Nonlinear Analysis of Concrete Face Rockfill Dam

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Abstract: The concrete membrane face rockfill dam, in spite of its increasing popularity with construction engineers, is perhaps one structure where the design is still largely empirical and is based on past experience rather than theory. Studies were carried out to understand the prototype behavior of this type of rockfill dam and to identify the various factors influencing the development of stresses and strains in the face slab and the rockfill embankment. Different types of elements are used to simulate the prototype behavior of various components of the dam in the sequential nonlinear analysis and some representative results are presented. The behavior of rockfill embankment on application of reservoir water load in case of face dam is compared with that for the case of central earth core dam. The effect of valley abutment slopes on the movement of rockfill and consequent development of stresses in the embankment in cross-valley transverse direction is brought out. Creep deformations in the rockfill occurring after filling of reservoir are found to be a major factor governing the development of stresses and deformations in the upstream face membrane.

Introduction

The concrete membrane face rockfill dam, due to its inherent advantages and greater suitability at many sites, over the central earth core type, has become increasingly popular during the last 15 years. It is in most cases the first choice of many dam-building agencies throughout the world, but it is perhaps the one structure least understood by the design engineers.

In contrast to the development of earth dams, the present state of art of concrete membrane face rockfill dam design has evolved through a process that is more evolutionary than revolutionary, in which engineering progress occurred principally as a consequence of cautious trials and errors on the basis of performance evaluation of existing dams (Cooke 1984). Thus, although several high face rockfill dams have been constructed in different parts of the world and are operating satisfactorily, notable among them being Cethana (Wilkins et al. 1973), Anchicaya (Regalado et al. 1982), and Areia (Pinto et al. 1982), each of which has contributed to the state of art of rockfill dam design through improved performance measurements, a comprehensive mathematical analysis, which could correctly predict the prototype behavior of such dam, is not yet available. The design of the concrete face is still largely empirical and is based on experience and judgment, modification to design practice being made in the light of performance of each new dam. For proposed concrete face rockfill dams of still greater height, even this experience is not available.

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Basically, the dam consists of a rockfill embankment with an upstream face membrane, usually of concrete, laid over a dense semipervious and nonerodible base made up of finer well-graded material. For face dams, the stability of the mass is generally not a problem as the rockfill embankments are inherently stable under all operating conditions. It is the concrete face membrane that is the critical member of this type of rockfill dam. For high dams, the main problems are cracking of the face and leakage, resulting from movement of underlying rock.

Studies were carried out to understand the prototype behavior of this type of rockfill dam and to identify the various factors influencing the development of stresses and strains in the face slab and the rockfill embankment. A variety of isoparametric, numerically integrated, curved, parabolic elements are used to correctly simulate the true prototype behavior of various components of the dam. Thus, while quadrilateral elements are used for the rockfill and joint elements for the face membrane-rockfill and rockfill-foundation interface, slender incremental elements are adopted to represent the thin concrete face membrane in the finite element analysis. The method of analysis was gradually developed from simple single-lift linear analysis to the more realistic analysis involving nonlinear material behavior for the rockfill and for the joint element at the membrane-rockfill interface, sequential construction, and incremental reservoir loading to simulate the construction process and filling of reservoir. Four different methods of predicting creep deformations in the rockfill embankment were proposed and a solution algorithm developed and incorporated in the computer program (Khalid 1983).

A brief outline of the various elements used in the analysis and of sequential nonlinear approach is presented in this paper, along with some representative results. The studies involving creep deformations in rockfill are being reported separately.

Design of Face Dam—A Review

A chronicle of modern rockfill dam design, including a description of the present practice of concrete face rockfill dam design is presented by Cooke (1984). Precedence has played an important part in the design of face dam, and attempts to evolve a rational method of analysis have been few.

Wilkins (1968) suggested a design method based on analysis of rockfill deformations. Noting that a mathematical analysis from fundamental principles is not yet possible, and using the fundamental work of Rowe (1962) and his own experience and observations, he attempted to provide an empirical basis for the prediction of settlements and deflections of rockfill under load.

However, Wilkins' calculations of movements in the plane of the face use an empirical relationship based on the observed deformation of Salt Springs dam. The latter are suspect, as face deformations might have been influenced by rockfill settlement resulting from high leakage (Boughton 1970). The very important Poisson's ratio effect also is not explicitly included. The assumptions for derivation of many of the formulas are arguable.

A somewhat different approach was adopted by Boughton (1970), who used the finite element method for the analysis of 36-m-high rolled rockfill Wilmot dam. His analysis, however, suffers from the lacuna of creep deformation of the rockfill not having been considered for the reservoir loading

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case. Further, he made no allowance for slippage along the face membrane-rockfill interface. The compatibility of deformations at the interface in his analysis, therefore, has a tendency to magnify load transfer effects from the rockfill to the concrete slab. The use of triangular elements for the rockfill and thin concrete face membrane further adversely affected the efficacy of his analysis, since the constant strain triangle, in addition to making the structure inherently stiff, cannot truly simulate the high strain gradients resulting from strain transfer between zones of differing stiffness in and around the concrete slab. His results indicated a maximum tensile strain of 0.00015, which is about the concrete failure strain. Much higher tensile strains were indicated on the higher dam analyzed.

Finite Element Method

Most applications of finite element method in soil and rock mechanics have been made by adopting, or suitably modifying, formulations developed for structural and continuum mechanics. Majority of nonlinear formulations by finite element method have been written in terms of displacements. The displacement model employing basic variational principle of minimum potential energy was therefore adopted for the analysis. The quadrilateral parabolic element used for the rockfill is described in the literature (Zienkiewicz 1979).

Joint Element

In order to allow for differential displacements along material interfaces, i.e., planes along which the boundaries of different materials with markedly different stiffnesses meet, joint elements as suggested by Zienkiewicz et al. (1970) and Goodman et al. (1968) are used. The element used in the analysis is isoparametric and numerically integrated and has not so far been used in the analysis of concrete face rockfill dam (Khalid 1983).

Incremental Elements

In case of elements with higher aspect ratios, presence of large off-diagonal terms associated with adjacent nodes across the thickness create ill-conditioning of stiffness matrix in the usual isoparametric elements. This results in loss of accuracy and eventually the round-off errors produce non-positive definite matrices. This can be avoided by eliminating the rigid body displacements across the thickness by making suitable modifications in the shape function and the strain displacement matrices **N** and **B** locally (Nayak and Agarwala 1976).

In case of the concrete face membrane, isoparametric elements become slender due to small thickness of face slab. In order to avoid ill-conditioning of the stiffness matrices in case of such elements, adopted to model the thin concrete face slab in the analysis, the displacements on the top surface are expressed as difference in the displacements across the thickness and the nodal variables on the bottom surface, adjacent to the rockfill embankment, are the usual displacements. With the aforementioned modification for re-conditioning of element, the appliled loads and reactions are accordingly transformed consistent with virtual work equation (Khalid 1983).

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TABLE 1. Hyperbolic Stress Strain Parameters for Rockfill Material

S1. number (1)	Parameters (2)	Rockfill material (3)
1	Unit weight, in kN/m^3	20
2	Cohesion c, in kN/m^2	—
3	Friction angle, (°)	38
4	Modulus number, K	2,500
5	Modulus exponent, n	0.25
6	Failure ratio, R_f	0.76
7	Poisson's ratio parameter, G	0.43
	Poisson's ratio parameter, F	0.19
	Poisson's ratio parameter, d	14.80

Nonlinear Constitutive Relationships

The procedure usually in vogue for incorporating the nonlinear material properties of the rockfill in the analysis, using functional form, is based on hyperbolic stress-strain function proposed by Kondner (1963), developed by Duncan and Chang (1970), and extended by Kulhawy and Duncan (1972). It uses Mohr Coulomb failure criterion and, utilizing the experimental studies by Janbu (1963), develops relationship for tangent modulus, E_t, which in the functional form is given by

$$E_t = \left[1 - \frac{R_f(1 - \sin\phi)(\sigma_1 - \sigma_3)}{2c\cos\phi + 2\sigma_3\sin\phi}\right]^2 Kp_a\left(\frac{\sigma_3}{p_a}\right)^n \quad \text{(1)}$$

where, R_f = failure ratio with a value invariably less than unity; p_a = atmospheric pressure expressed in same units as E_i and σ_3; K = a modulus number; and n = exponent determining the rate of variation of initial tangent modulus E_i with σ_3.

Kulhawy et al. (1969) have developed an expression for obtaining the variation of tangent Poisson's ratio with stress as a means for reproducing volume changes that occur in the triaxial compression test, which in the function form is given by

$$\nu_t = \frac{G - F\log\left(\frac{\sigma_3}{p_a}\right)}{\left\{1 - \frac{d(\sigma_1 - \sigma_3)}{Kp_a\left(\frac{\sigma_3}{p_a}\right)^n\left[1 - \frac{R_f(1 - \sin\phi)(\sigma_1 - \sigma_3)}{2c\cos\phi + 2\sigma_3\sin\phi}\right]}\right\}^2} \quad \text{(2)}$$

The actual determination of the stress-strain parameters for Cethana dam materials would require performance of tests on large-size triaxial testing machine to be representative of the prototype, where the main body of the embankment consisted of well-graded rockfill, with maximum size of 600 mm placed in layers 0.9-m thick (Wilkins et al. 1973). Since results of any such tests for Cethana rockfill were not available, the values used by Sharma (1976), for the analysis of the proposed 260.5-m-high Tehri Dam, were adopted (Table 1).

The effect of intermediate principal stress is allowed for by taking the

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confining pressure as the average of the minor principal stress (σ_3) and the intermediate principal stress (σ_2). The finite element analysis is not applicable to materials with Poisson's ratio equal to 0.5 as all the stiffness coefficients become infinite for $\nu = 0.5$. It has, however, been observed with problems in linear media that the finite element method agrees reasonably with elastic theory for values of ν up to 0.485. In the computer program, the upper and lower limits of Poisson's ratio are, therefore, fixed as 0.485 and 0.18, respectively.

In the absence of any data on joint material parameters for Cethana dam, a somewhat simplified form of expression was adopted for the tangential shear stiffness as

$$K_{st} = K_j \cdot \gamma_w \left(\frac{\sigma_n}{p_a}\right)^n \qquad (3)$$

where K_j and n = dimensionless constants, γ_w = the unit weight of water; p_a = the atmospheric pressure expressed in the same units as normal stress σ_n. The value of exponent n was taken as unity and so the tangential shear stiffness of the joint element at the face membrane-rockfill interface, K_{st}, is directly proportional to the normal stress, σ_n, at that point due to water load. Since under the action of normal water load friction on the underside of concrete face slab was considered sufficient to provide restraint against membrane movement relative to the rockfill (Wilkins et al. 1973), a high value of tangential shear stiffness with K_j = 8,000 was initially adopted for the analysis.

Solution Algorithm for Sequential Nonlinear Approach

The residual force approach presented by Nayak and Zienkiewicz (1972) in the context of plasticity problems is modified for the sequential analysis. In this method, the total equilibrium of the assembled structure at any stage of construction is considered at every stage of iteration along with the assumption that the constitutive law allows incremental stress at any point, $\Delta(\sigma)$, to be determined uniquely in terms of incremental strain, $\Delta(\epsilon)$ and consequently in terms of the incremental nodal displacements, $\Delta(\delta)$.

The nodal forces (R), of total assembled structure of volume V, at any stage of construction are equilibrated and the principle of virtual work yields

$$d(\delta)^T(R) - \int_V d(\epsilon)^T(\sigma)dV = 0 \qquad (4)$$

or

$$[\psi(\delta)] = (R) - \int_V \mathbf{B}^T(\sigma)dV = 0 \qquad (5)$$

This expresses the residual vector, ψ, as an implicit function of displacements. Any problem with nonlinear stress-strain laws involves the solution of equilibrium Eq. 5, together with appropriate constitutive law. In sequential construction the volume of the body also changes from layer to layer. At any stage of construction the residuals ψ, are made nearly zero by assuming a set of iterative displacements

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TABLE 2. Percent Norm of Residuals Obtained at End of Each Iteration

Iteration number (1)	Layers I (2)	II (3)	III (4)	IV (5)	V (6)	VI (7)
(*a*) End of Construction Condition						
1	17.01	50.18	63.97	26.45	13.36	2.48
2	2.44	45.03	15.41	12.03	6.69	0.89
3	0.72	5.35	10.73	9.25	4.65	0.44
4	—	1.63	2.06	9.13	27.74	0.31
5	—	4.50	1.10	2.32	9.48	0.21
6	—	0.98	0.67	1.39	5.49	0.12
7	—	—	—	0.73	4.94	0.05
8	—	—	—	—	0.25	—
(*b*) Reservoir Full Condition						
1	1.46	23.75	4.53	7.53	4.06	—
2	0.56	12.39	1.86	1.85	3.40	—
3	—	14.37	1.09	2.04	0.60	—
4	—	3.78	0.80	0.08	1.29	—
5	—	1.49	—	—	1.19	—
6	—	0.61	—	—	1.72	—
7	—	—	—	—	0.04	—

$$\Delta(\delta)_i = \mathbf{K}'^{-1}\psi \quad (6)$$

where $\mathbf{K}'$ = a suitable matrix.

The norm of the residual is calculated for the iteration i, as $\|\psi\|_i = (\psi_i^T\psi_i)^{1/2}$ and compared with the norm of load applied that is given as $\|R\| = [(R)^T(R)]^{1/2}$. The convergence factor is given by percent factor as $(C_R)_i = (\|\psi\|_i \times 100)/\|R\|$. The convergence factors obtained at the end of each iteration, for Cethana dam's deepest cross section, for both are end-of-construction and reservoir-full conditions, are shown in Table 2. A rapid and good convergence is obtained in both cases (Khalid 1983).

Results and Analysis

General

Cethana dam was chosen as a case study, to facilitate comparison with the results of measurements of extensive instrumentation system installed at Cethana for monitoring the prototype performance of face dam (Fitzpatrick et al. 1973). The concrete face slab was assumed to be constructed in the analysis, after the construction of rockfill embankment section.

Rockfill Dam Cross Section

The details of discretization of Cethana dam's deepest cross section are shown in Fig. 1. The number of parabolic elements is 35 for the end of construction and 47 for the reservoir full condition.

End of Construction Condition

For the end of construction condition the stresses and deflections in the upstream half of the dam could be expected to mirror those in the down-

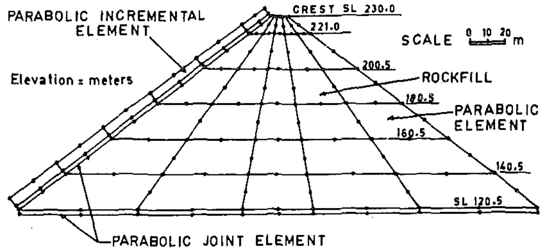

FIG. 1. Cethana Dam Section Showing Discretization (Concrete Face Slab Thickness Is Not to Scale)

stream half. This is reflected in the contours of calculated stresses and deflections (Figs. 2–4).

For corresponding points at equal depth below the top of the dam, horizontal stress is greater at locations close to the abutments as compared to that at center of valley. This is logical, since in case of dam cross section through the abutments the lateral movement of rockfill at any point is relatively restrained, as indicated by comparatively greater magnitude of shear stress on account of its proximity to the rigid foundations; whereas a much greater depth of flexible rockfill material, in case of dam cross section through the center of valley, allows more lateral movement of rockfill at that point, resulting in reduction of horizontal stress (Figs. 2 and 3).

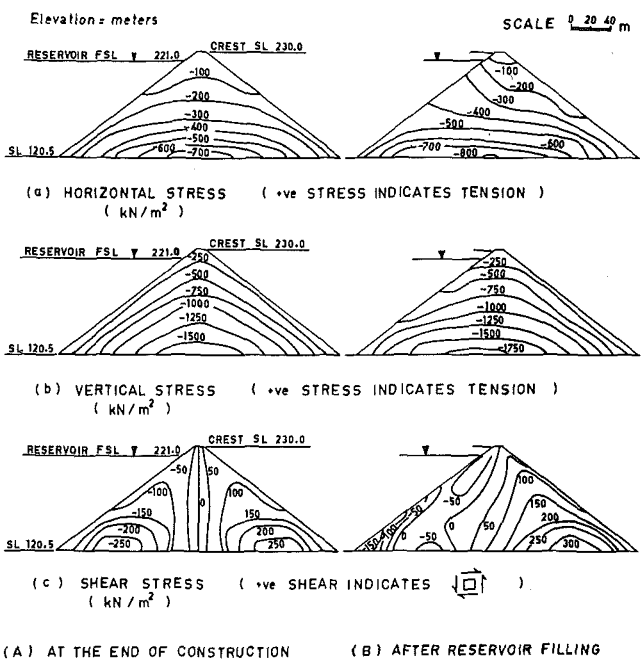

FIG. 2. Contours of Stresses in Cethana Dam Deepest Cross Section

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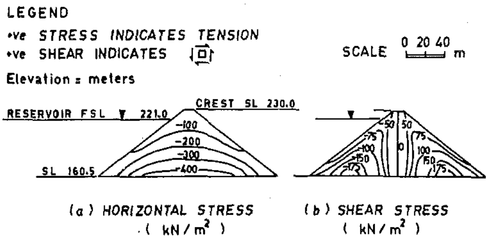

FIG. 3. Contours of Stresses in Cethana Dam Cross Section of 69.5-m Height at End of Construction

For points at the same horizontal elevation, the ratio of vertical stress to the depth of overburden rock is less in central region and increases for points near the two dam faces, thereby indicating that in case of embankments made of flexible material, part of the weight of embankment material coming over to the central portion is thrown to the sides (Fig. 2).

The shear stress is zero at the central line of dam and increases towards the two faces, until the maximum value of both negative and positive shear is reached at points midway between center line and embankment slopes (Figs. 2 and 3).

The horizontal displacement is zero at the center line of dam section and increases towards either face, indicating spreading of rockfill from the central region towards both the slopes. The maximum calculated horizontal displacement occurs at the dam faces, at $0.45H$ above the base, where H is the height of the dam. The calculated maximum vertical settlement is 0.275% of height and occurs at $0.65H$ height, on the center line of dam section. Settlement of rockfill interests the builder, who must add extra rock to make up for reduction in height. Trouble may also be experienced if concrete face is constructed concurrently with the fill (Fig. 4).

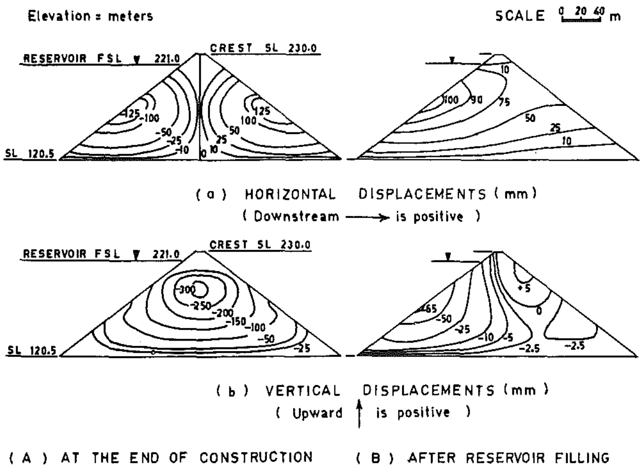

FIG. 4. Contours of Displacements in Cethana Dam Deepest Cross Section

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Reservoir Full Condition

The stresses for reservoir loading case also include the stresses at the end of construction, which were fed as initial condition in the computer for the analysis of reservoir full condition. The displacements due to water load are, however, calculated separately.

Both the horizontal and vertical stresses in the upstream half of rockfill embankment, increase considerably on application of reservoir water load. In the downstream half of the dam cross section, however, this increase is only marginal. This is in contrast to the behavior of central earth core type of rockfill dam, where the reservoir water load causes reduction of both the horizontal and vertical stresses in the upstream shell and increases the stresses in the downstream shell (Sharma 1976). The reduction of stresses in the upstream shell in case of central earth core dams is attributed to buoyancy effects (Nobari and Duncan 1972). As the water load is applied on the core face, it is passed in the downstream direction, thereby increasing the stresses in the downstream shell and relieving the upstream shell portion. In case of membrane face dams, on the other hand, the water pressure is resisted on the upstream face and the total force exerted by the reservoir on the dam is directed downwards with a much greater inclination, thereby increasing the stresses in the upstream portion, with only marginal effect in the downstream portion of the dam (Fig. 2).

The reservoir load pushes the dam in the downstream direction resulting in development of positive shear in the whole of embankment cross section, increasing the initial positive shear in the downstream portion and considerably decreasing the initial negative shear in the upstream portion of the embankment cross section (Fig. 2).

Both the maximum horizontal and vertical displacements due to water load occur at the upstream face at about midheight of dam (Fig. 4).

Cross-Valley Transverse Dam Section

For analysis in the cross-valley transverse direction, vertical longitudinal sections, through the concrete face, and parallel to the dam centerline are considered. Vertical water load is applied on the thin horizontal strip of concrete slab, at the top of such cross-valley transverse dam sections. The contours of stresses and displacements in the rockfill, for reservoir full condition, in case of one such representative cross-valley section, which intersects the upstream concrete face slab at SL 210.5, are presented (Fig. 5).

The calculated horizontal and vertical displacements of the rockfill in the cross-valley transverse dam section indicate movement of the rockfill along an inclined path from the abutment towards the center of the valley. Such cross-valley movement of rockfill, as predicted by the analysis, has also been reported earlier in the case of clay core (Wilson and Squire 1969; Dolezalova 1970).

The horizontal component of movement of rockfill adjacent to the face slab, due to application of reservoir load, would transfer the movements to the concrete slab. These horizontal displacements of the face membrane in the cross-valley direction, are distributed in such a way that strains change their sign leading to compression in the central portion of the face membrane and tension along the entire perimeter of the contact of the face slab with the sloping abutments.

At any horizontal elevation, the horizontal stress in the rockfill is seen to

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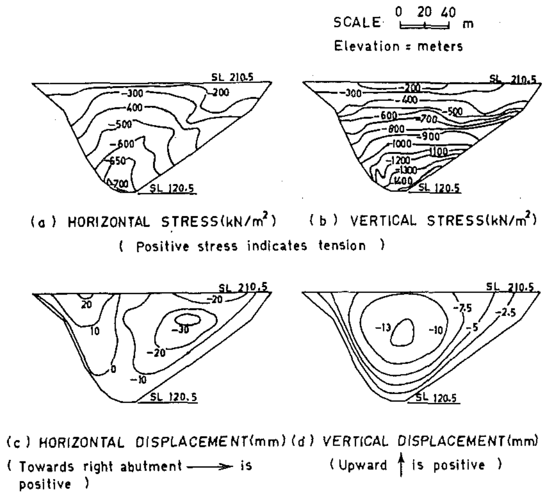

FIG. 5. Contours of Stresses and Displacements for Reservoir Full Condition, in Cross-Valley Transverse Dam Section that Intersects Concrete Face Membrane at SL 210.5

be minimum at the abutments and increases gradually towards the center of the valley. The contours of vertical stress that are generally horizontal everywhere, are inclined in the downward direction in the region close to steep left abutment indicating that for points having constant overburden rock, the minimum value of vertical stress occurs in region close to steep left abutment. This reduction in vertical stress is attributed to the fact that in case of steep left abutment, the rock can more easily slide towards the deepest point of the valley, and thus relieve to some extent the vertical stress at that point in the rockfill, while the vertical movement of rockfill elsewhere is relatively restrained, resulting in higher value of vertical stress, in spite of equal depth of overburden rock.

Concrete Face Membrane

Calculated displacements of the upstream and downstream faces of the dam and membrane slope stress for the deepest dam cross section, due to application of reservoir water load, are compared with the available observed values for the period since commencement of filling of reservoir on February 4, 1971–December 8, 1971. The reservoir filling at Cethana was completed on April 25, 1971.

The term *slope deflection* used here indicates the deflection in the direction from the crest to the toe in the plane of the membrane and normal to dam axis, while the term *normal deflection* indicates the deflection in the direction normal to the plane of the face.

The calculated slope deflection curves for the concrete face slab and the upstream face of rockfill embankment are shown in Fig. 6, for different stages of reservoir filling. The intercept between the two curves at any point gives the amount of slippage between the underside of the concrete slab and the rockfill upstream face.

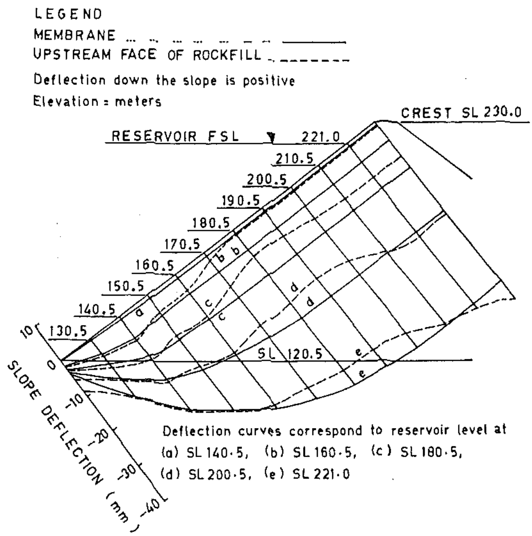

FIG. 6. Slope Deflection of Concrete Face Membrane and of Upstream Face of Rockfill for Different Stages of Reservoir Filling

The pattern of calculated and measured normal deflection of the upstream face membrane is generally similar (Fig. 7). The normal deflection, which is small for lower reservoir elevation, increases rapidly as the reservoir level rises. When the reservoir is filled to two-fifth of its normal depth, in the analysis, a slight upward bulging out of the concrete face membrane is indicated in a short stretch close to the toe of the slab. The kink formed is also indicated in the calculated normal deflection profiles of the membrane for higher reservoir elevations, although the normal deflection is everywhere

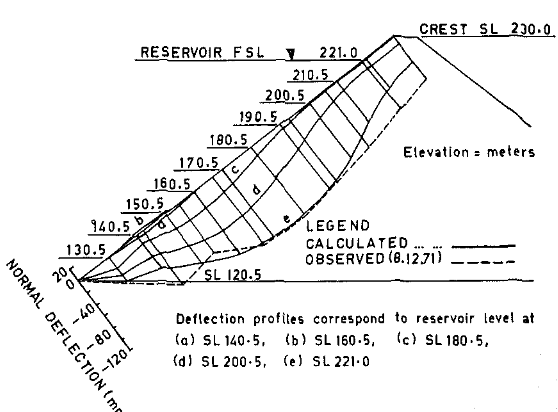

FIG. 7. Normal Deflection of Concrete Face Membrane for Different Stages of Reservoir Filling

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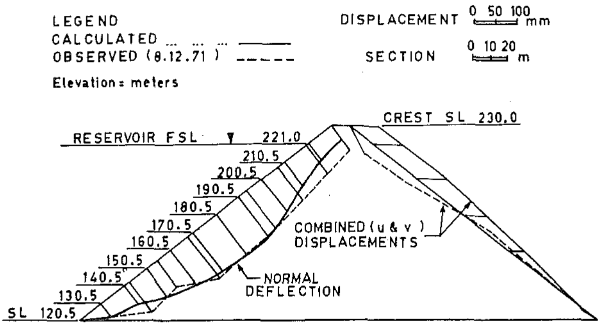

FIG. 8. Normal Deflection of Concrete Face Membrane and Displacement of Downstream Face of Rockfill Embankment

in the downward direction. The pronounced link is also evident at about the same location in the observed normal deflection profile of the concrete face membrane for the deepest section, as well as in case of the other two adjoining slabs for which measurements are available (Fitzpatrick et al. 1973).

The nonlinear sequential analysis, however, failed to correctly predict even the true nature of displacements of the downstream face as well as that of the top of dam. The calculated displacements of the downstream face due to reservoir water load are, all along the downstream face in the downstream direction, whereas observations at Cethana indicate downward settlement of the downstream face except for slight bulging out of the lower portion of the downstream face (Fig. 8). Again, the nonlinear analysis indicated that on application of reservoir water load the downward settlement of the top of the dam is practically zero. This is contrary to the truth. Measured deflections at Cethana, and a large number of other dams, indicate continued downward settlement of the top of the dam both during and after reservoir filling period (Fitzpatrick 1972).

Further, with this scope, the analysis indicated that the slope deflection of the face membrane due to reservoir water load, increases from a negligible value at the toe to the maximum value at the crest (Fig. 6), thereby causing stretching of the face membrane and consequently resulting in development of calculated tensile stresses all along the membrane length in slope direction, contrary to observations at Cethana, which indicate compression in the central portion of the membrane (Fig. 9) (Fitzpatrick et al. 1973).

Thus it became apparent that the nonlinear analysis has ignored and not taken into account some important major factors governing the development of compressive stresses in the concrete face membrane. A major problem thus faced in the course of this study was to identify and include these factors in the analysis.

During the initial stages of the study it was considered that at least a part of the compressive stresses in the central portion of the concrete face could probably be attributed to the bending stresses in the face slab that might develop on application of reservoir load. If this were true, the face slab could be treated for the purpose of finite element analysis, as a plate on elastic foundation. To ascertain the true facts in this regard, Cethana dam cross section was analyzed under reservoir full condition and a very small value

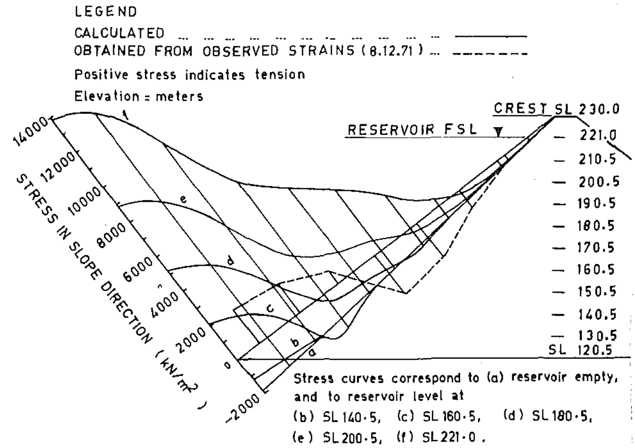

FIG. 9. Slope Stress in Concrete Face Membrane for Different Stages of Reservoir Filling

of stiffness constant, K_j, equal to 0.08 was assigned to the joint element at the face slab-rockfill interface. This had the effect of making the interface comparatively very smooth, and the slab was thus not constrained to follow the rockfill strains in the plane of the face by development of shear force between the underside of the slab and the rockfill. The in-plane stresses thus eliminated in the analysis, the remaining stresses in the slab would be due to bending only and these were found to be of negligible order. The absence of any bending stresses in the slab is due to the reason that the slab is assumed to be uniformly supported against the normal water load. This assumption seems reasonable in case of well-compacted rolled rock face of small graded rock that would provide a firm uniform-bearing surface for the slab, in contrast to the earlier practice of using a placed rock face to support the concrete where one unstable rock fragment could remove support from a significant area of the slab.

The discrepancy in the computed and observed values of membrane slope stress is, therefore, attributed to creep settlements of rockfill occurring after the application of reservoir water load. The downslope component of this downward creep settlement would result in development of compressive stresses in the face membrane in slope direction. Time-dependent (creep) deformations of rockfill were, therefore, included in the analysis. Four different methods of creep analysis were proposed and incorporated in the computer program (Khalid 1983).

In the absence of availability of rockfill properties for Cethana, the use of lower values of elastic modulus for rockfill in the analysis gave high values of tensile stresses near the toe, as compared to the reasonable values obtained from observed strains at Cethana. Again, a rather high value of stiffness constant, K_j, used in the analysis, prevented slippage at the face slab-rockfill interface, and so the de facto compatibility of deformations at the interface, had a tendency to magnify load transfer effects from the rockfill to the concrete slab. These two factors are (1) Reduced stiffness of the rockfill embankment; and (2) the excessive friction at the face slab-rockfill interface combined in the analysis to greatly magnify the calculated values of tensile stresses in the concrete face slab near the toe. Smaller value of stiffness constant, K_j, and improved properties of rockfill, when used in the analysis, considerably reduced the high values of calculated tensile stresses in the face membrane (Khalid 1983). While considering the results of Cethana dam, therefore, it was considered necessary to carry out parametric studies for the interfacial conditions of the membrane and the rockfill, and for the rockfill material constants, as the actual material constants for Cethana dam were not available. The creep and parametric studies are being reported separately.

Limitations of Two-Dimensional Analyses

The conventional two-dimensional (plane strain) analysis of an embankment is applicable to the case of a dam in a wide valley with large length–height ratio. The method will result in conservative values for an embankment section in a narrow valley due to load transfer to the sides. The extent of such load transfer, if known, can lead to large economy with greater safety margins. Three-dimensional analysis is, therefore, necessary to study the extent of load transfer to the abutments, and magnitude of vertical stresses to ensure adequate abutment contact for different valley slopes and embankment heights.

Conclusions

In this study, an attempt has been made to understand the various factors governing prototype behavior of concrete face rockfill dam. A variety of isoparametric, numerically integrated, curved, parabolic elements are used in the analysis to correctly model the various components of the dam, such as the thin concrete face slab, rockfill embankment, and the concrete face slab-rockfill interface. Sequential construction and incremental reservoir loading are adopted to truly simulate the construction process and filling of reservoir. Nonlinear material behavior is considered for the rockfill and for the joint element at interface. Residual force approach is used in the analysis and the specified value of convergence factor is kept at 0.1%. A rapid and good convergence is obtained in both cases.

The behavior of rockfill embankment on application of reservoir water load in case of face dam is compared with that for the case of central earth core dam. The effect of valley abutment slopes on the movement of rockfill and consequent development of stresses in the embankment in the cross-valley transverse direction is brought out.

The incremental element used for modeling face dam can model bending and inplane behavior. The overall action of the dam with membrane introduces negligible bending stress in the concrete slab. Creep deformations in the rockfill occurring after filling of reservoir are found to be a major factor governing the development of stresses and deformations in the upstream face membrane.

The study has brought into sharp focus the paucity of laboratory data for nonlinear material behavior of rockfill and interface joint element. Much work is, therefore, expected on this aspect from material scientists.

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A review of present state of knowledge of the design of concrete membrane face rockfill dam has been included. It is indicated that at present, there is no theoretical procedure sufficiently tested by practice that could permit the computation of various aspects of the design of face membrane.

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Appendix I. References

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APPENDIX II. NOTATION

The following symbols are used in this paper:

$\mathbf{B}$ = strain displacement matrix;
$(C_R)_i$ = convergence factor for iteration i;
c = cohesion;
E_t = tangent modulus;
G, F, d = Poisson's ratio parameters;
K = modulus number;
K_j = joint stiffness constant;
K_{st} = tangential joint shear stiffness;
$\mathbf{K}'$ = assembled stiffness matrix;
N = shape function;
n = exponent;
p_a = atmospheric pressure;
R = nodal forces of total assembled structure of volume V;
R_f = failure ratio;
γ_w = unit weight of water;
ν_t = tangent Poisson's ratio;
ϕ = friction angle;
$\Delta(\sigma), \Delta(\epsilon)$ = incremental stress and strain at any point;
$\Delta(\delta)$ = incremental nodal displacement;
σ_1, σ_3 = major and minor principal stress;
σ_n = normal stress; and
$\boldsymbol{\psi}$ = residual force vector.