INDIVIDUAL - INSTITUTIONAL DYNAMICS

$$\frac{d}{dt} G_{i,\ell}^{(ind)} = (n-i+1)\Big\{ f(\alpha(\ell)-1) + \beta(i-1) + e\beta\bar{i} \Big\} G_{i-1,\ell}$$

(in-group: benefit, cost, imitation; out-group)

$$+ (i+1)\Big\{ f(1-\alpha(\ell)) + \gamma(n-i-1) + e\gamma(n-\bar{i}) \Big\} G_{i+1,\ell}$$

(imitation)

$$- (n-i)\Big\{ f(\alpha(\ell)-1) + \beta i + e\beta\bar{i} \Big\} G_{i,\ell}$$

$$- i\Big\{ f(1-\alpha(\ell)) + \gamma(n-i) + e\gamma(n-\bar{i}) \Big\} G_{i,\ell} \qquad (1)$$

where $\bar{i} = \sum_{i,\ell} i\, G_{i,\ell}$

$\alpha(\ell)$: $\frac{d\alpha}{d\ell} > 0$; $f(x)$: $\frac{df}{dx} > 0$, $f(0) = \frac{1}{2}$ (indifference about strategy)

$$\frac{d}{dt} G_{i,\ell}^{(group)} = g(bi - c\ell)\Big\{ \mu + e\frac{Z_\ell}{Z_{\ell-1}} \Big\} G_{i,\ell-1}$$

(condition for required resources: resources, cost; spontaneous; imitation)

$$+ \Big\{ \mu g(c(\ell+1) - bi) + e\, g(bi - c\ell)\frac{Z_\ell}{Z_{\ell+1}} \Big\} G_{i,\ell+1}$$

$$- g(bi - c(\ell+1))\Big\{ \mu + e\frac{Z_{\ell+1}}{Z_\ell} \Big\} G_{i,\ell}$$

$$- \Big\{ \mu g(c\ell - bi) + e\, g(bi - c(\ell-1))\frac{Z_{\ell-1}}{Z_\ell} \Big\} G_{i,\ell} \qquad (2)$$

where $g(x)$: $\frac{dg}{dx} > 0$, $g(0) = \frac{1}{2}$ (indifference about strategy)

$$Z_\ell = \frac{\sum_i G_{i,\ell}\, \tilde{g}(bi - c\ell)}{\sum_i G_{i,\ell}} \quad ; \text{ possibly } \tilde{g} \equiv g .$$

$$\frac{d}{dt} G_{i,\ell} = \frac{d}{dt} G_{\vartheta,\ell}^{(ind)} + \frac{d}{dt} G_{i,\ell}^{(group)}$$

$$\left\{ \mu\, g(c\ell - bi) + e\, g(bi - c(\ell-1)) \frac{z_{\ell-1}}{z_\ell} \right\} G_{i,\ell} = g(bi - c\ell) \left\{ \mu + e \frac{z_\ell}{z_{\ell-1}} \right\} G_{i,\ell-1}$$

Summing over i:

$$\mu \sum_i g(c\ell - bi)\, G_{\vartheta,\ell} + e \frac{z_{\ell-1}}{z_\ell} \sum_\vartheta g(bi - c(\ell-1))\, G_{\vartheta,\ell} = \left(\mu + e \frac{z_\ell}{z_{\ell-1}}\right) \sum_i g(bi - c\ell)\, G_{i,\ell-1}$$

$$G_{i,\ell} \simeq G_\ell\, \delta_{i,0} + h_{i,\ell}\, I \sim \vartheta(1) + \vartheta(I) \qquad \text{for } I \to 0$$

$$\Rightarrow \mu\, g(c\ell)\, G_\ell + e \frac{\tilde{g}(-c(\ell-1))}{\tilde{g}(-c\ell)} g(-c(\ell-1))\, G_\ell = \left(\mu + e \frac{\tilde{g}(-c\ell)}{\tilde{g}(-c(\ell-1))}\right) g(-c\ell)\, G_{\ell-1}$$

$$\Rightarrow G_\ell = \frac{\left[\mu + e \frac{\tilde{g}(-c\ell)}{\tilde{g}(-c(\ell-1))}\right] g(-c\ell)}{\mu\, g(c\ell) + e \frac{\tilde{g}(-c(\ell-1))}{\tilde{g}(-c\ell)} g(-c(\ell-1))}\, G_{\ell-1}$$

$$\Rightarrow G_\ell = G_1 \prod_{K=1}^{\ell-1} \frac{\left[\mu + e \frac{\tilde{g}(-cK)}{\tilde{g}(-c(K-1))}\right] g(-cK)}{\mu\, g(cK) + e \frac{\tilde{g}(-c(K-1))}{\tilde{g}(-cK)} g(-c(K-1))} \qquad \checkmark$$