c='blue', marker='x',␣
 ↪label='Ciudades Aleatorias')
axes[1].legend()

plt.show()
```