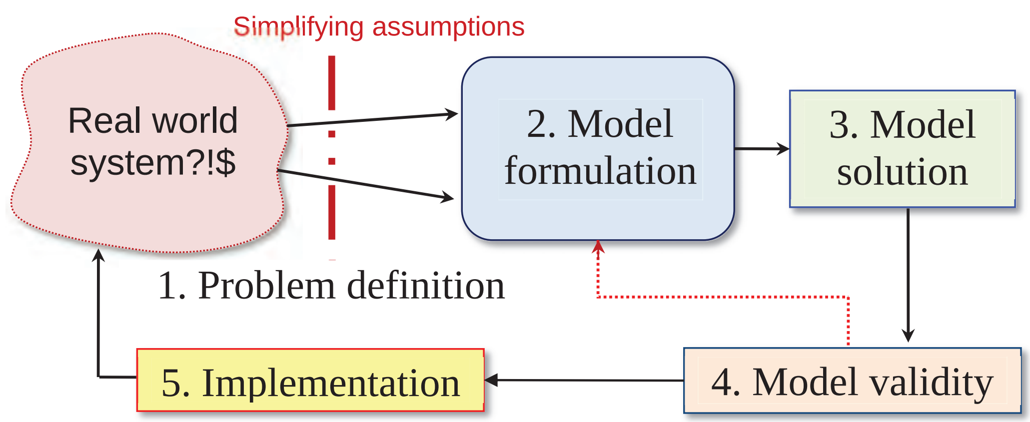
Session 1. Introduction

"A theory is just a mathematical model to describe the observations."

* Nature of Modeling

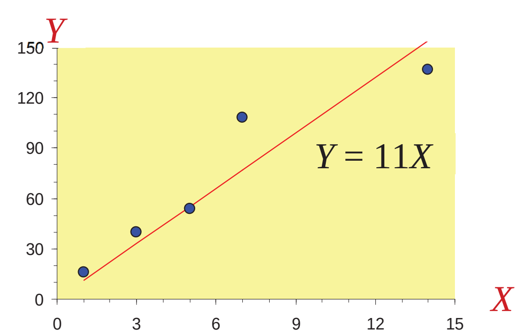
Optimal decision making in, and modeling of, *deterministic* and *probabilistic* systems that originated from real life.



Ex] Mileage (Y) and age (X) of your BMW!

- Problem: How to predict the *mileage* on the car?
- Assumption: *Linear* relationship between Y and X.
- Model: Linear regression model: $Y = \beta X + \varepsilon$
- Solution: The slope β is estimated to be $\hat{\beta}=11$.
- Prediction: If X=5 years, then $\hat{Y}$=55 thousand miles!

i	Age, X_i	Mileage, Y_i
1	1	16
2	3	40
3	5	54
4	7	108
5	14	137



A. Deterministic Models

How to build models of business problems that lead to making better decisions?

* Effect of Data Availability on Business Modeling:

A. Deterministic models: Data is known *with certainty*.

B. Probabilistic (stochastic) models

* Linear Programming (*LP*)

A farmer can plant up to 8 acres of land with wheat and barley. He can earn $5,000 for every acre he plants with wheat and $3,000 for every acre he plants with barley. His use of a necessary pesticide is limited by federal regulations to 10.5 gallons for his entire 8 acres. Wheat requires 2 gallons of pesticide for every acre planted and barley requires just 1 gallon per acre. What is the maximum profit he can make?

- Decision variables
  - x_1 = the number of acres of wheat
  - x_2 = the number of acres of barley

- Objective function
  - $Max\ z =$
- Constraints

- Model solution: $(x_1^*, x_2^*) = (2.5, 5.5)$ and z^*=$29,000.
  - Graphical method
  - Simplex method

* Integer Programming (*IP*)

Ex 1] Knapsack problem: Given weights and values of 4 items, put these items in a knapsack of capacity 45 pounds to get the maximum total value in the knapsack.

i	Item	Weight (lbs)	Value ($)	$/lbs
1	Item 1	20	400	20
2	Item 2	30	450	15
3	Item 3	10	100	10
4	Item 4	40	200	5

- Decision variables

$$x_i = \begin{cases} 1 & \text{if item } i \text{ is taken} \\ 0 & \text{if item } i \text{ is not taken} \end{cases} \qquad \text{where } i=1, 2, 3, 4.$$

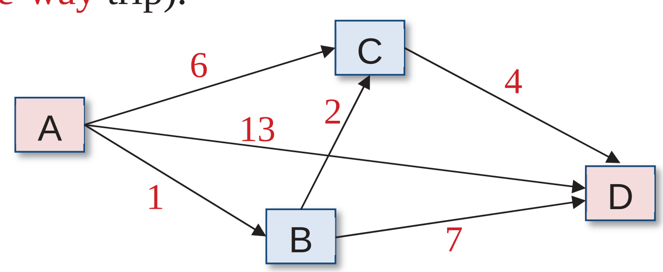
- Objective function

  Max z =

- Constraints

- Model solution: (0, 1, 1, 0) and z^*=550

Ex 2] Shortest path problem: Find the shortest path from **A** to **D** (One-way trip).

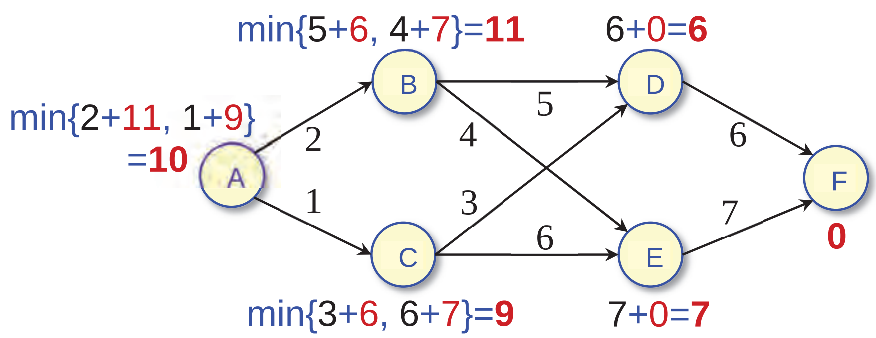


Ex 3] Traveling salesman problem (*TSP*): Leave the headquarter **A**, visit all the branch offices, and return to **A**.

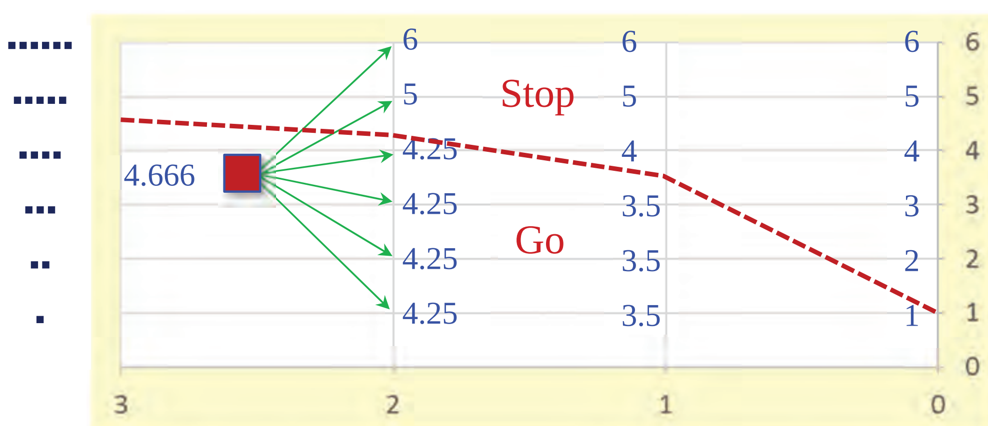
* Dynamic Programming (*DP*)

Dynamic programming obtains solutions by working *backward* from the end of a problem toward the beginning, thus breaking up a large unwieldy problem into a series of smaller, more tractable problems.

Ex 1] ***Deterministic*** **dynamic programming**: Find the shortest path from city A to city F.

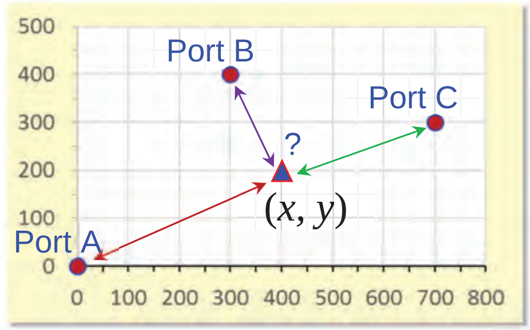


Ex 2] ***Probabilistic*** **dynamic programming**: You can roll a die up to 3 times and receive $1 for each dot. You may stop and collect the money or give up the money and continue.



▪ Decision process: Stages, States, Rewards, and Decisions

* Non-linear Programming (*NLP*)



ExxonMobil wants to build a refinery that will be supplied from three port cities. Port B is located 300 *km* east and 400 *km* north of Port A, while Port C is 400 *km* east and 100 *km* south of Port B. Determine the location of the refinery so that the total amount of pipe required to connect the refinery to the three ports is *minimized*.

- Variables

  (x, y) = coordinates of the refinery.

  z = sum of the *distances* between the refinery and the three ports.

- Non-linear Objective function

$$\begin{aligned} Min \quad z = & \sqrt{(x-0)^2 + (y-0)^2} \\ & + \sqrt{(x-300)^2 + (y-400)^2} \\ & + \sqrt{(x-700)^2 + (y-300)^2} \end{aligned}$$

- Constraints? Unconstrained optimization!
- Model solution: Search method (e.g., Gradient descent)
  Microsoft Excel - Solver
- Optimal solution: Center of gravity or centroid? Nope!

  $z^* =$ with $x^* =$ and $y^* =$

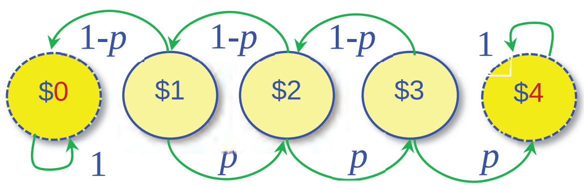
Calculus? First-order derivative of the objection function!

B. Probabilistic Models

* Markov Chain

Ex 1] The Gambler's ruin: At time 0, I have \$2. At time 1, 2, ..., ∞, I play a game in which I bet \$1. With probability p (or $1-p$), I win (or lose) the game. My goal is to increase my capital to \$4, and as soon as I do, the game is over. The game is also over if my capital is reduced to \$0.

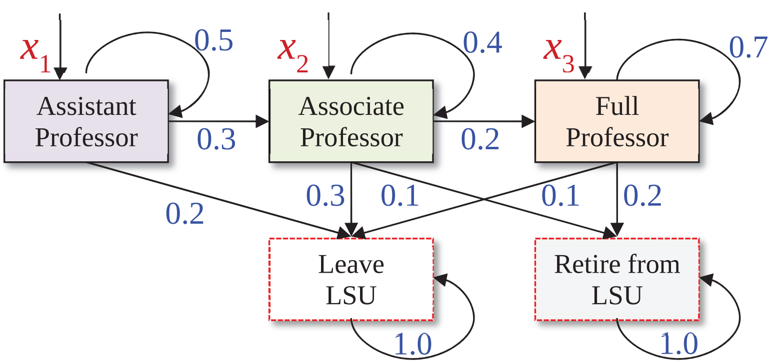
- Transition diagram



- Transition probability

	0	1	2	3	4
0	1				
1	$1-p$		p		
2		$1-p$		p	
3			$1-p$		p
4					1

Ex 2] Publish or perish! Miserable life of a college professor



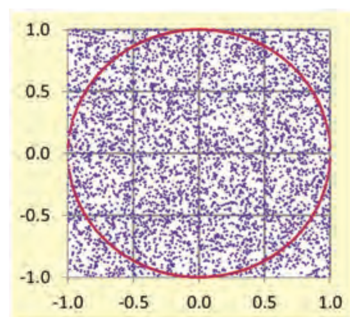
p_{ij}	Asst.	Asso.	Full	Leave	Retire
Asst.	0.5	0.3		0.2	
Asso.		0.4	0.2	0.3	0.1
Full			0.7	0.1	0.2
Leave				1.0	
Retire					1.0

* Monte Carlo Simulation

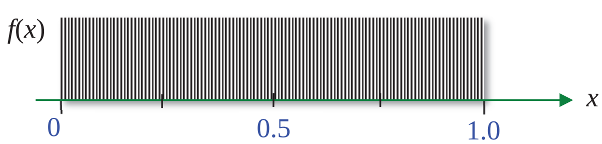
- Real-world phenomenon is too *complex*.
- No *analytical* solution can be obtained from the model.
- In such a case, simulate the real-world phenomenon!
- *Monte Carlo* methods (or Monte Carlo experiments) are a class of computational algorithms that rely on repeated random sampling to compute their results.

Ex] "Hit or miss" method for π (=3.14...)

- Draw a circle inscribed in a square.
- The area of the square is 4.0 and the area of the circle is πr^2 where r=1.
- Throw your dart 1,000 times and count the number of hits inside the circle.



- Instead of a real dart, you may generate standard *uniform* random numbers, U, between 0 and 1.



- And then transform U_i into a *uniform* random variable X_i between -1 and +1.

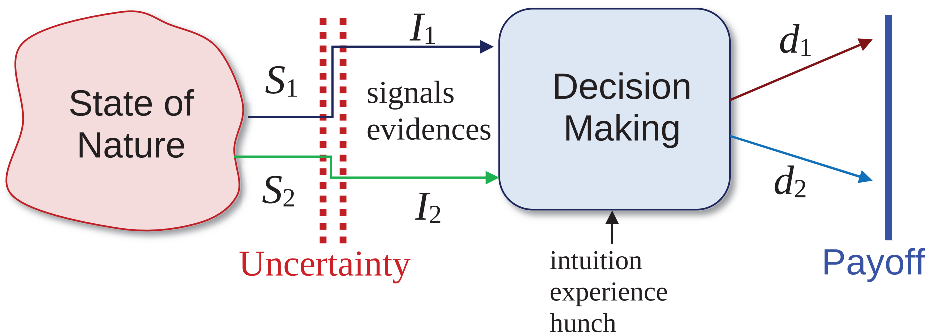
$$X_1 = 2\,U_1 - 1 \text{ and } \quad X_2 = 2\,U_2 - 1$$

i

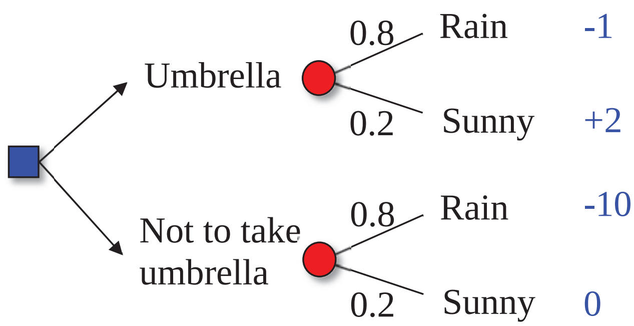
- Note that the circle is $x_1^2+x_2^2=1$. If $x_1^2+x_2^2 < 1$, we assume that the dart hits inside the circle.
- π is estimated to be the proportion of the hits × 4.

C. Decision Models

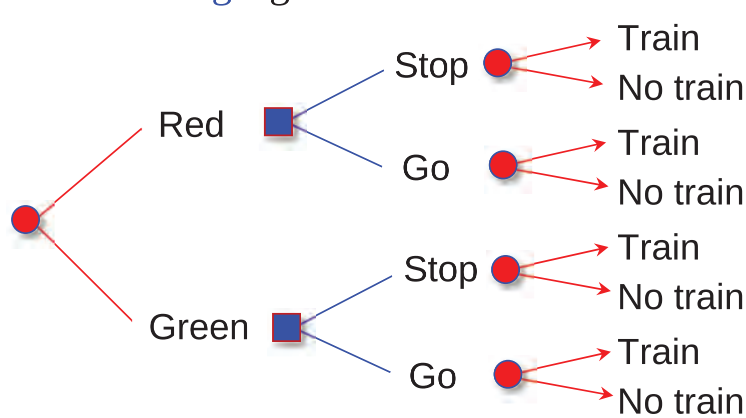
* Statistical Decision Analysis



(a) Decision making *without* sample information (*EVPI*)
Take an umbrella or not?



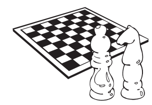
(b) Decision making *with* sample information (*EVSI*)
Railroad crossing signals.



* Game Theory

Game theory attempts to mathematically capture behavior in *strategic situations*, in which an individual's success in making choices depends on the choices of others.

1. Two-person zero-sum game:



Ex 1] Coach's dilemma

LSU has the ball!		Alabama	
		b_1: Run	b_2 Pass
LSU	a_1: Run	-4 yards	+6 yards
	a_2: Pass	+15 yards	-8 yards

- The *optimal* strategy for LSU?
- The *optimal* strategy for Alabama?
- The *value* of the game?

2. Two-person general-sum game:

Ex 2] What to wear for the Halloween party?

		Your Friend	
		b_1: Wears the same costume	b_2: Doesn't wear the costume
You	a_1: Wear your favorite costume	You both look like copycats. (**-10**, -10)	You look cool, your friend doesn't. (**8**, -2)
	a_2: Don't wear your favorite costume	Your friend looks cool, but you don't. (**-2**, 8)	Neither of you look cool or dorky. (**0**, 0)

- The *optimal* strategy for each player?

3. *n*-person zero-sum game:

AT&T, Sprint, Verizon, T-Mobile, etc.

* Multi-Objective Decision Analysis

In many situations, the action chosen depends on how each possible action affects *more than one* attribute or variable.

Ex 1] Buying a new car?

1. Price, 2. Size, 3. Fuel economy, and 4. Style

Ex 2] Looking for a job?

1. Starting salary,
2. Location,
3. Degree of interests you have in doing the work involved in the particular job, and
4. Long-term opportunities associated with job.

Ex 3] Top 7 criteria chosen by LSU tigers who are looking for

Rank	Princess Charming	Mr. Right
1	Body	Physical attractiveness
2	Personality	Personality
3	Money	Good general attitude
4	Intelligence	Trustworthiness
5	Lack of #4	Wit
6	Wit	Intelligence
7	Maturity	Money

* *Reproduced from the 1987 LSU yearbook without permission*

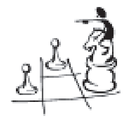
▪ **Solution methods:**

- Multi-attribute utility theory
- Goal programming (*GP*)
- Analytical hierarchical process (*AHP*)

D. Operations Research / Management Science

* Origin of Operations Research

- The first formal activities of *Operations Research* were initiated in England during World War II, when a team of British scientists set out to make scientifically based decisions regarding the best utilization of war materiel.
- After the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector.



* Definition of Operations Research

- "Operations research has come to describe the scientific, quantitative study of the operations of war." (*old*)
- "Operations research is a discipline that deals with the application of advanced analytical methods to help make better decisions." (*new*)
- Also known as operational research, operational analysis, management science, systems analysis, decision science, etc.
- Employing techniques from other mathematical sciences, such as mathematical modeling, statistical analysis, and mathematical optimization, operations research arrives at optimal or near-optimal solutions to complex decision-making problems.
- Operations research is often concerned with determining the maximum (of profit, performance, or yield) or minimum (of loss, risk, or cost) of some real-world objective.

Ex 1] Tramcar Problem

During World War II, the allies had very little idea of how many tanks the enemy was capable of producing in a year. Without this information, they were unsure whether any invasion of the continent on the western front could succeed.

Statisticians had one key piece of information, which was the serial numbers on captured German tanks. The statisticians believed that the Germans had logically numbered their tanks in the order in which they were produced.

And this deduction turned out to be right. It was enough to enable them to make an estimate of the total number of German tanks that had been produced up to any given moment.

(After the war, the allies captured German production records, showing that the true number of tanks produced in those three years was almost the same as what the statisticians had calculated.)

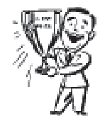
Ex 2] Selection Bias

During World War II, a lot of bombers were not returning from their missions, so the Royal Air Force wanted to put armor on the bombers.

A mathematician looked at the planes that returned and noted where they had holes from enemy anti-aircraft fire. These holes were distributed more or less randomly throughout the plane except for two regions where there was nothing. His recommendation was to place the armor only in those two areas where no enemy fire was found, which seems counterintuitive.

* Applications of Operations Research

Operational Research (*OR*) has been used intensively in business, industry, and government. Many new analytical methods have evolved, such as: mathematical programming, simulation, game theory, queuing theory, network analysis, decision analysis, multi-criteria analysis, etc., which have powerful application to practical problems with the appropriate logical structure.



- "There are too many trucks on the road"

  A bakery used OR to devise an efficient scheduling system for its delivery vehicles. The new system reduced delivery truck mileage, road congestion and pollution as well as saving money for the bakery.

- "I had to wait all morning in hospital"

  Great pressure on consultant time coupled with some patients who don't always keep their appointment times can cause real problems for hospitals. But by using OR, appointment systems have been designed that substantially reduce waiting times while keeping highly qualified medical staff fully occupied.

- "We've just got to increase our sales"

  Easier said than done, but OR proved equal to the task for a mail order firm. The OR model helped boost catalogue sales by designing an ideal mix of discounts, special promotions and customer incentives.

- "Bottlenecks - the bane of my life"

  One production manager wanted to ensure the efficient operations of his new automated warehouse by simulating the operation of alternative material handling equipment. This meant that a selection could be made which eliminated any bottlenecks and delivered the required output.

* Professional Organizations

- IFORS (*International Federation of Operational Research Societies*): The umbrella organization for OR societies worldwide.
- INFORMS (*Institute for Operations Research and Management Science*): *Operations Research**, *Management Science*, *OR/MS Today**, *Interfaces*, etc.
- EURO (*Association of European Operational Research Societies*): *Journal of the Operational Research Society**, *European Journal of Operational Research**, *INFOR**, etc.
- DSI (*Decision Sciences Institute*): *Decision Science**.

* Analytics Conferences

- *INFORMS Conference on Business Analytics and Operations Research*
- *SAS Analytics Conference*

* Concerned about Your Math Skills?

No mathematical background beyond "high school" calculus and algebra will be required for an understanding of *business modeling and decision making*.

The emphasis here will be (i) on providing students with solid and effective evidence concerning the power and applicability of modern optimization models, (ii) on making sure that they can use these solution algorithms, and (iii) on indicating the assumptions underlying these models, as well as their limitations.

To accomplish these purposes, it is neither necessary nor appropriate to deluge students with mathematics.