The actual talk:

\*\* Introduction \*\*

Hi! My name is Justin, and I graduated from TAS 2013. I'm currently a senior at

Columbia University studying mathematics and computer science, with a minor in

architecture and art history.

I'm very underqualified to be "giving a talk," so instead I'd like to just chat

with you guys about a couple of things. I had three things I wanted to discuss:

how to learn math, why studying math is important, and my personal experience

studying mathematics.

\*\* How to learn math \*\*

So let's get started with the first segment: how to learn math.

This is a section from the textbook I was using for my Modern Algebra class in

my sophomore year. Don't worry about not understanding the specific bits,

between you and me I barely understand the specific bits anymore. I put this

here because I really like the wording- I think it's hilarious.

"surrender easily" evokes this image of this old British dude brandishing a

theorem at a Big Bad Polynomial in a valiant mathematical duel. I liked the idea

of fighting every theorem, every math problem, of seeing solving problems as a

way to make them yield & surrender.

I like this comic and this quote a whole lot. In my freshman year, when I was

first studying proof based math, I was taking and getting ruined by a linear

algebra course. I went to my math professor Mu Tao Wang's office hours in a

desperate but unsuccessful attempt to make some sense of the mess that was proof

based linear algebra in my head. I wanted some advice as to how to study, and

how to begin properly learning the material and really understanding it. He gave

me some advice that has slowly made more sense as I've studied and learned more

math, and he told me that the way to properly read the material was to engage

it. The way to learn a theorem is to test it and try it and challenge it, to see

under what general cases it holds true, under what special cases it might not,

what the edges cases are, what part of the theorem uses which of the given,

which conditions are necessary, to grapple with it until you've proven to

yourself without a doubt that it is true. In my experience, that's a very

important part of learning math. It's easy to accept certain things as true by

handwaving and accepting just the right amount of fuzziness to avoid hurting

your brain, but I think to properly learn a subject you must fight it. Why is

this the case? Can you prove that it is not? What counter examples come to mind,

and why are they not counter examples? In my opinion, proper math learning is a

very active endeavor- it is all too easy to let things go and accept a theorem

to be true, but then you cheat yourself out of real understanding.

I'd like to demonstrate this concept with a fairly simple theorem from number

theory.

The most exciting phrase to hear in science, the one that heralds new

discoveries, is not 'Eureka!' but 'That's funny...' Isaac Asimov

This idea extends to most other things. The best way to learn something is to do

battle with it, otherwise it goes in one hole and comes out the other.

This is a nice segway into the second part of the talk: why learning math is

useful.

\*\* Discussing Definitions \*\*

An important skill that mathematicians develop is comfort with definitions, and

fluidity and agility in learning and grappling with new definitions. After all,

mathematics is primarily concerned with "proving" things to be definitively

wrong or right, and in order to do so, you need to have strict and clear

definitions. In addition, since mathematics works in a space of more abstract

concepts, rigid definitions help us reason about more abstract things with more

logical precision. A good example of this is the definition of real numbers.

Real numbers as irrational, rational combined.

Important part of modern analysis and set theory was the rigid mathematical

definition of real numbers.

Still not a clear benefit. Important in arguments about race, about gender,

in your relationships, political,

We have strong evidence of weapons of mass destruction in Iraq

\*\* Finding counterexamples \*\*

Another important part of discourse is finding examples and counterexamples

an important part of evaluating abstract claims is by looking at specific

examples and counterexamples

really important in mathematics to be able to evaluate edge cases, to be able to

study a claim and support it via examples or disprove it via counterexamples

we make a lot of unproven claims and we hear a lot of unproven claims, and being

able to identify edge cases is important. I say this also as a programmer, it's

very important to be able to identify where an algorithm or a program might go

wrong and the important examples to check. After all, not all examples are

equally important. I can say that x\*y where x and y are integers will

always be positive, and you can list as many positive integers as you like, but

ultimately the important edge case is where x or y is a negative integer and the

other is not.

\*\* Embracing wrong-ness \*\*

Mathematical discourse is very unique in that most parties (or parties that are

good are it or at least reasonable about it) easily switch claims and discourse

is based entirely upon the validity of the argument. This is very different from

most other forms of discourse, where

this is because mathematicians are primarily concerned with the empirically

provable truth beyond doubt through logic and building upon assumptions.

Therefore, we're most interested in truth, and arguments are not personal and

emotions are not attached as often as they are. This is an important quality to

embrace in all discourse- to free yourself and your emotions from your argument

and be open and willing to discuss new points of view in the pursuit of truth.

\*\* Evaluating consequences \*\*

We saw this earlier in the theorem we looked at together, but an important

aspect of mathematics is evaluating consequences. When we look at a theorem,

what are the consequences and extensions of this new theorem? This examination

is important for two reasons:

We either discover new interesting areas to explore, or we discover logical

extensions that are inconsistent with existing knowledge, in which we've

disproved the theorem.

The same idea is useful in evaluating general claims. Oftentimes, without

evaluating the validity of the argument itself, by examining its consequences,

we can evaluate the claim itself.

\*\* Exploring Underlying assumptions \*\*

Mathematics is often regarded as rigor incarnate, but just like when we discovered

2 other types of geometry different from Euclidean geometry, the

underlying claims and assumptions of arguments are very important. As such, when

we evaluate new theorems, an important technique is to evaluate the assumptions

behind the theorem, and try to understand the theorem in context to its

assumptions. It also lends itself to extensions as we change and manipulate the

assumptions. The idea is to figure out the building blocks upon which the

theorem is constructed.

Another aspect of this is that in mathematical proofs, each step has an

underlying assumption or intent behind it (we call this motivation). By

understanding the \*\*big idea\*\* and the motivation behind proofs and theorems, we

can often better understand the theorem.

This is also applicable to the real world, since oftentimes when we find

ourselves confused, it is the mathematical and most helpful thing to do to just

evaluate the assumptions and understand the motivation behind things.

\*\* abstracting and extracting \*\*

Mathematics is, with perhaps the exception of philosophy, the most abstract

field. It is an important skill as a mathematician to be able to scale the

ladder of abstraction, and extract the essence of what is important in order to

understand and make arguments.

A good analogy is networking. When I type in google.com, a lot of stuff happens,

and all that stuff happens at different "layers of abstraction." I can discuss

hardware, I can discuss machine code, I can discuss bytes, I can discuss packets, I

can discuss higher level software, and these all exist at different levels of

abstraction. I don't have to understand hardware to do networking, and in fact I

myself do a fair amount of web development without understanding a lot of the

inner workings. I can turn on the lights without knowing exactly how the lights

are turned on.

In the same way, in mathematics, mathematicians often have to "scale" this

ladder of abstraction, from understanding definitions and theorems, to the

higher level argument, to the paper in relation to other papers, to the paper in

relation to what's interesting in the field.

This is important, because to understand certain things there is no need to

understand it to its absolute detail- you can extract what you need by

abstracting and black boxing what you don't.

I actually write my papers in the same way that I write my mathematical proofs-

the ability to evaluate consequences, explore assumptions, find counterexamples,

provide definitions, and abstract arguments is really useful. most of my

"brainstorming" and sketching actually comes in a really similar form as a math

proof.

\*\* my personal experience \*\*

The last thing I want to talk about is my personal experience into mathematics

I liked math in high school and I was always pretty good at it

Math was kind of boring to me before though, and it was an ok class but never

that interesting

numbers and geometric shapes and basic algebra was easy to pick up, and it made

sense, but the number chugging system of equations and find the length of the

third side problems were super boring to me.

I spent most of 11th grade mathematics drawing penises on my TI-84 by graphing

them. In fact, I dropped ms connors honors class because I thought it was too

hard and math was not that interesting.

I started enjoying math in my senior year. I took calculus in IB math and I

thought it was much more interesting, and I mentioned to Ms. Connor that I found

that type of math interesting. The wonderful teacher that she is, Ms. Connor

gave me a ton of her own material and I started working through all the calculus

notes. Soon I was doing a ton of math outside of school, and it took up all of

my time. I was bored for a long time, and math was deeply fascinating to me. I

enjoyed learning at my own pace and being able to move as quickly or as slowly

as I wanted to. For the first time, I saw how interesting mathematics was, and

how astonishing it was that everything just worked. It was beautifully shocking

every time I learned a new concept and saw how it just fit. I was convinced that

mathematics was art, and happily spent hours learning and appreciating the

beauty of mathematics. When I went to college, I was set on studying

mathematics, which was what was wanted (W5).

But in venturing forth into what I would describe as "real" mathematics, I

started stumbling into some roadblocks. Mathematics was (and rightfully so) no

longer the joyous, self-paced journey it was before, and instead made rigorous

and competitive by difficult material, constant problem sets, impossible exams,

and smarter classmates. Drowned in the feeling that I was bad at math, I slowly

began to lose my love of math, and I slowly forgot why I enjoyed mathematics at

all. I came in as an intended pure math major, wanting to get a Ph.D. in math,

and started second semester in a joint math/computer science major, goals now

very far from a math professorship. Worst of all, when people asked me "why do

you still do so much math?" I could not find a good answer.

When I began studying for my final in May, all I could think about and worry

about was my grade. Math was no longer enjoyable or interesting, but instead an

ordeal that I had to get through. The awe and respect I had for mathematics was

now replaced by fear and trepidation, and I regarded the final much like a

dreaded visit to the dentist's office- painful, yet unavoidable. I got a 60% on

my first college final and it was curved to a 69%, and I just had this drowning

feeling like I was bad at math, something that I found so deeply enjoyable.

But once I actually started studying the material, I underwent an amazing

change- I once again began to appreciate mathematics. The way in which the

material would expand in a wholly unexpected direction, and yet remain

undeniable true and verifiable once again drew my awe and fascination. The way

Greene's Theorem, Stoke's Theorem, and Gauss's Theorem related made intuitive

sense to me, and the picture that was so hard to keep clear slowly lost its fog.

By the time I finished studying, I re-found my love of mathematics, but also a

deep sense of guilt that I had once forgotten it.

The drive to do anything for the purpose of pursuing grades or prestige or

boosting your college resume is a bad reason to do anything. It becomes less

about the purity and enjoyment of the subject and more about what comes with the

subject. so my encouragement for you all is to see mathematics for its purity

and its beauty and enjoy it as it is, not as a proof of your intelligence, not

as a way to get into college, not as a "smart thing to do after school." IF your

interest is in math, then follow your passion

For me, personally, my interest was math, but now I'm more engaged in computer

science. And thats ok. if your interest change like mine did, then that's also

ok, just change your direction

but I say this because I want to encourage you to follow what you love and find

a direction that is meaningful and interesting to you. And if you aren't sure

what that is, just do interesting things until you figure it out.