→ What is the time complexity of vectors
→ for adding or removing an element ??

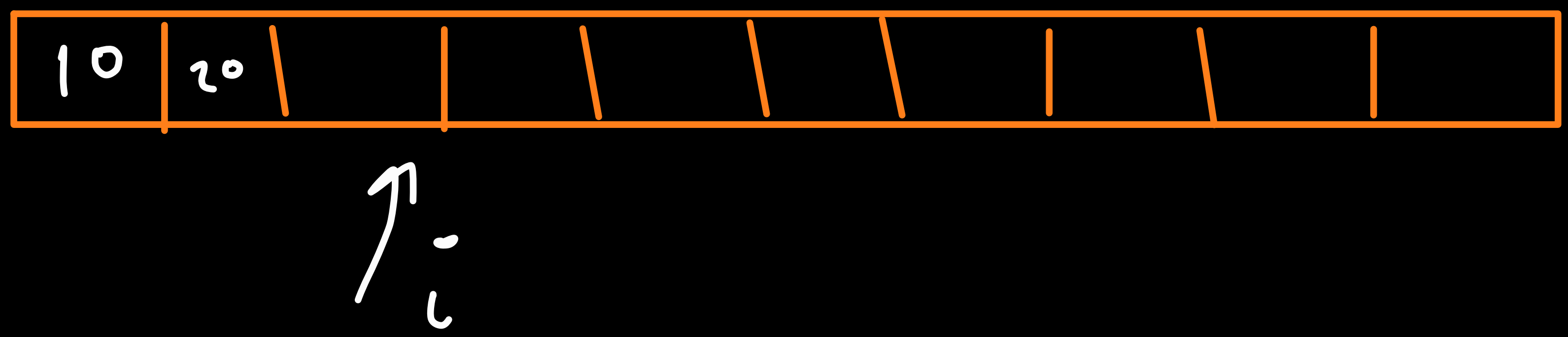
How vectors are internally implemented ??

```
std::vector<int> v;
```

internally vectors are also implemented with basic arrays

fixed size arrays

Approach 1 → Create a very big array, with mostly empty spaces.



push_back(10)
push_back(20)

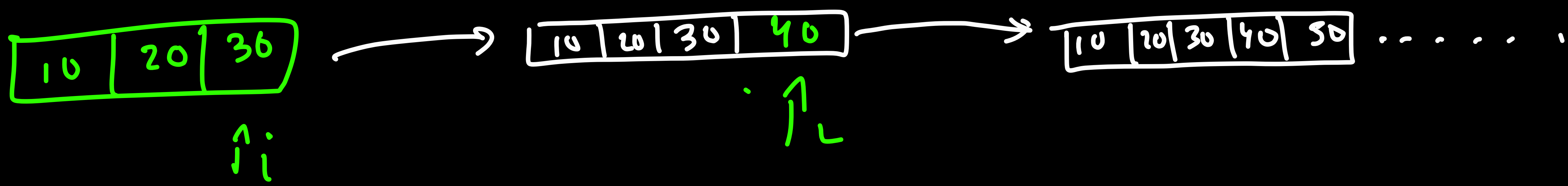
Approach 2 → Create a n length array, if you get more than n

2

Some starting length

elements to insert, then create a new array of $n+1$ length, copy the old n elements & add the new element.

Do it for every new element.



| 1 | 2 | 3 | n $\rightarrow$ | 1 | 2 | 3 | 4 | $n+1$ $\rightarrow$ | 1 | 2 | 3 | 4 | 5 | $n+2$

n copy ops

1 insert

$\downarrow$

$(n+1)$

$\downarrow$

$(n+2)$

if we are inserting K elements then for every element insertion we take $O(n)$ time.

Approach 3 →

We create a fixed size array. We keep on adding elements in the array & the moment it is full,

we will create a double length array, copy all the prev elements & then insert new one.

We want to insert → {10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120} ......

array

array		capacity	size	ops
[10]	1	1	~~0~~ 1	1
[10 | 20]	2	2	2	2 → (2^0+1)
[10 | 20 | 30 |]	4	4	3	3 → (2^1+1)
[10 | 20 | 30 | 40]	4	4	4	1
[10 | 20 | 30 | 40 | 50 | | |]	8	8	5	5 → (2^2+1)
[10 | 20 | 30 | 40 | 50 | 60 | |]	8	8	6	1
[10 | 20 | 30 | 40 | 50 | 60 | 70 |] 80	8	8	7	1
		8	8	1
[| | | | | | | - - - - - |]	16	16	9	9 → (2^3+1)
		⋮	⋮	⋮ ⋮

Total ins → Sum of all the operations we will do for inserting n elements.

Total Sum = $(1 + 2 + 3 + 1 + 5 + 1 + 1 + 1 + 9 + 1 + 1 \ldots\ldots)$

Best Case → $\Omega(1)$

Worst Case → $O(n)$

as soon as array goes big worst case will be way lesser than best cases.

Amortized Analysis $\rightarrow$ Avg no. of ops per insertion $= \left(\text{Total no. of operation done for inserting } n \text{ elements}\right) / n$

$$\text{avg} \rightarrow \frac{(1+2+3+1+5+1+1+1+9+1+1+1\ldots\ldots)}{n}$$

$$\frac{(1+(2^0+1)+(2^1+1)+1+(2^2+1)+1+1+1+(2^3+1)+1+1+1\ldots)}{n}$$

How many occ of 1 is there ?? $\rightarrow$ n ones.

$$\frac{(\underbrace{1+1+1+1+1\ldots\ldots+1+1}_{n \text{ ones}}) + (2^0+2^1+2^2+2^3\ldots)}{n}$$

$$\underbrace{(1+1+1+1+1 \ldots\ldots +1+1)}_{n \text{ times}} + \underbrace{(2^0 + 2^1 + 2^2 + 2^3 \ldots)}_{\log_2 n \text{ terms}}$$

$$n$$

$$n + \underbrace{(2^0 + 2^1 + 2^2 + 2^3 \ldots\ldots\ldots)}_{\log_2 n \text{ times}}$$

$$n$$

$a = 2^0$

$r \to 2$

$n \to \log_2 n$

$\hookrightarrow \quad a, ar, ar^2, ar^3, ar^4 \ldots\ldots \quad$ GP

Sum of GP $\rightarrow \dfrac{a(r^n - 1)}{r-1}$

$r \to$ multiplicative factor

$n \to$ total terms

$a \to$ first term

$r > 1$

$$2^0 + 2^1 + 2^2 \ldots\ldots = \frac{2^0 \times (2^{\log_2 n} - 1)}{2-1} \Rightarrow \frac{1 \times (n-1)}{1}$$

$$= n-1$$

$$\boxed{a^{\log_a b} = b}$$

Avg $\rightarrow$ $\dfrac{n + (n-1)}{n}$ $\Rightarrow$ $\dfrac{2n-1}{n}$ $\longrightarrow$ constant

$\Theta(1)$

$2 \to 2^0+1$

$3 \to 2^1+1$

$5 \to 2^2+1$

$9 \to 2^3+1$

$17 \to 2^4+1$

$\vdots$

$K \to 2^m+1$

K is the last valid value less than n.

$K \approx n$

$2^m+1 \approx n$

$2^m \approx n-1$

taking log

$\log_2 2^m \approx \log_2(n-1)$

$m\log_2 2 \approx \log_2(n-1)$

$m \approx \log_2(n-1)$

m is $O(\log n)$

m is having approx log n value

must have

Ap Cup

logs

linear eqn

permutation combination

basic probability

70-80

```
f11(n) {
    for( j = 1; j <= n; j++) {
        for( i = 0; i < n; i = i + j) {
            // some ops
        }
    }
}
```

$\rightarrow$ $\underline{\underline{O(n\log n)}}$

Total ops

$j = 1$ $\quad i \rightarrow [0, n-1]$ $\quad i = i + 1$ $\rightarrow$ n

$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \ldots\ldots n-1$

$j = 2$ $\quad i \rightarrow [0, n-1]$ $\quad i = i + 2$ $\quad n/2$

$0 \rightarrow 2 \rightarrow 4 \rightarrow 6 \ldots\ldots n-1$

$j = 3$ $\quad i \rightarrow [0, n-1]$ $\quad i = i + 3$ $\quad \underline{\underline{n/3}}$

$0 \rightarrow 3 \rightarrow 6 \rightarrow 9 \ldots\ldots$

$\vdots \qquad \vdots$

total $\rightarrow$ $n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \frac{n}{5} \cdots\cdots\cdots 1$

$$= n\underbrace{\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \cdots\cdots \frac{1}{n}\right)}_{\log n}$$

(imp)

$O(n \log n)$ harmonic progression

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d} \cdots\cdots \frac{1}{a+(n-1)d}$$

$$S_n = \frac{1}{d} \log_e \left(\frac{2a + (2n-1)d}{2a-d}\right)$$

$d = 1 \qquad a = 1$

$$= 1 \times \log_e (2 + (2n-1)) \approx \log n$$