Tutorial Machine Learning

Viscoelasticity

Outline

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Theoretical model
Viscoelasticity

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From the second law of thermodynamics we derive the Clausius-Duhem inequality:

$$\sigma\dot{\varepsilon} - \dot{e} \geq 0 \tag{1}$$

Where we mention that:

- $e = e(\varepsilon, \gamma)$, with γ an internal state variable
- Change in temperature is neglected

Using the definition for e, we get:

$$\left(\sigma - \frac{\partial e}{\partial \varepsilon}\right)\dot{\varepsilon} - \frac{\partial e}{\partial \gamma}\dot{\gamma} \geq 0 \tag{2}$$

$$\Rightarrow \qquad \sigma = \frac{\partial e}{\partial \varepsilon} \quad (3) \qquad\qquad D = -\frac{\partial e}{\partial \gamma}\dot{\gamma} \quad (4)$$

Theoretical model
Maxwell model

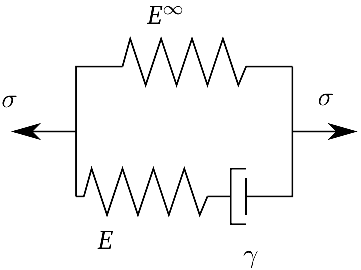

Figure: Maxwell modell

The overall energy is defined with:

$$e = \frac{1}{2}E^\infty\varepsilon^2 + \frac{1}{2}E(\varepsilon-\gamma)^2 \tag{5}$$

We therefore get for the stress and dissipation:

$$\sigma = E^\infty\varepsilon + E(\varepsilon-\gamma) \tag{6}$$

$$D = E(\varepsilon-\gamma)\dot{\gamma} \tag{7}$$

For the evolution equation we obtain:

$$\dot{\gamma} = \tilde{f}(\varepsilon,\gamma)(\varepsilon-\gamma), \quad \tilde{f}(\varepsilon,\gamma) = \frac{E}{\eta} > 0 \tag{8}$$

Generalized standard materials (GSM)

A more general framework

Now: generalized motivation for the viscoelastic model.

Introduction of thermodynamic potentials:

$$\Psi = \Psi(\mathbf{F}, \Lambda) \qquad (9) \qquad\qquad \Phi = \Phi(\mathbf{F}, \Lambda, \dot{\Lambda}) \qquad (10)$$

This implies the constitutive equations:

$$\mathbf{S} = \frac{\partial \Psi(\mathbf{F}, \Lambda)}{\mathbf{F}} \qquad (11) \qquad\qquad \frac{\partial \Psi(\mathbf{F}, \Lambda)}{\partial \Lambda} + \frac{\partial \Phi(\mathbf{F}, \Lambda, \dot{\Lambda})}{\partial \dot{\Lambda}} = 0 \qquad (12)$$

Derivation of the Maxwell model

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We then identify for the 1D case $\Lambda \to \gamma$ and $\mathbf{F} \to \varepsilon$ and define for the dissipation:

$$\Phi(\mathbf{F}, \Lambda, \dot{\Lambda}) = \phi(\varepsilon, \gamma, \dot{\gamma}) = \frac{1}{2g(\varepsilon, \gamma)}\dot{\gamma}^2, \; g(\varepsilon, \gamma) > 0 \tag{13}$$

Using the constitutive equation 12 , rewritten for the 1D case and the definition 13 for the dissipation energy:

$$\left(\sigma - \frac{\partial e(\varepsilon, \gamma)}{\partial \varepsilon}\right)\dot{\varepsilon} - \frac{e(\partial \varepsilon, \gamma)}{\partial \gamma}\dot{\gamma} \geq 0 \tag{14}$$

$$\Leftrightarrow \left(\sigma - \frac{\partial e(\varepsilon, \gamma)}{\partial \varepsilon}\right)\dot{\varepsilon} + \frac{\partial \phi(\varepsilon, \gamma, \dot{\gamma})}{\partial \dot{\gamma}}\dot{\gamma} = \left(\sigma - \frac{\partial e(\varepsilon, \gamma)}{\partial \varepsilon}\right)\dot{\varepsilon} + \frac{1}{g(\varepsilon, \gamma)}\dot{\gamma}^2 \geq 0 \tag{15}$$

GSM

Derivation of the Maxwell model

Then, we obtain the evolution equation for γ from eq. 12 with:

$$\frac{\partial e(\varepsilon,\gamma)}{\partial \gamma} + \frac{1}{g(\varepsilon,\gamma)}\dot{\gamma} \Leftrightarrow \dot{\gamma} = -g(\varepsilon,\gamma)\frac{\partial e(\varepsilon,\gamma)}{\partial \gamma} \tag{16}$$

Using the overall energy $e(\varepsilon,\gamma)$ we obtain the maxwell model:

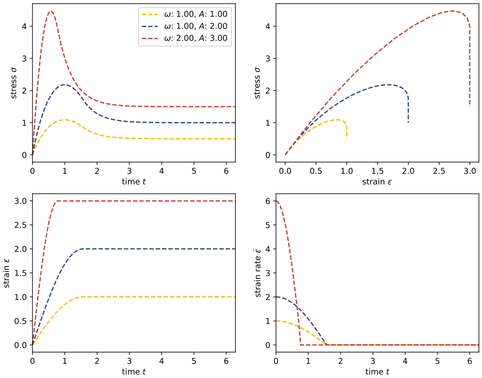
$$\sigma = E^{\infty}\varepsilon + E(\varepsilon - \gamma) \tag{17}$$

$$D = -E(\varepsilon - \gamma)\dot{\gamma} \tag{18}$$

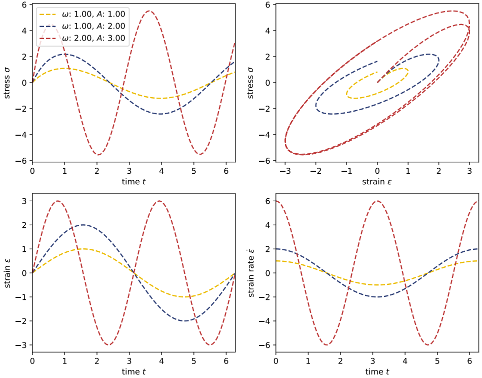
$$\dot{\gamma} = \frac{E}{\eta}(\varepsilon - \gamma) \tag{19}$$

Training data

Used for models with 3 layers, 8 nodes, 4000 training epochs



(a) Relaxation training data

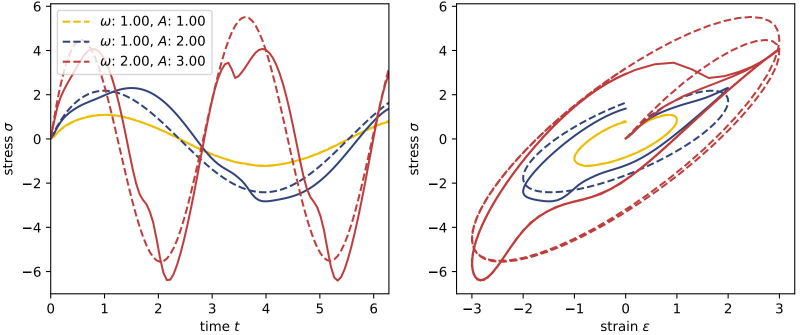


(b) Cyclic training data

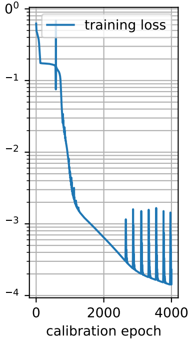
Simple RNN

Training without physical information

Data: dashed line, model prediction: continuous line



(a) Trained on both data sets ($\omega = 1, A = 1$)

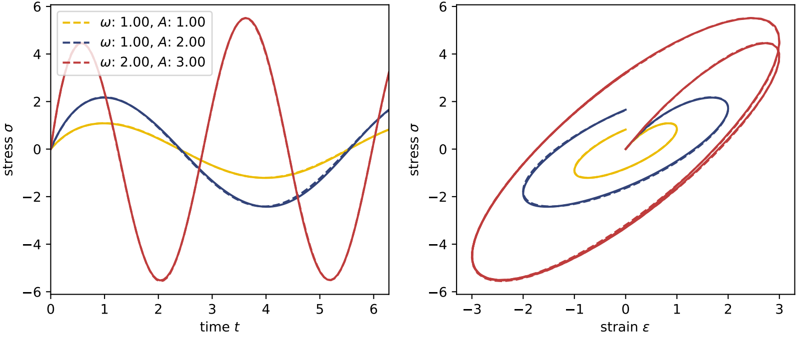


(b) Loss

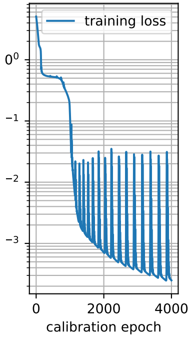
Simple RNN

Training without physical information

Data: dashed line, model prediction: continuous line



(a) Trained on both data sets ($\omega \in \{1,2\}, A \in \{1,3\}$)



(b) Loss

Maxwell model results

Evolution equation as FFNN

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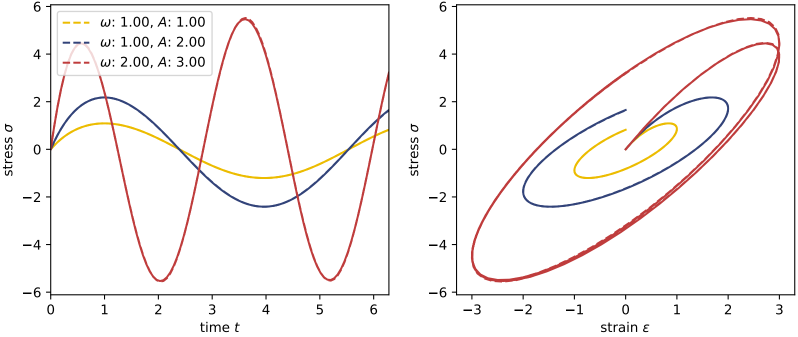
Data: dashed line, model prediction: continuous line



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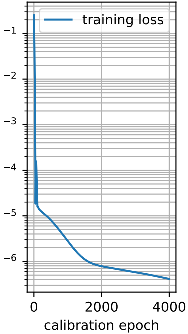

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Maxwell model results

Evolution equation as FFNN

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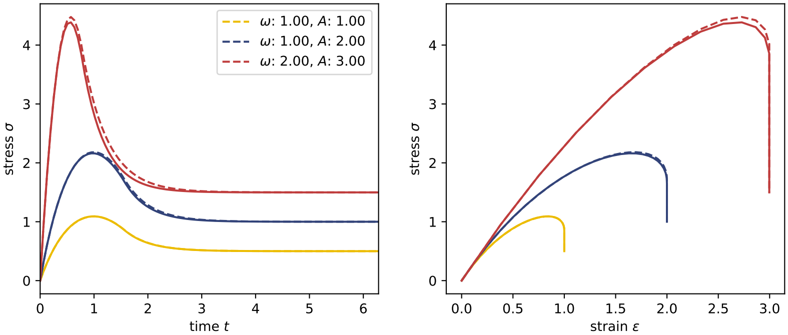
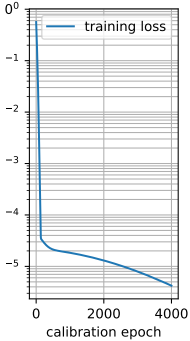


(a) Trained on cyclic data ($\omega = 1, A = 1$)



(b) Loss

Maxwell model results

Evolution equation as FFNN

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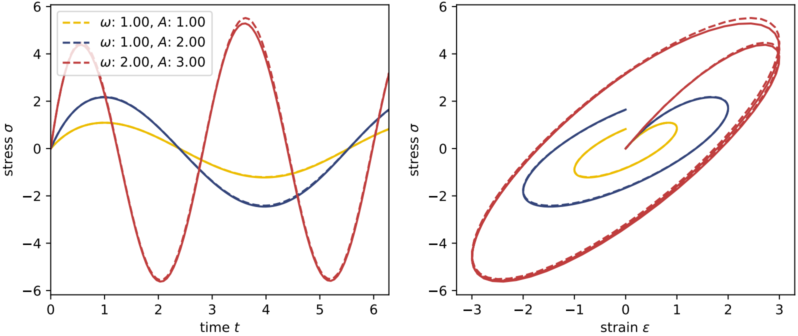
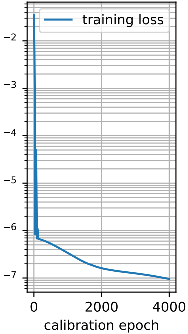


(a) Trained on relaxation data ($\omega = 1, A = 1$)

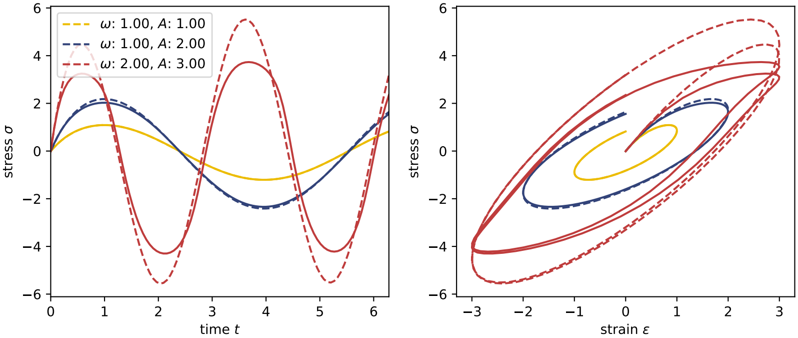


(b) Loss

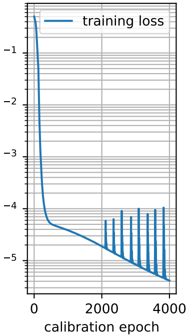
GSM results

Strain energy as FFNN

Data: dashed line, model prediction: continuous line



(a) Trained on both data sets ($\omega = 1, A = 1$)

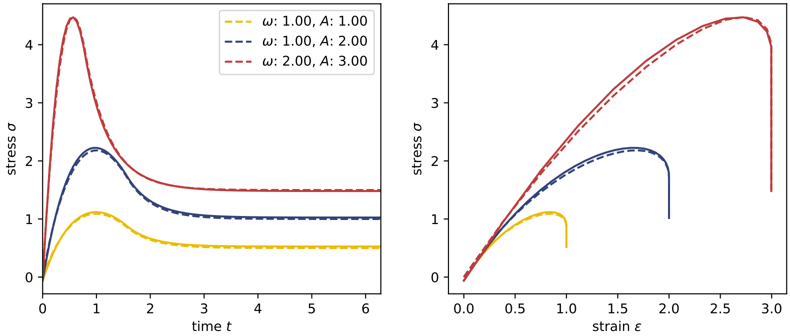


(b) Loss

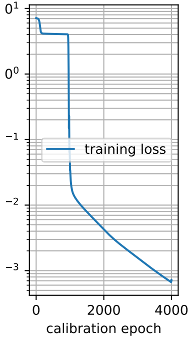
GSM results

Strain energy as FFNN

Data: dashed line, model prediction: continuous line



(a) Trained on cyclic data ($\omega \in \{1, 2\}, A \in \{1, 3\}$)



(b) Loss

GSM results

Strain energy as FFNN

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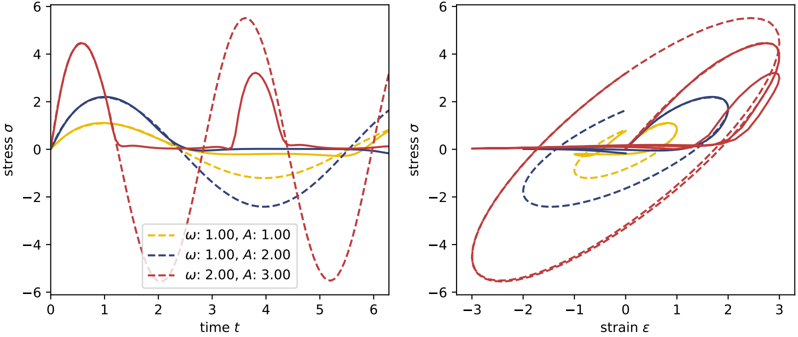

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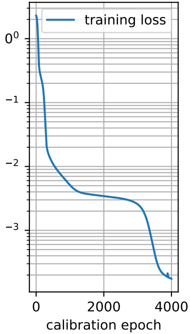

(b) Loss

Conclusion

→ Physics, e.g. the second law of thermodynamics can be included in RNN models

→ Simple RNN models without any physics included only work for carefully chosen training data (interpolation instead of extrapolation)

→ Modeling evolution equations with FFNNs works well already for only a few training data points (interpolation **and** extrapolation)

→ Energy as FFNN in GSM model yields reasonable results only in interpolation cases

⇒ Try to model only the evolution equation whenever possible using FFNNs, otherwise model the strain energy by using a wide range of training data points