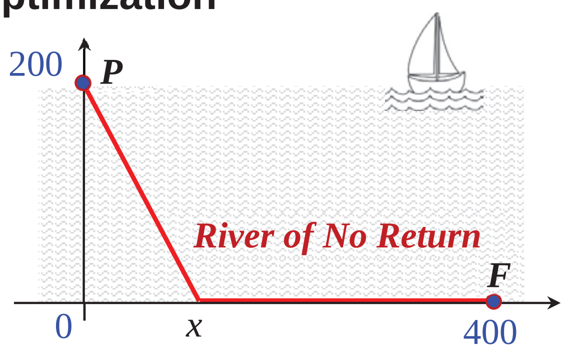
Session 2. Basic Math and Probability

We first need to review some basic topics in mathematics and probability which will be useful in later sessions.

Ex 1] Single-Variable Optimization

A powerhouse, *P*, is on one bank of a straight river 200 *m* wide, and a factory, *F*, is on the opposite bank 400 *m* downstream from *P*.



The cable has to be taken across the river, under water at a cost of \$6/*m*. On land the cost is \$3/*m*. What path should be chosen so that the total cost is minimized?

- Total cost $f(x)$ =
- Optimal solution x^* =

Ex 2] Chuck-a-Luck

Three fair dice are rolled in a wire cage. You place a bet on any number from 1 to 6. If any one of the three dice comes up with your number, you win the amount of your bet. (You also get your original stake back.) If more than one die comes up with your number, you win the amount of your bet for each match. (e.g., if you had a \$1 bet on number 6, and each of the three dice came up with 6, you would win \$3.) If you bet \$1, what is your expected payoff?

Matches, X	0	1	2	3	Expected payoff
P[X]	125/216	75/216		1/216	
Payoff, Y	0	\$2	\$3	\$4	

A. Review of Basic Math

* Derivative of a Function

	Function, $f(x)$	Derivative of Function
1	a	0
2	x	1
3	$a\,f(x)$	$a\,f'(x)$
4	$f(x) + g(x)$	$f'(x) + g'(x)$
5	x^n	$n\,x^{n-1}$
6	$f(x)\,g(x)$	$f'(x)\,g(x) + f(x)\,g'(x)$
7	$[\,f(x)\,]^n$	$n\,[\,f(x)\,]^{n-1} f'(x)$
8	$e^{f(x)}$	$e^{f(x)}\,f'(x)$
9	$ln\ f(x)$	$f'(x)\,/\,f(x)$

* Finding Maxima and Minima using Derivatives

- In a *smoothly* changing function, a low point (a minimum) or a high point (a maximum) is where the function flattens out.

- Where does it flatten out and the slope is zero?
  The derivative $f'(x)$ of the function at that point is zero!

- When a function's slope is zero at x, and the second-order derivative $f''(x)$ at x is:

  - less than 0, it is a local maximum.
  - greater than 0, it is a local minimum.
  - equal to 0, then the test fails.

(There are other ways of finding out in Session 7.)

Ex 1] Minimization Problem

Consider the function of x: $f(x) = x^2 - 4x + 5$.

- *First-order* derivative of $f(x)$

  $f'(x) = \frac{d}{dx} f(x) =$ Thus, $x^* =$

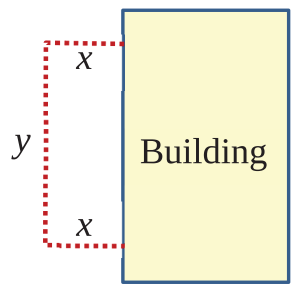
- *Second-order* derivative of $f(x)$

  $f''(x) = \frac{d}{dx} f'(x) =$

  Thus, $x^*=2.0$ is

Ex 2] Maximization Problem

We need to enclose a field with a fence. We have 500 feet of fencing material, and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area.



- Maximize the area, $f(x, y) = xy$

  subject to a constraint, $2x+y=500$ (or $y = 500 - 2x$)

- *Max* $f(x) = x\,(500-2x) = 500x - 2x^2$

- *First-order* derivative of $f(x)$

  $f'(x) = \frac{d}{dx} f(x) =$ Thus, $x^* =$

- *Second-order* derivative of $f(x)$

  $f''(x) = \frac{d}{dx} f'(x) =$

  Thus, $x^*=125$ (and $y^*=250$) is

* Review of Integral Calculus

In the following formulas, *f* and *g* represent *functions* of *x*, while *a*, *C*, and *n* represent fixed real numbers.

1. $\int a\, dx = ax + C$

2. $\int a\, f(x)dx = a \int f(x)dx$

3. $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$

4. $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$, except $n = -1$

5. $\int x^{-1}\, dx = \ln x + C$

6. $\int e^{ax}\, dx = \frac{1}{a} e^{ax} + C$

* Distribution of Random Variable

(a) Probability density function (*pdf*): $f(x)$

$$f(x) = \frac{d}{dx} F(x)$$

▪ It represents the *probability density* (i.e., height) evaluated at a certain point *x*.

(b) Cumulative distribution function (*cdf*): $F(x)$

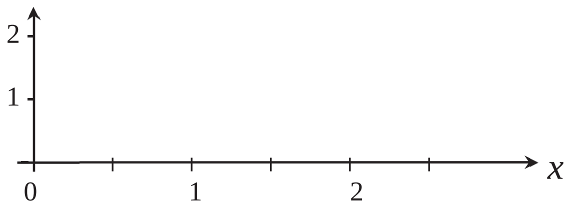
$F(x) = P[X < x]$ or $F(x) = \int f(x)dx$

▪ It represents the probability that a random variable *X* will be less than a certain value *x*.

Ex] Triangular Distribution: Suppose the probability density function (*pdf*) of a random variable *X* is given by

$$f(x) = \begin{cases} 4 - 2x & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the *probability density function f*(*x*).



(b) Find the *cumulative distribution function F*(*x*).

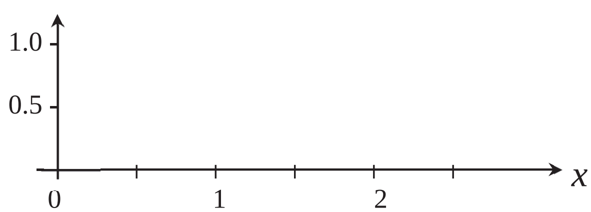
$F(x) = \int f(x)dx = \int (4 - 2x)dx =$

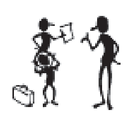
- At the *lower* limit x=1, $F(x)$ should be 0
  or $4x$-x^2+constant = 0 if x=1.

- At the *upper* limit x=2, $F(x)$ should be 1
  or $4x$-x^2+constant = 1 if x=2.

Thus, $F(x)$ =

(c) Sketch the *cumulative distribution function F*(*x*).





\# **Genius**? Find the *mean* and the *median* of the distribution!

B. Review of Linear Algebra

I. Vectors

- **Definition**

A vector **x** of order (m×1) is an ordered set of real numbers ($x_1, x_2, \ldots, x_m$), and is defined by a boldfaced, lowercase letter.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ . \\ x_m \end{bmatrix}$$

- **Length** of a vector

The *length* (or magnitude) of a vector **x** of m elements emanating from the origin is given by the Pythagorean formula:

$$L_{\mathbf{x}} = \|\mathbf{x}\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_m^2}$$

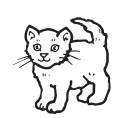
- **Vector addition**

The sum of two vectors **x** and **y**, each having the same number of entries, is the vector

$\mathbf{z} = \mathbf{x} + \mathbf{y}$ with ith entry $z_i = x_i + y_i$.

Ex] Let **x**'= [3, 4] and **y**' = [7, 1].

- $L_{\mathbf{x}}$ =
- **x**+**y** =
- **x**-**y** =



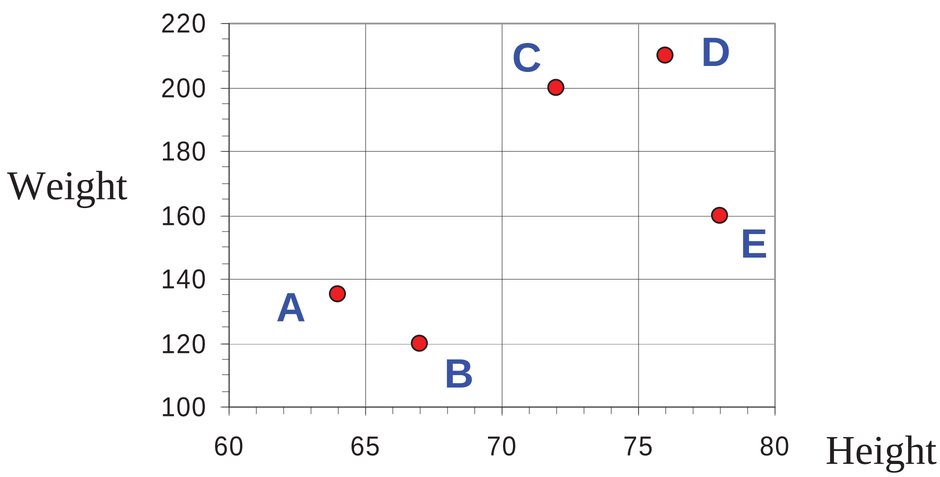
- Length of the vector **x**-**y** = ||**x**-**y**|| =

Ex] Cluster Analysis

Suppose that five students in the class have the following characteristics:

		Variables					
		Height	Weight	Gender	Eye color	Hair color	Handedness
Items	Amber	64	135	F	green	blond	right
	Brad	67	120	M	brown	brown	right
	Cindy	72	200	M	blue	blond	right
	Doug	76	210	M	brown	brown	right
	Emily	78	160	F	brown	brown	left

Using only the heights and weights in the table, calculate the *Euclidean* (i.e., straight line) *distances* between pairs of students.



	Amber	Brad	Cindy	Doug	Emily
Amber	0	15.30	65.49		
Brad	15.30	0	80.16	90.45	41.48
Cindy	65.49	80.16	0	10.77	40.45
Doug	75.95	90.45	10.77	0	50.04
Emily	28.65	41.48	40.45	50.04	0

▪ Possible clusters?

II. Matrices

▪ **Definition**

A matrix $\mathbf{A}$ of order $m \times n$, generally denoted by a boldface uppercase letter, is a rectangular array of elements arranged into m rows and n columns:

$$\underset{(m\times n)}{\mathbf{A}} = \begin{bmatrix} a_{11} & a_{12} & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{m1} & a_{m2} & \cdot & a_{mn} \end{bmatrix}$$

- If $m = n$, then the matrix is called a *square* matrix.

- If $a_{ij} = a_{ji}$ for all elements of a square matrix, then the matrix is called a *symmetric* matrix.

- If $a_{ij} = 1$ for all on-diagonal elements and $a_{ij} = 0$ for all off-diagonal elements of a square matrix, then the matrix is called an *identify* matrix and is usually denoted $\mathbf{I}$.

▪ **Addition** of two matrices

Let $\mathbf{A} = \{a_{ij}\}$ and $\mathbf{B} = \{b_{ij}\}$ be two matrices with the same order. Then, the matrix $\mathbf{C} = \mathbf{A} + \mathbf{B}$ is defined to be the $m \times n$ matrix where ijth element is $a_{ij} + b_{ij}$.

▪ **Transpose** of a matrix

Given any $m \times n$ matrix $\mathbf{A} = \{a_{ij}\}$, the transpose of $\mathbf{A}$, written $\mathbf{A}^T$ or $\mathbf{A}'$, is the $n \times m$ matrix that is obtained from $\mathbf{A}$ by letting row 1 of $\mathbf{A}$ be column 1 of $\mathbf{A}^T$, letting row 2 of $\mathbf{A}$ be column 2 of $\mathbf{A}^T$, and so on.

- **Matrix multiplication**

The product **AB** of an $m\times n$ matrix $\mathbf{A}=\{a_{ij}\}$ and an $n\times k$ matrix $\mathbf{B}=\{b_{jk}\}$ is the $m\times k$ matrix **C** whose element c_{ij} is given by

$$c_{ij} = \sum_{s=1}^{n} a_{is}b_{sj}, \text{ for } i = 1, 2,\ldots, m \text{ and } j = 1, 2,\ldots, k.$$

- **Inverse** of a square matrix

Let **I** be the $m\times m$ *identity* matrix. The square $m\times m$ matrix **B** such that **AB** = **BA** = **I** is called the *inverse* of the square $m\times m$ matrix **A** and is denoted by $\mathbf{A}^{-1}$.

- **Trace** of a square matrix

Let $\mathbf{A} = \{a_{ij}\}$,be an $m\times m$ *square* matrix. The *trace* of the matrix **A**, written $tr(\mathbf{A})$, is the sum of the *diagonal* elements; i.e., $tr(\mathbf{A}) = \sum_{j=1}^{n} a_{jj}$.

Ex] Let $\underset{(2\times 2)}{\mathbf{A}} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ and $\underset{(2\times 2)}{\mathbf{B}} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

- Trace of **A** $tr(\mathbf{A})$ =

- Addition **A+B** **A+B** =

- Product **AB** **AB** =

C. Applications of Linear Algebra

* Systems of Linear Equations

- Consider the *system of linear equations* given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$
$$\dots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

- Then, it may be written as **Ax** = **b**, where

$$\mathbf{A}=\begin{bmatrix} a_{11} & a_{12} & . & a_{1n} \\ a_{21} & a_{22} & . & a_{2n} \\ . & . & . & . \\ a_{m1} & a_{m2} & . & a_{mn} \end{bmatrix}, \quad \mathbf{x}=\begin{bmatrix} x_1 \\ x_2 \\ . \\ x_n \end{bmatrix}, \quad \text{and} \quad \mathbf{b}=\begin{bmatrix} b_1 \\ b_2 \\ . \\ b_m \end{bmatrix}$$

- The solution to a *square* system of *m* linear equations in *m* unknown variables is $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$ which is a set of values for the unknown variables that satisfies each of the system's *m* equations.

Ex] If a farmer has a total of 30 cows and chickens, and the animals have 74 legs in all, how may chickens are in the coop?

$$\begin{cases} x_1 + x_2 = 30 \\ 4x_1 + 2x_2 = 74 \end{cases} \Rightarrow \mathbf{A} = \begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \text{and } \mathbf{b} = \begin{bmatrix} 30 \\ 74 \end{bmatrix}$$

Thus, the solution vector **x** is

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} =$$

* Linear Regression Model

$$Y_1 = \beta_0 + \beta_1 X_1 + \varepsilon_1$$

$$Y_2 = \beta_0 + \beta_1 X_2 + \varepsilon_2$$

$$\ldots..$$

$$Y_n = \beta_0 + \beta_1 X_n + \varepsilon_n$$

- Then, it may be written as $\underset{n\times 1}{\mathbf{y}} = \underset{n\times 2}{\mathbf{X}}\underset{2\times 1}{\boldsymbol{\beta}} + \underset{n\times 1}{\boldsymbol{\varepsilon}}$, where

$$\underset{n\times 1}{\mathbf{y}} = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdots \\ Y_n \end{bmatrix}, \quad \underset{n\times 2}{\mathbf{X}} = \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ \cdots & \cdot\cdot \\ 1 & X_n \end{bmatrix}, \quad \underset{2\times 1}{\boldsymbol{\beta}} = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}, \quad \underset{n\times 1}{\boldsymbol{\varepsilon}} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdots \\ \varepsilon_n \end{bmatrix}.$$

- The error term $\boldsymbol{\varepsilon}$ is a vector of independent normal random variables with

$$\underset{n\times 1}{E[\boldsymbol{\varepsilon}]} = \begin{bmatrix} 0 \\ 0 \\ . \\ 0 \end{bmatrix} = \underset{n\times 1}{\mathbf{0}} \quad \text{and} \quad \underset{n\times n}{\boldsymbol{\sigma}^2[\boldsymbol{\varepsilon}]} = \begin{bmatrix} \sigma^2 & 0 & . & 0 \\ 0 & \sigma^2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \sigma^2 \end{bmatrix} = \underset{n\times n}{\sigma^2 \mathbf{I}}.$$

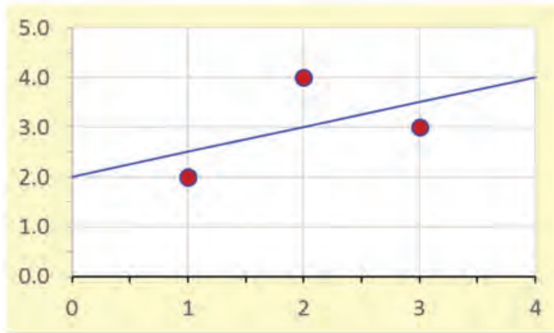
- How to find the estimates of $\boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$?

  - The least squares estimator **b** that minimizes the sum of squared errors (*SSE*) is shown to be

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\,\mathbf{X}'\mathbf{y}.$$

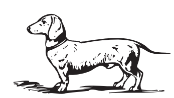
Ex] Use Microsoft Excel functions and find the least square estimates (*LSE*), **b**.

X_i	Y_i
1	2
2	4
3	3



- Matrix representation: $\underset{3\times 1}{\mathbf{y}} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$ and $\underset{3\times 2}{\mathbf{X}} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$

$$\underset{2\times 3}{\mathbf{X}}' \underset{3\times 2}{\mathbf{X}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} =$$



$$(\underset{2\times 3}{\mathbf{X}}' \underset{3\times 2}{\mathbf{X}})^{-1} =$$

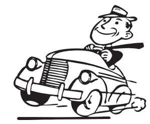
$$(\underset{2\times 3}{\mathbf{X}}' \underset{3\times 1}{\mathbf{y}}) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} =$$

$$\mathbf{b} = (\underset{2\times 3}{\mathbf{X}}' \underset{3\times 2}{\mathbf{X}})^{-1} (\underset{2\times 3}{\mathbf{X}}' \underset{3\times 1}{\mathbf{y}}) = \frac{1}{6} \begin{bmatrix} 14 & -6 \\ -6 & 3 \end{bmatrix} \begin{bmatrix} 9 \\ 19 \end{bmatrix} =$$

- Thus, the *LSE* is $\hat{y}_i$ =

* Markov Chain

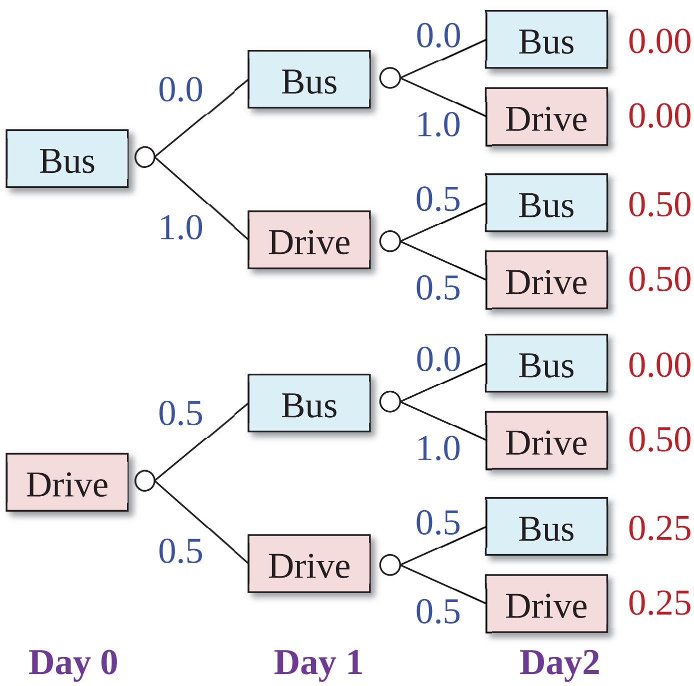
A student either drives his car or catches a bus to school each day. Suppose he never goes by bus two days in a row; but if he drives to school, then the next day he is just as likely to drive again as he is to travel by bus.



(a) Formulate a transition probability matrix.

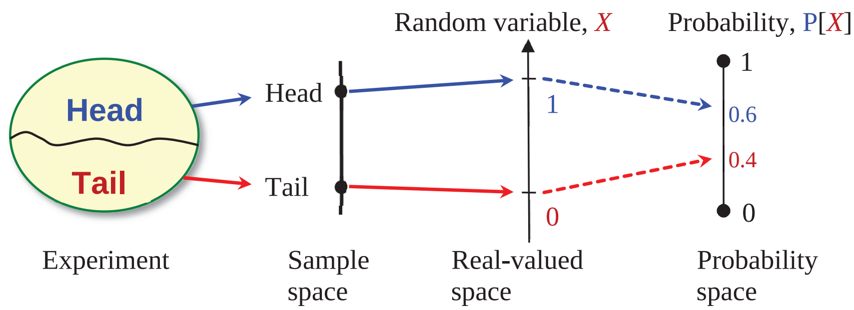
P =	Bus	Drive
Bus		
Drive		

(b) If he *drives* to school today, what is the probability that he will drive to school two days later?

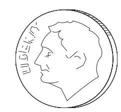


$\mathbf{P}^2$ =	Bus	Drive
Bus		
Drive		

D. Introduction to Probability



* Sample Space, *S*



The set of all possible experimental outcomes.

- Finite
- Infinite
  - Countably infinite
  - Uncountably infinite

* Random Variable, *X*

- *Discrete* random variable, $X = 0, 1, 2, \ldots$
- *Continuous* random variable, $-\infty < X < +\infty$

* Expectation of a Random Variable *X*

Single variable X	Expected value	$E[X]$	μ
	Variance	$Var[X]$	σ_x^2
	Standard deviation	$Stdev[X]$	σ_x
Two variables X and Y	Covariance	$Cov[X, Y]$	σ_{xy}
	Correlation coefficient	$Corr[X, Y]$	ρ_{xy}

* Probability Distributions

- *Discrete* probability distribution, $P[x]$
- *Continuous* probability distribution, $f(x)$

* Probability, *P*[*A*]

$P[A]$ = What is the probability that the event A will occur?

(a) *Relative frequency*

- If a trial is performed *a large number of times* in an independent manner, the *fraction* of times that event A occurs will approach, as a limit, the value $P[A]$.

(b) *Subjective probability*

- The probability $P[A]$ is the *degree of belief* one feels that event A will occur.
- One-time trials: unique, non-repeatable trials.

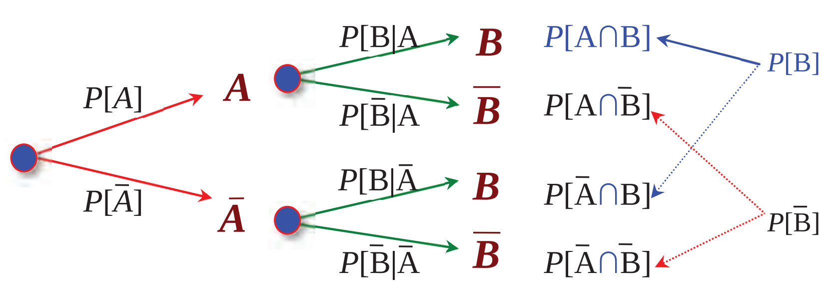
* Odds and Probability

- Odds *in favor of* the event A are 1 in $O(A\!:A^c) = \dfrac{P[A]}{P[A^c]}$.

* Conditional Probability, *P*[*A*|*B*]

- $P[A|B]$ = What is the probability of the unknown event A given that the event B has *occurred*?

$$P[A|B] = \frac{P[A \cap B]}{P[B]},$$ where $A \cap B \equiv AB \equiv A$ and B.

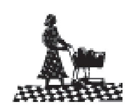


Ex 1] Market Basket Analysis

Consider the following transaction data of 100 customers.

	Wine	Cheese	Beer	Chips	-
1	Y	Y	Y	Y	.
2	Y		Y	Y	.
3	Y	Y			.
4		Y	Y	Y	.
5				Y	.
6	Y	Y		Y	.
7		Y		Y	.
8		Y			.
9			Y	Y	.
10			Y	Y	.
.	.	.	.	.	.
100	.	.	.	.	.
Total	**4**	**6**	**5**	**8**	.

Find the conditional probability (aka, confidence) of the following *association rules* in data mining.



(a) “Wine => Cheese” =

(b) “Cheese => Wine” =

		Cheese		Total
		Yes	No	
Wine	Yes			4
	No			96
Total		6	94	100

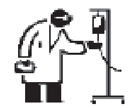
(c) “Beer => Chips” =

(d) “Wine => Beer” =

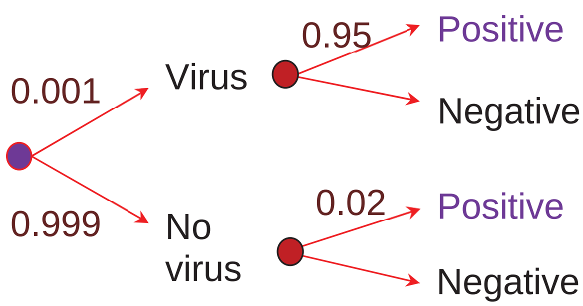
(e) “Wine & Cheese => Chips” =

Ex 2] Blood Test

Suppose that a laboratory blood test is 95% effective in detecting a certain disease when it is, in fact, present. However, the test also yields "false positive" result for 2% of the healthy persons tested (i.e., if a healthy person is tested, then, with probability 0.02, the test result will imply he/she has the disease.)



If 0.1% of the population actually has the disease, what is the probability a person has the disease given that his/her test result is positive?



- **Case 1**. Prior probability

  $P[\text{Virus}] = 0.1\%$

- **Case 2**. Posterior probability when tested positive *once*.

  $P[\text{Virus} \mid \text{Positive}]$

  $=$

  $=$

- **Case 3**. Posterior probability when tested positive *twice*.

  $P[\text{Virus} \mid \text{Positive}, \text{Positive}]$

  $=$

  $=$

E. Random Variables

* Two Types of Random Variables

Discrete random variable	Probability mass function (*pmf*)	$p(x)$, p_x, or $P[X=x]$
	Cumulative distribution function (*cdf*)	$P[X \le x]$
Continuous random variable	Probability density function (*pdf*)	$f(x) = \frac{d}{dx} F(x)$
	Cumulative distribution function (*cdf*)	$F(x) = P[X < x] = \int_{-\infty}^{x} f(y)\; dy$

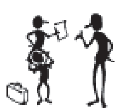
(a) Expected value, $E[X] = \mu$

- $E[X] = \sum_x x\; p[x]$ or $\int_{-\infty}^{\infty} x \;\; f(x)\; dx$

(b) Variance σ^2

- $Var[X] = E[(X-\mu)^2] = E[X^2] - \mu^2$

(c) Standard deviation σ



- $Stdev[X] = \sqrt{Var[X]}$

(d) Covariance σ_{xy}

- $Cov(X, Y) = E[(X-\mu_x)(Y-\mu_y)] = E[XY] - \mu_x \mu_y$

(e) Correlation coefficient ρ_{xy}

- $\rho_{xy} = \frac{Cov(X,Y)}{\sqrt{Var[X]\; Var[Y]}} = \frac{\sigma_{xy}}{\sigma_x\; \sigma_y}$

Ex 1] Discrete Case

Consider the following joint probability distribution of the numbers of purchases made by consumers during one day:

		Coffee, Y			Total
		0	**1**	**2**	
Soda, X	**0**	0.1	0.2	0.3	0.6
	1	0.3	0.1	0.0	0.4
Total		0.4	0.3	0.3	1.0

(a) Marginal distributions

- $E[X] = \mu_x =$ ▪ $E[X^2] =$ ▪ $Var[X] = \sigma_x^2 =$
- $E[Y] = \mu_y =$ ▪ $E[Y^2] =$ ▪ $Var[Y] = \sigma_y^2 =$

(b) Joint distribution of X and Y

- $E[XY] =$
- $Cov[X, Y] = \sigma_{xy} = E[XY] - \mu_x\mu_y = 0.1 - 0.4*0.9 =$
- $Corr[X, Y] = \rho_{xy} = \sigma_{xy}/(\sigma_x\ \sigma_y)$

  $=$

(c) New random variable: Let $Z = X+Y$ be the *total* number of purchases during one day.

- $E[Z] = E[X+Y] =$
- $Var[Z] = Var[X+Y] =$

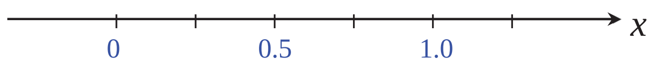
  $=$

Ex 2] Continuous Case

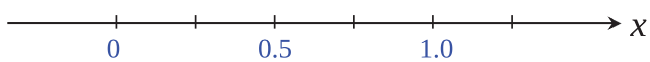
Suppose the density of X is given by

$$f(x) = \begin{cases} 2x & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the *probability density function* $f(x)$.



(b) Draw the *cumulative distribution function* $F(x)$.



(c) Find the *expected value* and *variance* of X.

- $E[X] =$
- Median =
- Mode =
- $E[X^2] =$
- $Var[X] = E[X^2] - \mu^2 =$

(d) Find the *expected value* and *variance* of $Y = 6X+3$.

- $E[Y] = E[6X+3] = 6\,E[X] + 3 =$
- $Var[Y] = Var[6X+3] = 6^2\,Var[X] =$

Appendix: Microsoft Excel

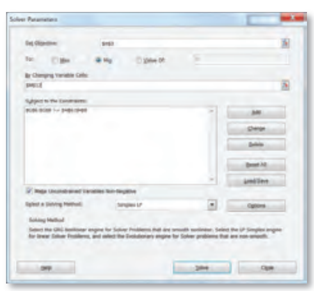
I. Optimization with Microsoft Excel

To use Solver, first make sure that the add-in is installed. To do so, select Data from the main menu. If the option Solver appears in Analyze tab, then Solver is already installed and you are ready to proceed. If the option Solver does not appear, then you must install Solver.

To install Solver, select File from the menu, then click Options. In the Excel Options dialogue box, select Add-Ins and click on Go... In the Add-Ins menu, check Solver Add-In and then click on OK. After a brief period, the installation will be complete.

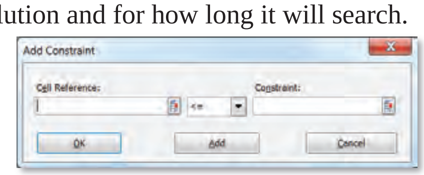
If you cannot find the Solver Add-In, try using the Find in Windows to locate the file. Search for "solver." Note the location of the file, return to the Add-Ins dialog box, click on Browse, and open the Solver Add-In file. Still can't find it? Then it is likely that your installation of Excel failed to include the Solver Add-In. Run your Microsoft Excel or Microsoft Office Setup again from the original CD-ROM or file and install the Solver Add-In.

You should now be able to use the Solver by clicking on the Data heading on the menu bar and selecting the Solver item. The Solver Parameters dialog box will appear.



The Solver Parameters dialog box is used to describe the optimization problem to EXCEL. The Set Objective box should contain the cell location of the objective function for the problem under consideration. Max or Min may be selected for finding the maximum or minimum of the set target cell. If Value of is selected, the Solver will attempt to find a value of the Target Cell equal to whatever value is placed in the box just to the right of this selection. The By Changing Variable Cells box should contain the location of the decision variables for the problem.

Finally, the constraints must be specified in the Subject to the Constraints box by clicking on Add. Change allows you to modify a constraint already entered and Delete allows you to delete a previously entered constraint. Reset All clears the current problem and resets all parameters to their default values. Options invokes the Solver options dialog box. We really shouldn't ever have to worry about the Solver Options dialog box. As you can see, a series of default choices are included that direct Solver's search for the optimum solution and for how long it will search.



When the Add button is clicked, the Add Constraint dialog box appears: Clicking on the Cell Reference box allows you to specify a cell location (usually a cell with a formula). The Constraint type may be set by selecting the down arrow (<=, >=, =, int, where int refers to integer, or bin, where bin refers to binary). The Constraint box may contain a formula of cells, a simple cell reference, or a numerical value. The Add button adds the currently specified constraint to the existing model and returns to the Add Constraint dialog box. The OK button adds the current constraint to the model and returns you to the Solver Parameters dialog box.

When you run Excel's Solver, it executes a series of "add-in" files and routines. Upon completion of the various programs, Excel presents the user with the Solver Results dialog box.

II. Matrix Operations with Microsoft Excel

▪ Adding or Subtracting Matrices

Excel has no function to add or multiply functions. To add or subtract matrices, both matrices must have the *same* number of rows and columns. In a cell that will be first row and first column of the matrix that will contain the sum, enter a formula to add the cells for the first row and first column of the matrices to be added. Once this formula is entered, click and drag the cell across to for the number of columns. Use Fill Right in the Editing menu. Next click and drag that row down for the number of rows. Use Fill Down in the Editing menu. This will yield the sum of two matrices.

▪ Finding the Transpose of a Matrix

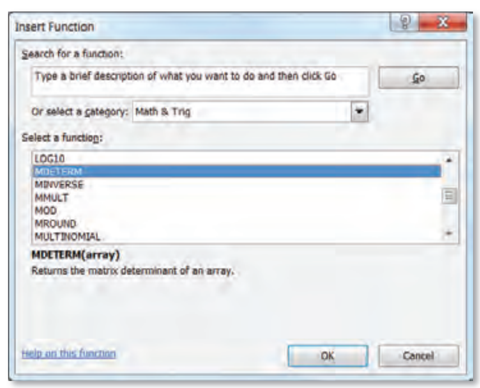
The transpose of a matrix has the rows and columns reversed. If the matrix A has *m* rows and *n* columns, then the transpose of A has *n* rows and *m* columns. First, highlight an area for the transposed matrix with the appropriate number of rows and columns. Select the function TRANSPOSE in Insert Function of Formulas menu. Enter the cells for the matrix in Array. Rather than merely hitting Enter or clicking on the OK button, you must hold down both the control [Ctrl] and [Shift] keys while hitting Enter or clicking on the OK button.

▪ Multiplying Matrices

To multiply matrices, the first step is to make sure that the number of columns of the first matrix must equal the number of rows of the second matrix. The product matrix will have the same number of rows as the first matrix and the same number of columns as the second matrix. In Excel, highlight an area for the product matrix with the appropriate number of rows and columns. Select the function MMULT in Insert Function of Formulas menu. Enter the cells for the first matrix in Array 1 and the cells for the second matrix in Array 2. Rather than merely hitting Enter or clicking on the OK button, you must hold down both the control [Ctrl] and [Shift] keys while hitting Enter or clicking on the OK button.

▪ Finding a Determinant of a Square Matrix

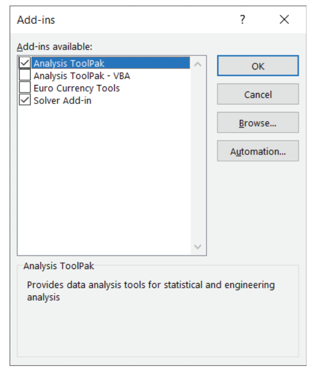
Select a cell where you want to put the determinant. Then select MDETERM from the functions under the menu for Math & Trig in Formulas menu. The determinant is a single value and not an array of values. In the window beside the word ARRAY, specify the array of values for the square matrix for which you want the determinant. Click OK and the value of the determinant will appear in the specified cell.



▪ Finding an Inverse of a Square Matrix

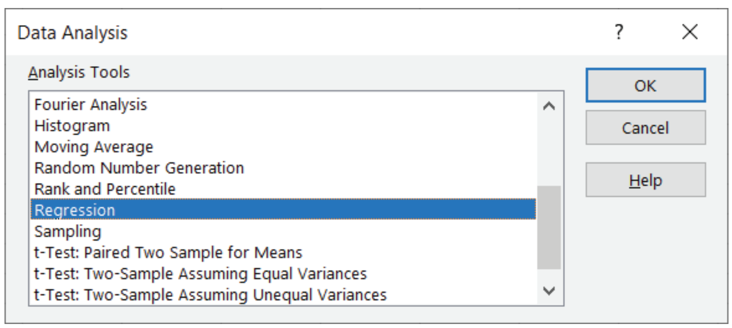
To find the inverse, first highlight the area corresponding with the order of the matrix. Select MINVERSE from the functions under the menu for Math & Trig. In the window beside the word ARRAY, specify the array of values for the square matrix for which you want the inverse. Rather than merely hitting Enter or clicking on the OK button, you must hold down both the control [Ctrl] and [Shift] keys while hitting Enter or clicking on the OK button. The inverse matrix will appear in the highlighted area.

III. Regression Analysis with Microsoft Excel



Microsoft Excel provides a set of data analysis tools — called the Analysis ToolPak — that you can use to save steps when you develop complex statistical or engineering analyses. You provide the data and parameters for each analysis; the tool uses the appropriate statistical or engineering macro functions and then displays the results in an output table. Some tools generate charts in addition to output tables.

To view a list of available analysis tools, click Data Analysis on the DATA tool bar. If the Data Analysis command is not on the DATA menu, you need to install the Analysis ToolPak. (Click FILE => Options => Add-Ins. On Excel Options window, click Go and check the box, Analysis ToolPak.)



Select Data from the toolbar to display the Data menu. Select Data Analysis. In the Data Analysis dialog box, select Regression. In the Regression dialog box, specify the cells that contain the response variable in the Input Y Range box, and the predictor variables in the Input X Range box. Click the Label checkbox if the first row contains labels. Click OK to run the results. By default, Excel generates the regression statistics on a new worksheet.