

Homework 2 Optional Tasks

Jacob Thielemier

19 February 2024

Question 1 Optional Task

- Take the PDF

$$f(x|\mu_k, \Sigma) = \frac{1}{(2\pi)^{\frac{p}{2}} |\Sigma|^{\frac{1}{2}}} \exp \left(-\frac{1}{2} (x - \mu_k)^T \Sigma^{-1} (x - \mu_k) \right)$$

- Take the log of the PDF

$$\log f(x|\mu_k, \Sigma) = -\frac{1}{2} \log((2\pi)^p |\Sigma|) - \frac{1}{2} (x - \mu_k)^T \Sigma^{-1} (x - \mu_k)$$

- Simplify the log

$$\delta_k(x) = -\frac{1}{2} (x - \mu_k)^T \Sigma^{-1} (x - \mu_k) + \log \pi_k$$

- Left with discriminant function

$$\delta_k(x) = x^T \Sigma^{-1} \mu_k - \frac{1}{2} \mu_k^T \Sigma^{-1} \mu_k + \log \pi_k$$

Question 2 Optional Task

- Take the PDF

$$f(x|\mu_k, \Sigma_k) = \frac{1}{(2\pi)^{\frac{p}{2}} |\Sigma_k|^{\frac{1}{2}}} \exp \left(-\frac{1}{2} (x - \mu_k)^T \Sigma_k^{-1} (x - \mu_k) \right)$$

- The log of PDF

$$\log f(x|\mu_k, \Sigma_k) = -\frac{1}{2} \log((2\pi)^p |\Sigma_k|) - \frac{1}{2} (x - \mu_k)^T \Sigma_k^{-1} (x - \mu_k)$$

- The discriminant function of $\delta_k(x)$

$$\delta_k(x) = -\frac{1}{2} \log |\Sigma_k| - \frac{1}{2} (x - \mu_k)^T \Sigma_k^{-1} (x - \mu_k) + \log \pi_k$$

- Expand the quadratic term within $\delta_k(x)$

$$\delta_k(x) = -\frac{1}{2} \log |\Sigma_k| - \frac{1}{2} (x - \mu_k)^T \Sigma_k^{-1} (x - \mu_k) + \log \pi_k$$