STEVENS
INSTITUTE of TECHNOLOGY
THE INNOVATION UNIVERSITY

Asset Replication via Kalman Filtering
FE 800 Special Problems in FE
Spring 2014 Semester

Asset Replication via Kalman Filtering

Introduction of Team Members

- Jason Gunther
- Maciej (Matt) Karasiewicz

Faculty Advisor

- Rupak Chatterjee

Asset Replication via Kalman Filtering

Literature Review

- The project and paper referenced the following technical papers and books:

1. "An Introduction to the Kalman Filter"
   by Gary Welch & Gary Bishop, 2006

2. "An Alternative Approach to Alternative Beta"
   by Thierry Roncalli & Jerome Teiletche, 2008

3. "Asset Replication"
   by Rupak Chatterjee, 2014

4. "Tracking Problems, Hedge Fund Replication and Alternative Beta."
   by Thierry Roncalli & Guillaume Weisang, 2011

5. "Introduction to Random Signals and Applied Kalman Filtering."
   by Robert Grover Brown & Patrick Y. C. Hwang, 2012

Asset Replication via Kalman Filtering

FE 800 Challenge from the Wolf of Wall Street

"Matt and Jason: If you guys think you're big time, then try to beat the returns of my hedge fund buddies on Wall Street by using only a simple portfolio of futures contracts and some fancy mathematics. At the end of the day, nothing matters but the money."

Asset Replication via Kalman Filtering

Introduction

- Asset replication strategies have recently emerged as a major research area in financial engineering and can used to generate a source of alternative alpha or beta for a portfolio.
- Replication strategies attempt to deliver returns similar to hedge funds with the added benefits of improved liquidity, transparency, and simplicity.
- Research has proven there is a tradeoff for using a more liquid and transparent replication strategy: returns tend to underperform actively managed hedge fund portfolios.
- This project examines how the Kalman Filter estimation algorithm can be used to select a portfolio of liquid futures contracts that attempts to replicate or beat the returns of a proprietary Hedge Fund Research Inc. (HFRI) index.
- The Kalman Filter Methodology Index (KMFI) portfolio will investigate whether the alpha returns of the HFRI index are possible while obtaining the added benefits of higher transparency, simplicity, and better liquidity.

Asset Replication via Kalman Filtering

Kalman Filter Basics

- The Kalman filter is a mathematical method used to estimate the true value of a hidden state given only a sequence of noisy observations.
- It is a recursive algorithm and generates optimal estimates of the state under the conditions that the state transition process and the mappings to observable measurements are linear.
- It supports estimations of past, present, and even future states of a modeled system even when the precise nature of the modeled system is unknown.

Asset Replication via Kalman Filtering

Discrete Linear Kalman Filter

- The Kalman filter attempts to estimate the state of a discrete-time controlled process ($x \in \mathbb{R}^n$) that is governed by the stochastic difference equation

  $x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}$ (Time Update Equation)

  with an observation or measurement ($z \in \mathbb{R}^m$) of the process occurring at discrete points in time according to the equation

  $z_k = H_k x_k + v_k$ (Measurement Update Equation)

- The optimization criteria used is the minimization of the mean-square estimation error of the random variable x.

Asset Replication via Kalman Filtering

The Discrete Kalman Filter Algorithm

- The Kalman Filter algorithm is a predictor-corrector algorithm. It processes discrete measurements into optimal estimates.
- Time Update Equations: Project forward (in time) the current state and error covariance estimates to obtain the initial estimates for the next time step (Predictor Equations)
- Measurement Update Equations: Incorporate a new measurement into the prior estimate to obtain an improved estimate for the next time step (Corrector Equations)

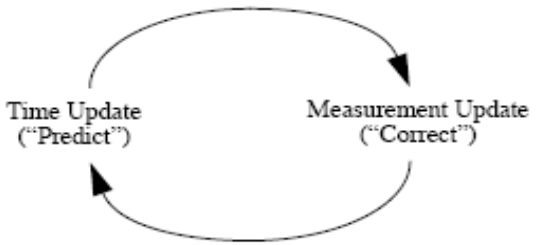
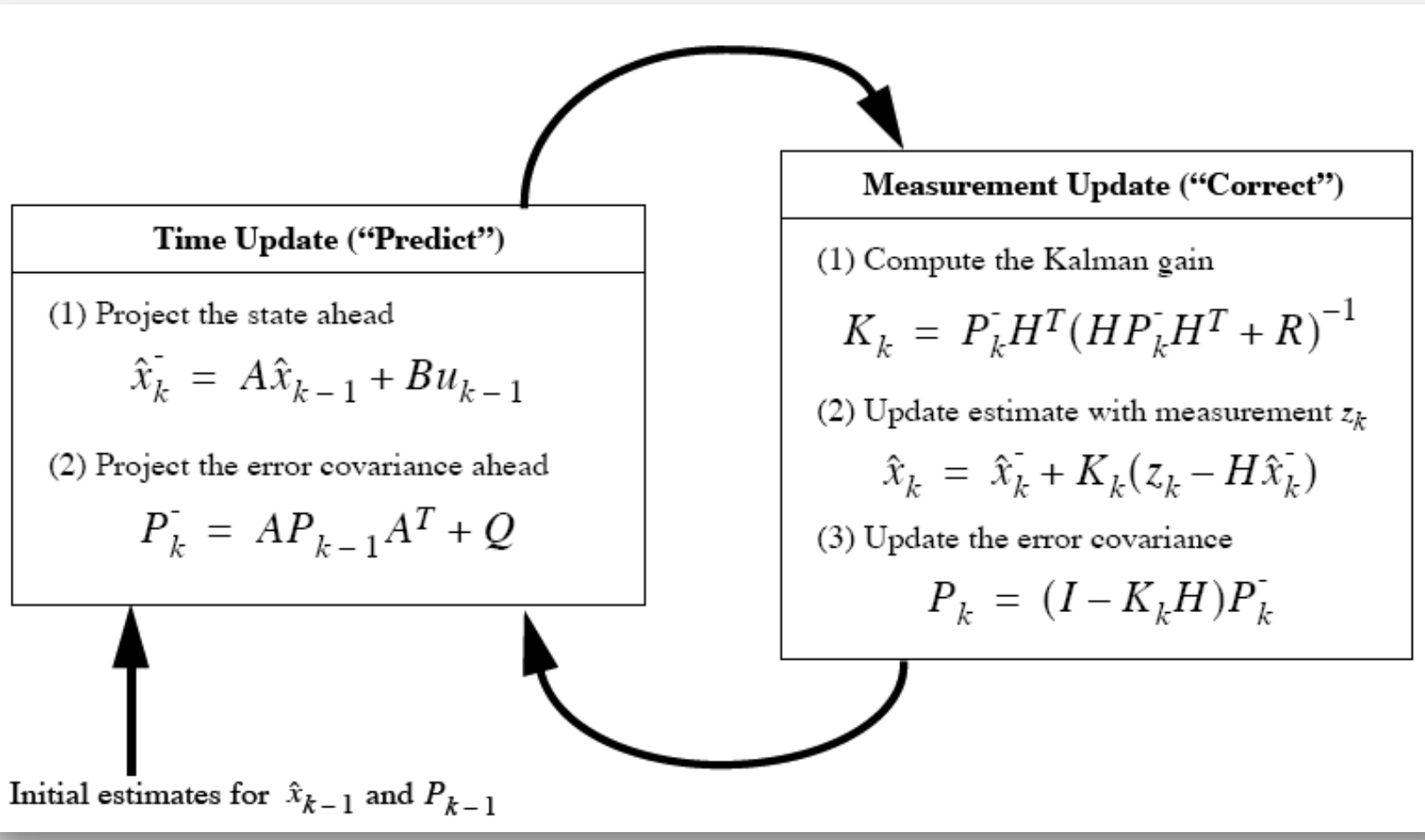


Figure 1-1. The ongoing discrete Kalman filter cycle. The *time update* projects the current state estimate ahead in time. The *measurement update* adjusts the projected estimate by an actual measurement at that time.

Asset Replication via Kalman Filtering

Kalman Filter Algorithm Process Flow



Asset Replication via Kalman Filtering

Asset Replication Basics

- Asset replication involves replicating the returns of one asset with the returns of another asset or several other assets

- Usually the asset whose returns the investor is trying to replicate may not be directly tradable or liquid so the investor needs to use cash instruments, derivatives, or exchange-traded funds (ETFs) to gain exposure to the replicated asset or index

- Examples include using Treasury and Corporate Bonds to replicate the returns of a bond index or using volatility (VIX) futures and options to replicate the returns of the spot VIX volatility index

Asset Replication via Kalman Filtering

Modeling of Index Returns

- Assume that m individual liquid assets or instruments are used to replicate the returns of an index that is not directly tradable

- The returns of this index can be described by a weighted average of factor returns:
  $r_t^{observed} = \sum_i x_t^i r_t^i + \epsilon_t$, i = 1, 2, ….m, t = 1, 2,….T
  x_t^i = the weight of the ith individual asset at time t
  r_t^i = the return of the ith individual asset at time t

- Assume that each weight (underlying state process x_t) evolves through time in the following way:
  $x_t = A_t x_{t-1} + B_t u_{t-1} + w_t$
  A_t is the weight transition model at time t
  B_t is the control input model at time t
  u_{t-1} is a possible control process
  w_t is the weight process noise assumed to be Gaussian with zero mean and covariance Q
  $w_t \sim N(0, Q)$

Asset Replication via Kalman Filtering

Kalman Filter Initial Values

- **A** = Identity Matrix sized by number of replicating assets
- **R** = Variance of residuals from the linear least squares solution
- **Q = (**Variance of HFRI index over the time period)*$\boldsymbol{A}$
- $\mathbf{x_{t-1}}$= linear least squares solution of $(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T y$
  - **X** is the matrix of assets
  - y is the vector of HFRI index
- **H** = current index values of the replicating assets

Asset Replication via Kalman Filtering

Modeling of Index Returns

- The weight process of the replicated index is not directly observable but can be mapped to an observable value of the index's returns at time t:

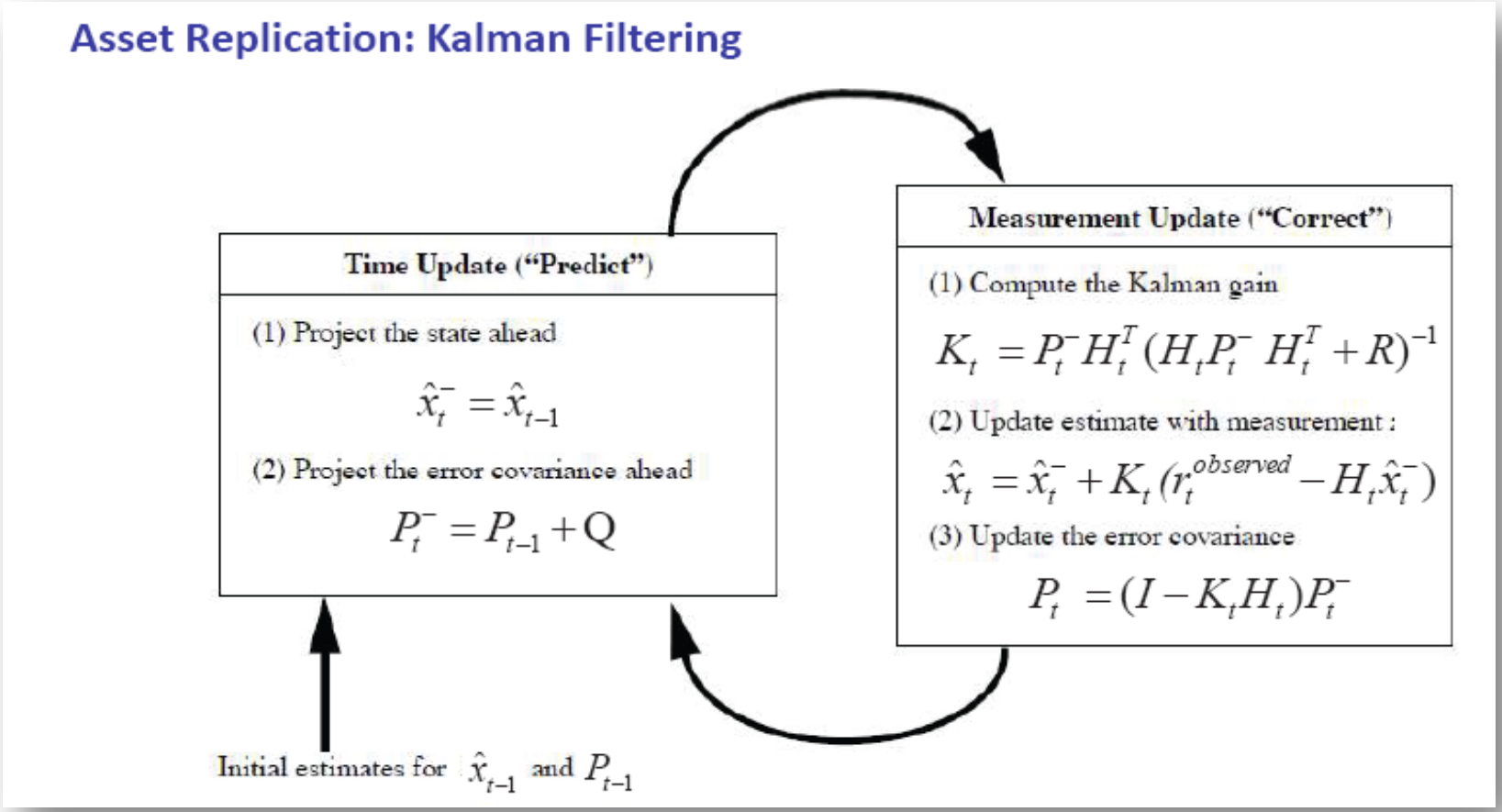
$r_t^{observed} = H_t x_t + v_t$, t = 1, 2,….T

H_t is the individual asset (factor) returns at time t

v_t is the observation process noise assumed to be Gaussian with zero mean and covariance R [$v_t \sim N(0, R)$]

Asset Replication via Kalman Filtering

Asset Replication Algorithm Process Flow



Asset Replication via Kalman Filtering

Kalman Filter Methodology Index (KFMI) Portfolio Construction

- The Kalman Filter Methodology Index (KFMI) portfolio in this project consists of a directional long/short portfolio of liquid futures contracts from multiple asset classes (equities, commodities, currencies, fixed income) where the weights of each futures contract are selected to minimize the tracking error of the selected HFRI index.

- The KFMI portfolios in this project attempts to replicate or beat the returns of the HFRI Relative Value, Macro, Event-Driven, Equity Hedge, and Emerging Markets (Total) indices.

- Time horizons chosen for the replication of the five HFRI indices were divided into 3 time periods: Pre-Crash (01/01/2003 to 09/30/2007), Credit Crunch-to-Crash Recovery (10/01/2007 – 12/31/2009), and Post-Crash (01/01/2010 to 01/31/2014).

Asset Replication via Kalman Filtering

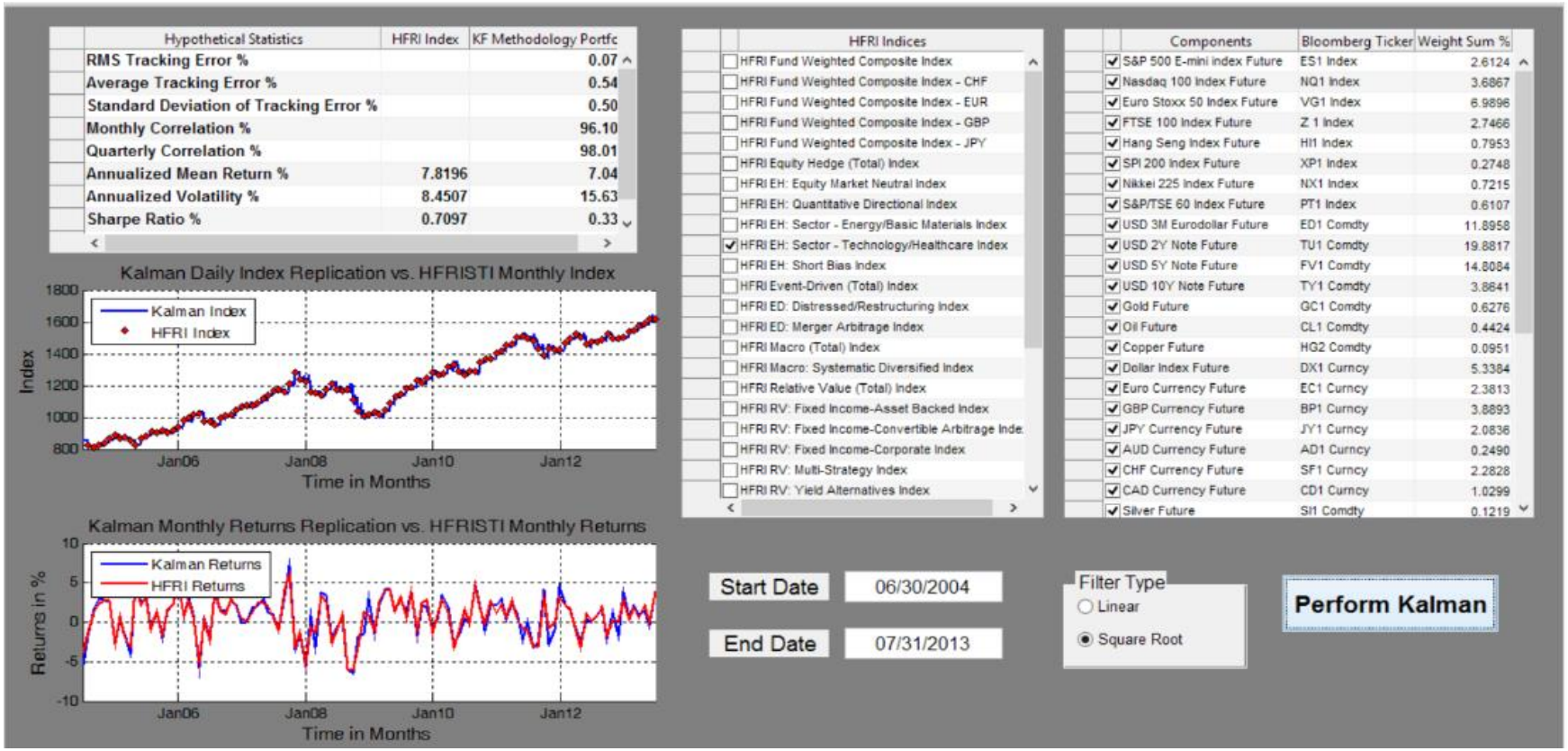
Kalman Filter Methodology Index (KFMI) Portfolio Construction

KFMI Relative Value, Macro, Event-Driven, and Equity Hedge Portfolio Components
S&P 500 E-mini index Future
Nasdaq 100 Index Future
Euro Stoxx 50 Index Future
FTSE 100 Index Future
Hang Seng Index Future
SPI 200 Index Future
Nikkei 225 Index Future
S&P/TSE 60 Index Future
USD 3M Eurodollar Future
USD 2Y Note Future
USD 5Y Note Future
USD 10Y Note Future
Gold Future
Oil Future
Copper Future
Dollar Index Future
Euro Currency Future
GBP Currency Future
JPY Currency Future
AUD Currency Future
CHF Currency Future
CAD Currency Future
Silver Future

KFMI Emerging Market Portfolio Components
S&P 500 E-mini Index Future
Nasdaq 100 Index Future
Hang Seng Index Future
USD 3M Eurodollar Future
USD 2Y Note Future
USD 5Y Note Future
USD 10Y Note Future
Gold Future
Oil Future
Copper Future
Dollar Index Future
Brazil Bovespa Equity Index Future
Mexican IPC Equity Index Future
Indian S&P CNX Nifty Equity Index
South Africa FTSE/JSE Top 40 Equity Index Future
France CAC 40 Equity Index Future
Germany DAX Equity Index Future
Poland WIG 20 Equity Index Future
Portugal PSI-20 Equity Index Future
Spain IBEX 35 Equity Index Future
Israel Tel Aviv 25 Equity Index Future
Turkey ISE 30 Equity Index Future
Singapore SGX Straights Times Equity Index Future
South Korea Kospi 200 Equity Index Future
Thailand SET 50 Equity Index Future

Asset Replication via Kalman Filtering

Kalman Filter Methodology Index (KFMI) Portfolio Construction GUI



Asset Replication via Kalman Filtering

KFMI Portfolio Replication Results

- Daily KFMI replication portfolio index values are graphed against their corresponding monthly HFRI index values to see how closely the KFMI portfolio tracks the HFRI index over each time horizon.

- Monthly returns of the KFMI replication portfolio are also graphed against the monthly returns of the corresponding HFRI index to compare the performance of the KFMI replication portfolio against its HFRI benchmark index over each time horizon.

- To get a sense of how well the KFMI replication portfolio is tracking its benchmark HFRI index, the following tracking statistics are calculated for each KFMI replication portfolio over each time horizon: RMS tracking error, average tracking error, standard deviation of tracking error, monthly correlation, and quarterly correlation.

- To compare the performance of each KFMI replication portfolio against its targeted HFRI index, the following performance attribution statistics are calculated for each of the three time horizons selected: annualized mean return, annualized volatility, Sharpe Ratio, and Modigliani-Modigliani Measure (M2).

Asset Replication via Kalman Filtering

KFMI Portfolio Tracking and Performance Statistics Pre-Crash (01/01/2003 to 09/30/2007)

Hypothetical Statistics	HFRI Relative Value (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.03%
Average Tracking Error		0.35%
Standard Deviation of Tracking Error		0.28%
Monthly Correlation		81.39%
Quarterly Correlation		92.77%
Annualized Mean Return	8.10%	7.89%
Annualized Volatility	2.21%	7.97%
Sharpe Ratio	2.28%	0.61%
M2	3.11%	3.11%

Hypothetical Statistics	HFRI Equity Hedge (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.04%
Average Tracking Error		0.39%
Standard Deviation of Tracking Error		0.33%
Monthly Correlation		94.99%
Quarterly Correlation		99.16%
Annualized Mean Return	12.33%	11.83%
Annualized Volatility	5.48%	11.27%
Sharpe Ratio	1.69%	0.78%
M2	3.16%	3.15%

Hypothetical Statistics	HFRI Macro (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.03%
Average Tracking Error		0.36%
Standard Deviation of Tracking Error		0.26%
Monthly Correlation		95.91%
Quarterly Correlation		99.42%
Annualized Mean Return	9.40%	8.95%
Annualized Volatility	5.16%	9.05%
Sharpe Ratio	1.23%	0.65%
M2	3.13%	3.12%

Hypothetical Statistics	HFRI Emerging Markets (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.07%
Average Tracking Error		0.68%
Standard Deviation of Tracking Error		0.58%
Monthly Correlation		93.57%
Quarterly Correlation		98.99%
Annualized Mean Return	23.94%	22.81%
Annualized Volatility	7.82%	18.36%
Sharpe Ratio	2.67%	1.08%
M2	3.27%	3.26%

Hypothetical Statistics	HFRI Event-Driven (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.05%
Average Tracking Error		0.52%
Standard Deviation of Tracking Error		0.36%
Monthly Correlation		90.37%
Quarterly Correlation		98.40%
Annualized Mean Return	13.80%	13.34%
Annualized Volatility	4.67%	11.51%
Sharpe Ratio	2.30%	0.89%
M2	3.17%	3.17%

Asset Replication via Kalman Filtering

KFMI Portfolio Tracking and Performance Statistics Credit Crunch-to-Crash Recovery (10/01/2007 – 12/31/2009)

Hypothetical Statistics	HFRI Relative Value (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.05%
Average Tracking Error		0.80%
Standard Deviation of Tracking Error		0.65%
Monthly Correlation		92.31%
Quarterly Correlation		98.55%
Annualized Mean Return	1.88%	2.07%
Annualized Volatility	9.15%	19.52%
Sharpe Ratio	0.05%	0.03%
M2	1.43%	1.43%

Hypothetical Statistics	HFRI Equity Hedge (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.10%
Average Tracking Error		1.25%
Standard Deviation of Tracking Error		1.49%
Monthly Correlation		86.69%
Quarterly Correlation		99.54%
Annualized Mean Return	-4.45%	-6.02%
Annualized Volatility	12.95%	24.93%
Sharpe Ratio	-0.45%	-0.30%
M2	1.37%	1.35%

Hypothetical Statistics	HFRI Macro (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.03%
Average Tracking Error		0.43%
Standard Deviation of Tracking Error		0.32%
Monthly Correlation		94.49%
Quarterly Correlation		97.13%
Annualized Mean Return	4.32%	3.76%
Annualized Volatility	5.37%	10.50%
Sharpe Ratio	0.54%	0.22%
M2	1.46%	1.45%

Hypothetical Statistics	HFRI Emerging Markets (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.18%
Average Tracking Error		2.30%
Standard Deviation of Tracking Error		2.75%
Monthly Correlation		82.96%
Quarterly Correlation		99.82%
Annualized Mean Return	-4.68%	-5.26%
Annualized Volatility	18.20%	35.99%
Sharpe Ratio	-0.34%	-0.19%
M2	1.37%	1.36%

Hypothetical Statistics	HFRI Event-Driven (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.05%
Average Tracking Error		0.71%
Standard Deviation of Tracking Error		0.69%
Monthly Correlation		94.25%
Quarterly Correlation		99.48%
Annualized Mean Return	-1.84%	-2.28%
Annualized Volatility	10.15%	16.72%
Sharpe Ratio	-0.32%	-0.22%
M2	1.39%	1.39%

Asset Replication via Kalman Filtering

KFMI Portfolio Tracking and Performance Statistics Post-Crash (01/01/2010 to 01/31/2014)

Hypothetical Statistics	HFRI Relative Value (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.08%
Average Tracking Error		0.83%
Standard Deviation of Tracking Error		0.87%
Monthly Correlation		54.76%
Quarterly Correlation		90.08%
Annualized Mean Return	6.75%	6.59%
Annualized Volatility	3.30%	13.27%
Sharpe Ratio	2.00%	0.49%
M2	0.20%	0.19%

Hypothetical Statistics	HFRI Equity Hedge (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.04%
Average Tracking Error		0.49%
Standard Deviation of Tracking Error		0.42%
Monthly Correlation		96.24%
Quarterly Correlation		98.77%
Annualized Mean Return	5.98%	5.65%
Annualized Volatility	8.27%	11.40%
Sharpe Ratio	0.71%	0.48%
M2	0.19%	0.19%

Hypothetical Statistics	HFRI Macro (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.04%
Average Tracking Error		0.40%
Standard Deviation of Tracking Error		0.34%
Monthly Correlation		94.27%
Quarterly Correlation		98.88%
Annualized Mean Return	1.27%	1.25%
Annualized Volatility	4.63%	8.71%
Sharpe Ratio	0.25%	0.13%
M2	0.14%	0.14%

Hypothetical Statistics	HFRI Emerging Markets (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.05%
Average Tracking Error		0.56%
Standard Deviation of Tracking Error		0.41%
Monthly Correlation		97.22%
Quarterly Correlation		99.13%
Annualized Mean Return	3.58%	2.96%
Annualized Volatility	10.23%	13.44%
Sharpe Ratio	0.34%	0.21%
M2	0.16%	0.16%

Hypothetical Statistics	HFRI Event-Driven (Total) Index	KF Methodology - Portfolio
RMS Tracking Error		0.07%
Average Tracking Error		0.73%
Standard Deviation of Tracking Error		0.70%
Monthly Correlation		82.61%
Quarterly Correlation		96.56%
Annualized Mean Return	7.04%	6.88%
Annualized Volatility	5.60%	13.04%
Sharpe Ratio	1.23%	0.52%
M2	0.20%	0.20%

KFMI Portfolio Tracking & Performance Observations

- The linear and square root KFMI replication portfolios generally have slightly smaller annualized mean returns, higher annualized volatilities, smaller Sharpe ratios, and almost identical M2 measures when compared against their respective HFRI indices in most of the three time horizons.

- The average tracking error for each of the linear and square root KFMI replication portfolios is less than 1% for the Pre-Crash and Post-Crash time horizons. The average tracking error for each of the linear and square root KFMI replication portfolios have a range of 0.43% to 2.3% for the Credit Crunch-to-Crash Recovery time horizon.

- For the Pre-Crash and Post-Crash time horizons, the linear and square root KFMI replication portfolios can generally capture at least 90% of the annualized returns of the corresponding Relative Value, Macro, Event-Driven, and Equity Hedge HRFI index that they are attempting to replicate.

Asset Replication via Kalman Filtering

Conclusions

- Despite having good tracking statistics, the KFMI portfolios generally underperformed their HFRI benchmarks while having higher average annualized volatility.
- The alpha or excess returns of the HFRI indices could not be reproduced through low frequency trading in a portfolio of liquid futures.
- The component hedge funds of each of the HFRI Indices typically generate their excess returns from investment in illiquid assets (real estate and private equity), non-linear exposures using OTC derivatives, or using high-frequency trading strategies.

Asset Replication via Kalman Filtering

Conclusions

- These hedge funds usually suffer from several criticisms: lack of transparency of the manager's strategy, poor liquidity, and fair pricing of their management fees.
- Recent declining average performance of the hedge fund industry as a whole along with interrogations on the levels of fees that they charge have led many investors to seek means of capturing hedge fund investment strategies and performance without direct investment in this alternative investment vehicle.
- The Kalman Filter Methodology Index HFRI replication portfolio turns out to be a more transparent and simpler investment strategy despite not being able to reproduce the full alpha of its benchmark HFRI index.

Asset Replication via Kalman Filtering

Questions and Answers