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The puzzle of Carbon Allowance spread

Financial Engineering Final Project
Mathematical Engineering - Quantitative finance

Authors:

Bechi Carlo
Gaspari Cecilia
Stringara Jacopo

Student ID:

10780853
10707989
10687726

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Introduction

The European Union Emission Trading Scheme (EU-ETS) is a fundamental component of the European Union's efforts to mitigate climate change. Launched in 2005, the EU-ETS has undergone several phases, each representing a step forward in the EU's approach to carbon emissions regulation.

Phase I, spanning from 2005 to 2007, served as an experimental period aimed at laying the groundwork for future phases. During this time, the majority of emission allowances were distributed freely, leading to a high volatility in prices which sometimes dropped to values close to zero.

Phase II, covering the window of time from 2008 to 2012, coincided with the initial commitment period of the Kyoto Protocol for the European Union. Despite some adjustments, such as a transition to more centralized regulation, Phase II largely maintained the free allocation model of Phase I, resulting in continued market instability and challenges in price discovery.

Phase III, which lasted from 2013 to 2021, marked a significant shift in the EU-ETS framework. Key reforms included the introduction of a single EU-wide emissions cap, the phasing out of free allocation in favor of auctioning and measures to enhance market liquidity and stability. These changes aimed to address the shortcomings of previous phases and create a more robust and effective carbon market.

The most recent phase, Phase IV, continues on the trajectory of increasing stringency and market efficiency. Building upon the foundations laid in Phase III, Phase IV introduces further measures to reduce emissions, expand the scope of regulated sectors, and align the EU-ETS with broader climate objectives outlined in initiatives such as the European Green Deal.

Despite the progress made in successive phases, one persistent challenge in the EU-ETS has been the existence of a cost-of-carry spread (C-spread) above the risk-free rate. This anomaly has puzzled researchers and policymakers alike, prompting investigations into its underlying causes and implications.

This paper aims to contribute to this ongoing discourse by examining the drivers of the C-spread and proposing potential policy interventions to address it. By understanding the dynamics of the EU-ETS and its impact on carbon markets, we can better inform efforts to combat climate change and transition to a more sustainable future. Clarity and transparency in the market are essential for fostering ecological transition. Therefore, gaining a clear understanding of the market dynamics and ensuring accountability in financial transactions are crucial steps towards achieving this transition.

1 | Preprocessing

In the **preprocessing** script, we read data from Excel and CSV files and processed them to make it easier to use in our analysis. In particular, among the available data, we have information on EUA futures, which are futures on European Unit Allowances (the contract size is 1000 EUA per lot and the contract price is denominated in Euro per EUA) and we have information about bonds issued by the European main polluters among the EU-ETS compliant entities. We extracted all available data for the phase III e and phase IV, i.e. from the beginning of 2013 to the end of October 2022 (the last available date in our data set). Moreover we created a function to process data from each Excel and CSV file, they can all be found in the **Preprocessing** folder. In this chapter, we will briefly describe the functions contained in this folder and how the data are organized after preprocessing.

1.1. Preprocessing OIS rates

In the **preprocess_OIS** function we read data about the OIS rates, filling missing data with the previous values, removed duplicates and divided the rates by 100, since they were stored as percentages. The result is a table called **OIS_Data** where OIS rates are available for various tenors, including short-term durations such as 1, 2, and 3 weeks, as well as 1-month increments up to 11 months. Additionally, rates are provided for 1 year, and further durations extending to 10 years, with intervals such as 15 months, 18 months, and 21 months.

1.2. Preprocessing March, June, September Futures

We implemented the **preprocessVolumesMonth** function for the futures with expiry in March, June and September. Giving one of this months as input to the function we can decide which of these futures' classes to process. In the scope of our study we were interested solely in the volumes of these futures, in particular, for each date, we considered the volumes of the future with closest expiry date, switching to the following future after the expiry date. The processed data are stored in three tables: **Volumes_june_front**, **Volumes_march_front** and **Volumes_sept_front**.

1.3. Preprocessing December and Overnight Futures

We built **preprocessDecember** to process futures data with expiry in December; for these we were interested not only in volumes but also in closing prices. We built three different tables, the first one considering the futures with the closest expiry (which we will refer to as Front), the second for those with the second closest expiry (which we will refer to as Next), and the last for those with the third closest expiry (which we will refer to as Second Next). Moreover for each row in the tables, we reported the expiry date of the futures from which we are taking the data. The processed data can be found in **Front_December**, **Next_December** and **Next_2_December** tables.

Regarding the overnight futures, we built the table **Daily_Future** with their closing prices with **preprocessDailyFuture** function.

Furthermore, since to compute the C-spread, we need both the December Futures and the daily prices, we restricted our dataset only to the dates on which both were available (we will refer to this set of dates as common dates).

1.4. Preprocessing Open Interest and Extra Variables

With the **preprocess_OI** function, we created a table with two time series, the first one for the open interest of the Front December futures and the second one for the open interest of the Next December futures. Instead with **preprocess_Extra_Variables** we extracted data about some financial control variables that we used in our analysis, the data are about the S&P 500 (column SPX in the table), VIX and the WTI; since the SPX and WTI are financial indexes we opted to take into consideration the log returns of both instead of the raw data set.

1.5. Preprocessing Bonds data

Lastly, we processed data about bonds with the **preprocessBonds** function. We extracted information about a list of valid bonds, such as the coupon rate, the maturity date, the original amount issued, the coupon frequency and the parent ticker, the latter is very important because it indicates which company issued the bond. Indeed, to restrict the scope of our study, we only considered the bonds emitted by Europe's largest polluters, nominally the following issuers:

- Arcelor Mittal (MT)
- ENEL (ENEI)
- ENGIE (ENGIE)
- Lafarge (LAFARGE)
- Heidelberg Materials (HEIG)
- EDF (EDF)
- ENI (ENI)
- Total Energies (TTEF)
- EON (EONG)
- AP Moeller (MAERS)
- CEZ (CEZP)
- Veolia Environment (VIE)

Thus, we filtered the valid bonds by selecting only those issued by the issuers above with an original amount of 500 million or above.

We built the cell array **Bonds** where each element is a struct referring to a valid bond, containing all the information above. In addition to the previous features, we also included in the struct all the dates on which the bond is traded in the market with their respective prices. If the bond is not traded even for a single day in phase III, we discarded it. Finally, we computed the coupon payment dates, starting from the maturity date and going back to the issuance date according to the coupon frequency.

2 | Chapter two

2.1. Bootstrap the risk-free interest rate from OIS rates

In order to obtain the risk-free rates from the OIS swap rates, we implemented the **bootstrapCurves** function. Firstly, we filled a matrix with all the dates on which we needed to find the rates and, after applying the modified follow convention, we calculated the respective year fractions with the European 30 360 convention. Each row of said matrix, represents the expiry dates for the rates curve on that date.

When building the discount factor curves, swaps with a maturity up to one year and swaps with longer maturities are managed in different ways:

- For swaps with a maturity up to one year the discount factors are computed as follows:
$$P^D(t_0, t_e) = \frac{1}{1 + \delta(t_0, t_e) \cdot R^{OIS}(t_0, t_e)}$$
- While if the OIS has a maturity t_i longer than one year the discount factor is equal to:
$$P^D(t_0, t_i) = \frac{1 - R^{OIS}(t_0, t_i) \cdot \sum_{k=i}^{i-1} \delta_k \cdot P^D(t_0, t_k)}{1 + \delta_i \cdot R^{OIS}(t_0, t_i)}$$

Subsequently we easily computed the zero rates curve as

$$y(t, T) = \frac{-\log B(t, T)}{\delta(t, T)}$$

To better observe the behaviour of the zero rates curve during the period under scrutiny, we created a small animated graph. During the animation of the zero rates, we observed that they are generally stable throughout the entire time period, with the exception of some peaks occurring in 2011. We identified some possible anomalies in the OIS rates corresponding to these dates. Nonetheless, we simply chose to ignore these possible anomalies since they fall outside of the period under study.

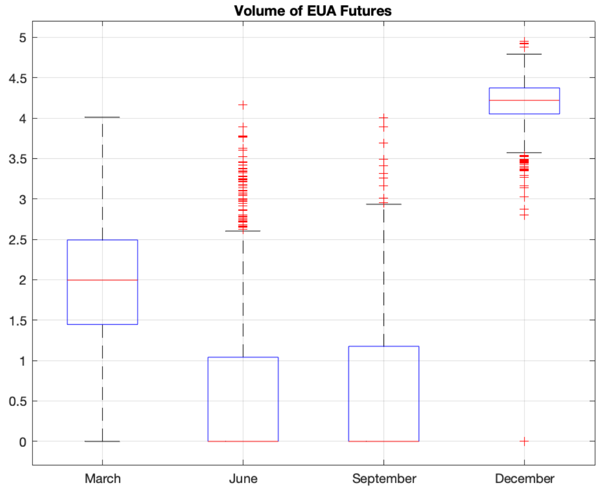
Lastly, we created the **fill_zrates** function to align the bootstrap curves align with the common dates mentioned above. If no curve was available on a given date, we simply used the previous one.

2.2. Futures liquidity analysis

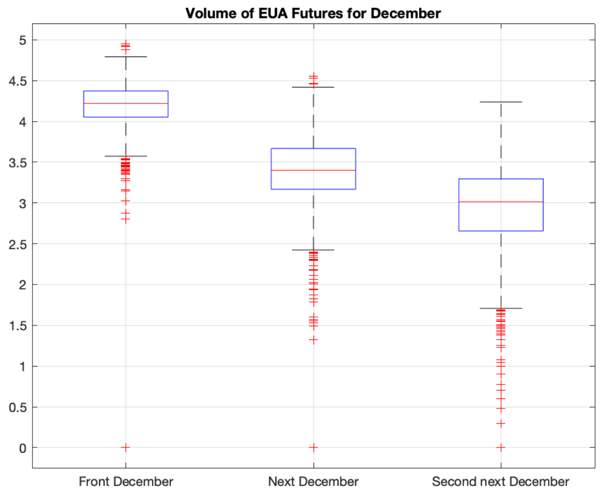
We analyzed liquidity for each future by examining the traded volumes of European Allowance (EUA) futures on the Intercontinental Exchange (ICE) Market. Our initial focus was on the March, June, September, and December EUA futures within the Phase III time window. Using boxplots, we aimed to determine which futures were the most liquid. This is particularly relevant; indeed, considering the most liquid instrument allows us to achieve more reliable results.

All futures traded on the ICE Market adhere to standard features: each contract represents 1000 EUAs, priced in Euros per EUA. December EUA futures expire on the penultimate Monday of the delivery month and additionally, all open futures contracts are marked-to-market with daily margins, remunerated at the EONIA/e-STR rate during Phase III.

To enhance visualization, we presented the data on a logarithmic scale, adding one contract to each data point of the volume to avoid zero values in the logarithm argument. Our analysis revealed that December futures consistently exhibited significantly higher liquidity compared to other months, often by at least two orders of magnitude.



Furthermore, it's worth mentioning that the liquidity comparison also applied to the December contracts with respect to their Front, Next, and Second Next counterparts. In this case as well, the difference in order of magnitude is approximately one, reaffirming the front contract's absolute liquidity. Here too, we chose to plot the volumes for the December contracts in a logarithmic scale.



2.3. C-spread

The C-spread (or "Carbon Allowance Spread") is a helpful metric for understanding the costs involved in trading carbon emissions within the EU Emissions Trading System (EU-ETS). It measures the difference between the price of futures on carbon allowances and their spot price, while also considering the risk-free interest rate. Understanding this spread is crucial for grasping market movements and financial implications for companies trading carbon emissions.

The fact that the C-spread is consistently positive, known as the "carrying cost puzzle", suggests that the spread is larger and more volatile than what might be expected solely from changes in market interest rates.

In our analysis, we focus on the most liquid contracts, particularly December Front contracts, as they provide more reliable and consistent data. This choice ensures that our results are robust and reflect real market conditions.

The C-spread at time t is defined as:

$$C_t = \frac{\log \frac{F_{t,T}}{S_t}}{T-t} - r(t,T)$$

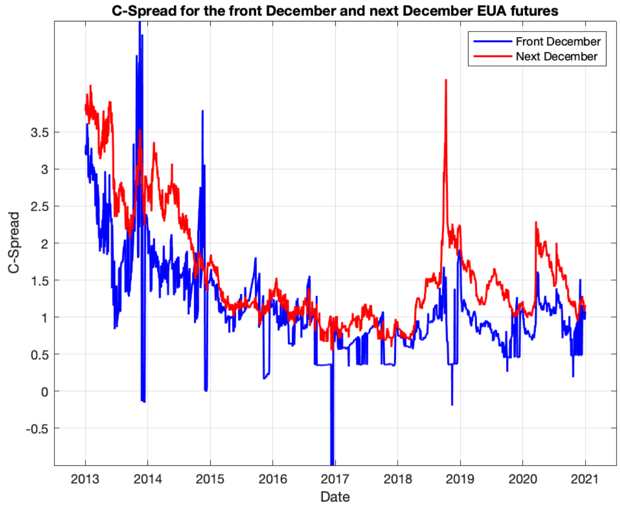
where $F_{t,T}$ is the future price of a contract maturing at time T, S_t is the spot price at time t, and $r(t,T)$ represents the risk-free interest rate over the period from t to T.

In order to calculate the C-spread, we use overnight futures as a reliable estimate for the spot price because the spot market for EU allowances on the European Energy Exchange (EEX) is not very liquid.

In summary, by using the C-spread and focusing on highly liquid contracts, we aim to provide a deeper understanding of cost-of-carry dynamics in the EU-ETS and to clarify underlying factors contributing to the observed spread.

2.3.1. C-spread for the Front and Next contracts

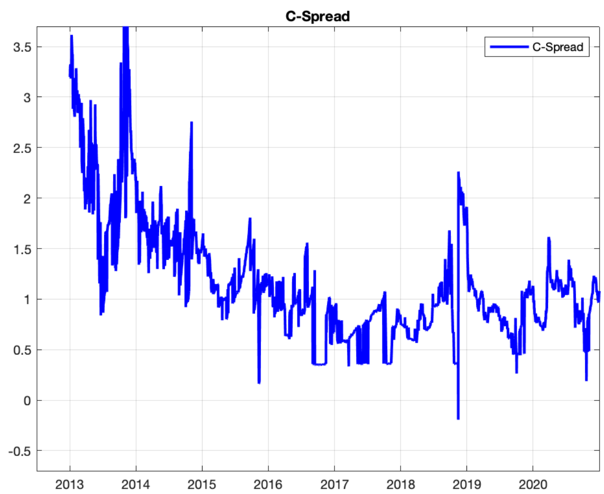
Practically, we began by interpolating the risk-free rates on the available dates using the **RiskFreeRate** function. Then, we computed the C-spread according to the formula above using the **compute_C_spread** function. We calculated them for both Front December futures and next December future. We stored these results respectively in two tables: **C_spread_front** and **C_spread_next**. Finally, we visualized the results in percentage in the following plot:



From this image, we can observe some peaks in the front plot corresponding to expiry dates. This occurs because the denominator tends to zero when we are close to the expiry date. Indeed, if we instead observe the curve computed using the Next December futures, it is more regular since the denominator never approaches zero.

2.3.2. Rollover rule

To prevent the denominator from becoming null, we implemented a rollover rule: we selected Front C-spread and switched to the Next C-spread starting from November 15th, which usually corresponds to approximately one month prior to the expiry. This regularization significantly smoothed out our plot, eliminating irregular behaviors as can be seen in the following plot:



2.4. Z-index

The Z-index is a measure of the spread between the rate $R(t, T)$, at which a player finances itself, and the risk-free rate, where with "player" we refers to one of the European main polluters among the EU-ETS compliant entities.

To represent the Z-spread for these companies, we calculated a representative index known as the Z-index, this is nothing more than the average credit spread in the EU ETS market. This process used bond data issued by these companies and involved two MATLAB functions: **compute_ZSpread_Bond** and **compute_ZSpread**. Let us look at what each function does:

compute_ZSpread_Bond:

- This function calculates the Z-spread for each bond on every date during Phases III and IV of the EU-ETS.
- The Z-spread is computed as the quantity to add to the risk-free zero rate curve to make discounted cash flows of the bond equal to its current market price.

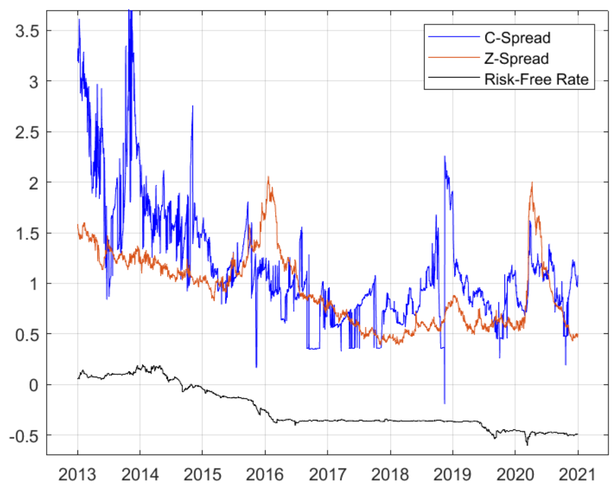
compute_ZSpread:

- This function constructs the Z-index by aggregating the Z-spreads calculated for each bond.
- It first calculates the Z-spread for each issuer as the weighted average of the Z-spreads of all the bonds they have issued and that are traded on a given date, with weights based on the bond issued amount.
- Then, it computes the Z-index as the average of the Z-spreads of all the issuers that have bonds traded on a given date.

In order to obtain a smoother behaviour in the Z-index, we chose to exclude from the analysis the bond with code "XS0877820422", since it displays very unusual behaviour compared to the others. We also set the Z-spreads of the bonds to 0 when all year fractions between the value date and coupon payments were less than 0.075; this year fraction corresponds to less than one month, thus introducing a roll-over rule similar to the employed for the C-spread above. Indeed very small time intervals often cause the Z-spreads to become very high when using root-finding algorithms to compute the Z-spread of a given bond. The results of this calculation were stored in the vector **Z_spread**.

2.5. Plots and correlograms

We plotted C-spread, Z-index and three month rate r_t, computed as the zero-rate associated to the three months OIS rate, for Phase III in percentage in order gain an understanding of their relations to one-another.

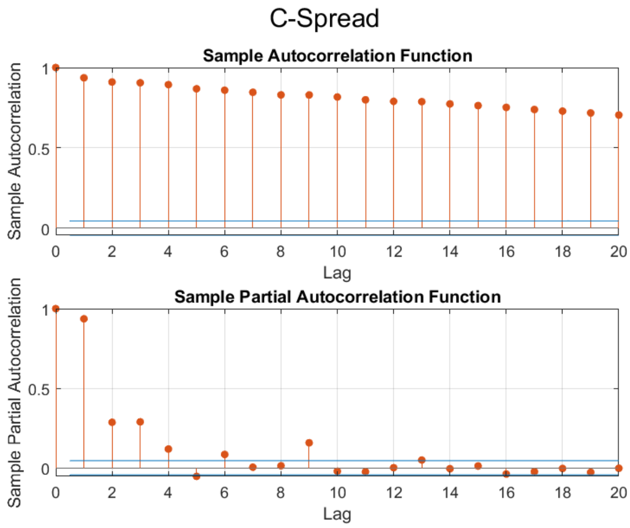


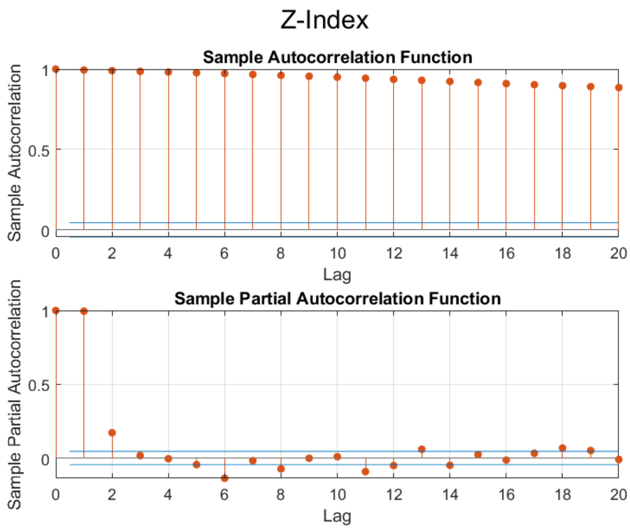
We observed that the C-spread remained positive throughout the entire phase, except for a negative spike at the end of 2018, moreover the C-spread and the Z-index seem to be of the same order of magnitude, while the rate is much smaller. In general, the C-spread and Z-index seem to display a high degree of correlation and often seem to move together. In the following chapters, we will investigate this initial intuition to determine if a long-run relationship actually exists between the C-spread and the Z-index. This could lead us to discover how and why the C-spread behaves as we have observed.
First of all, we computed the mean and standard deviation for all three time series:

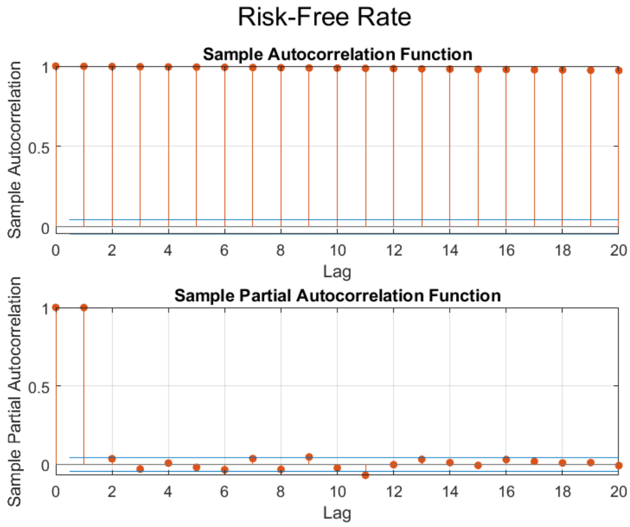
	C-spread	**Z-index**	**Risk Free Rate**
Mean	1.19	0.95	-0.24
Standard Deviation	0.62	0.36	0.22

Table 2.1: Summary table of the three time series. Values are expressed as percentages.

The means and standard deviations confirm what we had previously observed from the graph: the C-spread, on average, exceeds 1% and it appears to be quite volatile, the Z-Index shows similar behaviour. On the other hand, the risk-free rate shows negative average values and low volatility. This can be easily explained: during those years, interest rates were very low, sometimes even negative, and the curve was quite flat. This can be easily be seen in the animation provided before.







We plotted the ACF and PACF correlograms (confidence interval 95%) for the three time series: the C-spread, the Z-index, and the risk free rate. We were particularly interested in the PACF tests since they show the direct correlation that a time series has with its own lags, removing the effect of intermediate values. In this case, for all three time series, there was strong evidence that they were positively correlated with the previous step, having the first lag of autocorrelation very close to one. Furthermore, for all three series, the ACF displayed values close to one. This suggested to us that the time series might not be stationary and actually contain a unit root.

Thus, we wanted to check whether the three time series were integrated of order one. In order to do this, we implemented ADF-GLS tests for these series and their first differences. In the MATLAB code, we were only able to implement the ADF test, which is less efficient than ADF-GLS in handling residual correlation issues among the data and in handling roots close to unity, for further details we refer to Appendix B. The results are shown in the following table:

	P value	**Test Statistic**
C-Spread	0.1	-3.96
C-Spread Diff	0.1	-61.43
Z-Spread	12.94	-1.48
Z-Spread Diff	0.1	-53.19
Risk-Free Rate	90.83	0.94
Risk-Free Rate Diff	0.1	-46.21

Table 2.2: Summary table of the ADF tests computed in MATLAB. The p-values are expressed as percentages.

Both tests have as the null hypothesis that the time series be non-stationary and it is possible to reject it if the test statistic is sufficiently negative. For both the Z-index and the risk free rate we were able to reject the null hypothesis only for the first differences series. This allowed us to conclude that both the Z-index and the risk free rate are integrated of order one.

The test results would have led us to conclude that the C-spread and its first differences were stationary, as they had sufficiently negative statistics in both cases (with the difference series having a much more negative test statistic). However, implementing the ADF-GLS test in Python, we found that we have to reject the null hypothesis only for the first differences. This discrepancy is probably due to the fact that there is a root quite close to one, which the regular ADF test struggles to treat in a correct manner. In conclusion, we have statistical evidence that the C-spread, Z-index and risk free rate are integrated of order 1 in the whole of Phase III, this result is in accordance with the partial autocorrelation functions shown previously.

These results are very important, indeed to verify a long-term relationship between the C-spread and the Z-index, we cannot use a standard regression model with a C-spread not stationary since it would lead to a spurious regression. In the following section, we will explore the existence of a cointegration relationship among the three time series above and subsequently implement an error correction model.

2.6. Johansen test

Once we have verified that our time-series are integrated of order one, we can proceed with the Johansen test to verify whether a cointegration relationship exists between the three time-series. This test introduces a methodology to identify cointegration relationships.

In our case, we considered a set of three variables, hence ($g = 3$).

The method involves determining the number of cointegration relationships q, where $0 \leq q < g$. Johansen introduced two statistical tests for this purpose: the trace test and the eigenvalue test, both sharing the same null hypothesis. These tests are performed sequentially to identify the number of cointegration relationships q. Initially, the null hypothesis $q \leq 0$ is tested. If this hypothesis is rejected, the next test considers the null hypothesis $q \leq 1$ and the process continues until a test fails to reject the null hypothesis. Let us report the results of the Johansen test performed on the C-spread, Z-index and risk free rate:

	Trace	**MaxEigen**	**Trace Critical value**	**MaxEigen Critical value**
q <= 0	97.81***	90.67***	24.27	17.8
q <= 1	7.14	7.08	12.32	11.23
q <= 2	0.06	0.06	4.13	4.13

Table 2.3: Summary table of the Johansen test.

As we can see in the table above, we did not find evidence to reject the hypothesis of having one or less cointegration relationships. Instead the null hypothesis of no cointegration relationships is rejected by both the trace and eigenvalues test, this gave us the statistical evidence to state that only one cointegration relationship exists among the three time series.

We followed the Johansen procedure using the **computeECT** function and inside of it we recalled the MATLAB built-in function **jcitest**; we obtained the following cointegration vector for C_t, Z_t and r_t respectively:

$$\{1, -1.31, -0.45\}$$

This has allowed us to describe the non-stationary component of the C-spread by the following linear combination:

$$\gamma_1 Z_t + \gamma_2 r_t := 1.31 Z_t + 0.45 r_t$$

As noted before, these results confirm the relationship between the C-spread and the Z-index, moreover, we can also say that the influence of the risk free rate is moderate, identifying the Z-index as the primary determinant of the C-spread.

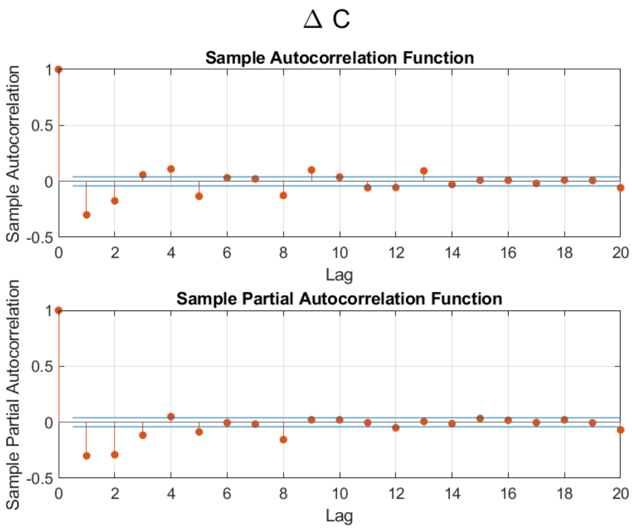
2.7. Error correction model and GARCH(1,1)

In the econometric literature, after determining that time-series are non-stationary and cointegrated, the standard approach is to compute the error correction model (ECM). This approach avoids the problem of using only one lag for first-order differenced variables in statistical estimations. Indeed, the introduction of the ECT lets the model have a long-run solution which does not exist for models based solely on first differences. For the first differenced time-series of the C-spread, we propose the following model:

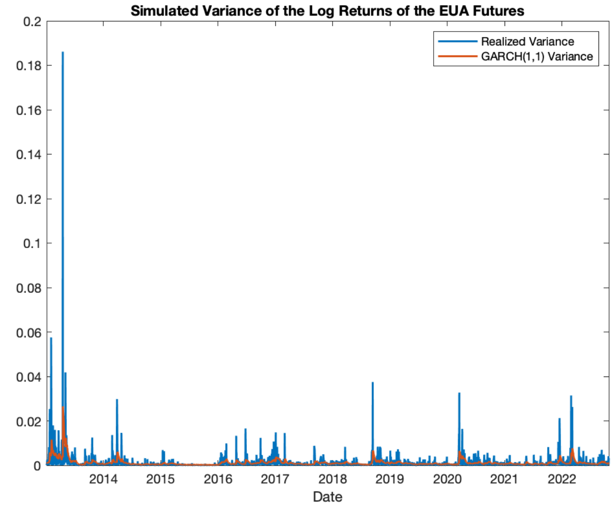
$$\Delta C_t = \alpha_0 + \sum_{i=1}^{3} \beta_i \Delta C_{t-i} + \alpha_1 \Delta Z_t + \alpha_2 \Delta r_t + \alpha_3 \psi_{t-1} + \sum_{j=1}^{4} \delta_j Y_{j,t} + \epsilon_t \tag{2.1}$$

where $\psi_{t-1} := C_t - \gamma_1 Z_t - \gamma_2 r_t$ is the previous cointegration relation, $Y_{j,t}$ are control variables and ϵ_t is the error term.

We considered three lags for the first differences of the C-spread since its PACF shows that three lages are the statistically significant ones.



Next, we implemented a GARCH(1,1) model in order to compute the variance of the log return of the spot price of EUA futures, obtaining the following variance estimation. For further details on the formulation of the GARCH model we refer to Appendix A.



As can be observed in the figure, the conditional volatility obtained with GARCH follows the pattern of the realized volatility, that we estimated with the square of log returns since their average is very close to zero.

As control variables $Y_{j,t}$ we selected the volatility and three broad-based financial indexes commonly considered benchmarks by financial investors worldwide: the S&P 500 (main equity index), the VIX (equity volatility index), and the WTI (reference for crude oil price). As standard convention for the S&P 500 and WTI, we took log return time series. We also performed a Pearson correlation test with the null hypothesis of zero correlation to justify their inclusion in our error correction model and avoid the trap of multicollinearity. We found statistical evidence that the considered control variables have low or no correlation. The Pearson coefficients are summarized in the table below:

	SPX	**VIX**	**WTI**	**Volatility**
SPX	1			
VIX	-0.17***	1		
WTI	0.27***	-0.08***	1	
Volatility	0.04*	0.09***	0.02	1

Table 2.4: Summary table of Pearson coefficients.

In order to prepare our data for the error correction model we implemented the function **prepareDataRegression** which filtered our time-series. Subsequently we constructed the model using the built-in MATLAB function **fitlm** giving as input the prepared data. Then, after these preliminary test, we can finally report the results for our error correction models:

Row	Model I	Model II	Model III	Model IV	Model V	Model VI
Delta_C_lag1	-0.41***	-0.41***		-0.41***		-0.41***
Delta_C_lag2	-0.33***	-0.33***		-0.33***		-0.33***
Delta_C_lag3	-0.11***	-0.11***		-0.11***		-0.11***
Delta_Z		0.38***		0.38***		0.40***
Delta_r		0.11		0.14		0.13
ect_lag1	-0.04***	-0.04***		-0.04***		-0.04***
WTI			-0.00	-0.00	-0.00	-0.00
SPX					0.00	0.00
VIX					0.00	-0.00
Volatility					0.00	0.00
(Intercept)	0.00	0.00	-0.00	0.00	-0.00	0.00
Obs	1993	1993	1993	1993	1993	1993
BIC	-19206	-19200	-18788	-19193	-18766	-19172
AIC	-19234	-19239	-18799	-19238	-18794	-19233

Table 2.5: Summary table of regression results.

The table above reveals several some key insights:

- The first model (I) includes the three lags of ΔC_t and the cointegration term ψ_{t-1} as independent variables. This model performs best in terms of Bayesian Information Criterion (BIC) and is nearly the best in terms of Akaike Information Criterion (AIC). Therefore, we can conclude that these quantities are the key to explaining the C-spread's behaviour.
- In models (II) to (VI), other variables such as the first differences of the risk-free rate (Δr_t), as well as control variables (SPX, VIX, WTI, and σ_t), do not achieve statistical significance at the 1% level. Conversely, the three lags of ΔC_t and the cointegrating term ψ_{t-1} and the Z-index (ΔZ_t) consistently show significance at the 1% level across all models, and the estimated coefficients maintain consistent signs. This confirms our findings that in the cointegration relationship found above, as well as in determining the value of the C-spread, the Z-index plays a major role.
- The negative coefficients for the lags of ΔC_t suggest a reverting mechanism towards equilibrium after a deviation, indicating a short-term autoregressive behavior.
- The negative coefficient of ψ_{t-1} implies that when the spread C_t deviates from its

long-term equilibrium relationship defined by the Z-index and the risk-free rate, it tends to revert back. Specifically, when C_{t-1} exceeds $\gamma_1 Z_{t-1} + \gamma_2 r_{t-1}$, the positive ψ_{t-1} indicates an over-extension that the model corrects in subsequent periods.

These results, overall, indicate that short-term fluctuations in the C-spread are primarily driven by its own past values and the deviation from the long-term equilibrium, rather than by contemporaneous changes in financial benchmarks or the volatility of spot EUA returns.

The detailed analysis above highlights the robust nature of the error correction mechanism and the minimal influence of external financial variables on the short-term movements of ΔC_t.

However, we proceeded to examine the normality of the standardized residuals using the Kolmogorov-Smirnov test, implemented through MATLAB's `kstest` function.

The test result obtained was $h = 1$, indicating that the null hypothesis, which states that the residuals follow a standard normal distribution, is rejected at the 5% significance level. This implies that the assumption of normality for the residuals is not satisfied.

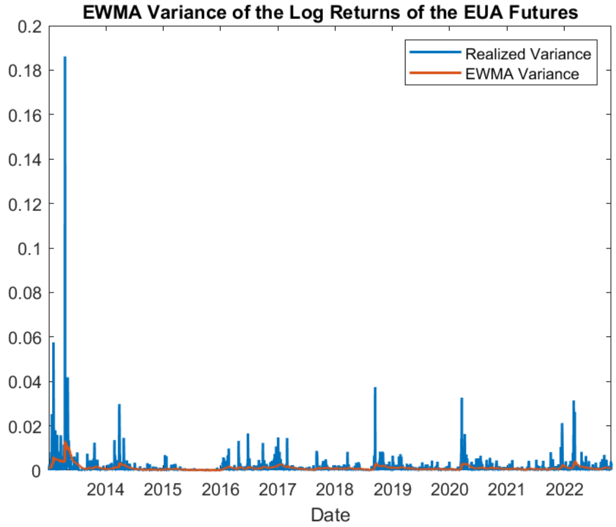
As a result, we must acknowledge that not all assumptions underlying the linear regression model are met. Specifically, the lack of normality in the residuals may affect the validity of our statistical inferences. This limitation should be considered when interpreting the results of our regression analysis, and further investigation or alternative modeling approaches may be required to address this issue.

2.8. Robustness checks

In this section, we conducted a robustness analysis on our model to demonstrate the accuracy and validity of our results. Specifically, we re-estimated the error correction model for five different new models by changing the method of variance estimation, the time period for data collection, and the roll-over rule. In all cases, we obtained results very similar to those derived in the previous section.

1. **EWMA volatility estimate**

   We re-estimated the variance of the log returns of the spot prices using an EWMA (Exponentially Weighted Moving Average) approach for EUA futures, with a parameter λ of 0.95. For further details on the EWMA model and its differences regarding the GARCH model, we refer to Appendix A.



In this figure, we can see the estimation results, which appear very similar to those of the GARCH model.

2. **Consider phase III and IV**

   In this new model, we took into consideration all available data points from phases III and IV, spanning from January 2013 to October 2022 (the last available date in our data set).

3. **Different rollover rules**

   We created three additional new models by changing the roll-over rule for the C spread. In the first model, we rolled over to the next future when the open interest of the next future exceeded the open interest of the front future for the first time in each year. In the second model, we rolled over to the next future one month before the expiry of the front future. In the third model, we rolled over to the next future one week before the expiry. We implemented these three new roll-over rules using the function **aggregate_C_Spread**, setting the variable flag to 2, 3, and 4, respectively.

The results and coefficients obtained for each model can be found in the table below:

	EWMA	**Phase IV**	**Open I.**	**Month**	**Week**	**Original**
Delta_C_lag1	-0.41***	-0.40***	-0.37***	-0.35***	-0.18***	-0.41***
Delta_C_lag2	-0.33***	-0.32***	-0.32***	-0.33***	-0.28***	-0.33***
Delta_C_lag3	-0.11***	-0.11***	-0.27***	-0.16***	-0.16***	-0.11***
Delta_Z	0.40***	0.33***	0.68***	0.42***	0.63***	0.40***
Delta_r	0.13	-0.01	0.71	0.19	0.77	0.13
ect_lag1	-0.04***	-0.04***	-0.05***	-0.04***	-0.13***	-0.04***
WTI	-0.00	-0.00	-0.00*	-0.00	-0.00*	-0.00
SPX	0.00	0.00	0.00	0.00	0.00	0.00
VIX	-0.00	-0.00	-0.00	-0.00	-0.00**	-0.00
Volatility	0.00	0.00	0.01	0.00	0.01*	0.00
(Intercept)	0.00	0.00	-0.00	0.00	0.00	0.00
Obs	1993	2463	1993	1993	1993	1993
BIC	-19171	-24139	-18407	-19164	-16743	-19172
AIC	-19233	-24203	-18468	-19225	-16805	-19233

Table 2.6: Summary table of regression results for various robustness checks.

The robustness checks involved re-estimating the error correction model under various conditions, and the results are summarized in the provided table. Here is an analysis of the outcomes:

- **EWMA Volatility Estimate**: The parameters for ΔC_{t-1}, ΔC_{t-2}, ΔC_{t-3}, ΔZ_t, and ψ_{t-1} are all statistically significant and have signs consistent with the original model. This indicates that using an EWMA approach for volatility estimation does not significantly alter the results and that overall the volatility of the spot prices, does not play a significant role in determining the C-spread.
- **Consider Phase III and IV**: The results are very similar to the original model, with minor variations in the coefficients' magnitudes. The signs and significance levels of the coefficients remain consistent, suggesting that extending the data range to include Phases III and IV does not substantially change the findings. This can be interpreted as further proof that a long-term relationship between the Z-index and C-spread actually does exist.
- **Different Rollover Rules**: Changing the rollover rules also yields results that are in line with the original model. The coefficients for ΔC_{t-1}, ΔC_{t-2}, ΔC_{t-3}, ΔZ_t, and ψ_{t-1} remain significant with similar signs. Notably the model which uses the rule of

rolling over one week before expiry, shows some variations in the magnitude of the coefficients, but the overall significance and direction remain consistent. This could be attributed to more erratic behaviour in the C-spread due to small values in the denominator, as discussed before.

2.8.1. Expected vs. Unexpected Results

Expected Results:

The consistency of the coefficients ΔC_{t-1}, ΔC_{t-2}, ΔZ_t, and ψ_{t-1} across different models is expected, as these are key components of the error correction model. The robustness checks confirm that the model's core relationships are stable across various estimation methods and data periods. The stability of the results when using different rollover rules suggests that the chosen rule does not heavily influence the model's outcome, which is a positive indication of robustness.

Unexpected Results:

The change in the magnitude of some coefficients, particularly in the model where the rollover occurs one week before expiry, is slightly unexpected. This might indicate some sensitivity in the model to the specifics of the rollover timing, although the overall consistency in sign and significance suggests it is not a critical issue.

2.8.2. Conclusion

The robustness checks confirm that the main findings of the original model are reliable. The key relationships hold true across different variance estimation methods, extended data periods, and alternative rollover rules, demonstrate the model's robustness and validity.

2.9. Quantile regression

We conducted a quantile regression analysis to investigate the relationship between the dependent variable and various independent variables across different quantiles. This approach provides a comprehensive understanding of the conditional distribution of the dependent variable, allowing us to examine the effects at different points in the distribution rather than focusing solely on the mean.

We computed it by using a quantile regression function taken by the MATLAB file exchange, we recall it in the function **estimateQR**.

The table below summarizes the quantile regression results from the 10th to the 90th percentiles. The coefficients and their significance levels provide insights into the heterogeneous impacts of the predictors across different points in the outcome distribution.

	10th	**20**th	**30**th	**40**th	**60**th	**70**th	**80**th	**90**th
(Intercept)	0*	0**	0***	0**	0	0	0	0
Delta_C_lag1	-0.11**	-0.04	-0.02	-0.02	-0.02**	-0.02***	-0.03***	-0.06***
Delta_C_lag2	-0.1**	-0.02	-0.01	-0.01	-0.01	-0.01	-0.02	-0.06***
Delta_C_lag3	-0.02	-0.02***	-0.01	-0.01	-0.02	-0.02***	-0.03***	-0.04***
Delta_Z	0.1	0.08*	0.05*	0.04	0.04	0.07*	0.12***	0.15
Delta_r	0.57	-0.59***	-0.8***	-0.86***	-0.9***	-0.91***	-1.11***	-1.11**
ect_lag1	-0.07***	-0.03***	-0.02***	-0.01***	0*	0.01***	0.01***	0
WTI	0	0	0**	0	0*	0*	0**	0**
SPX	0	0	0	0	0	0	0	0
VIX	0	0**	0**	0	0*	0	0**	0
Volatility	-0.02***	-0.01***	-0.01***	0***	0	0***	0.01***	0.03***

Table 2.7: Summary table of Quantile Regression results for Model VI.

	10th	**20**th	**30**th	**40**th	**60**th	**70**th	**80**th	**90**th
(Intercept)	0***	0***	0***	0***	0***	0***	0***	0***
Delta_C_lag1	-0.12***	-0.03	-0.02	-0.02**	-0.03**	-0.03**	-0.04***	-0.08***
Delta_C_lag2	-0.08	-0.03*	-0.02	-0.01	-0.01	-0.02	-0.03	-0.08***
Delta_C_lag3	-0.01	-0.02***	-0.02	-0.02***	-0.02***	-0.02***	-0.03***	-0.07***
ect_lag1	-0.08***	-0.03***	-0.01***	-0.01**	0*	0.01***	0.02***	0.03*

Table 2.8: Summary table of Quantile Regression results for Model I.

The results for Model VI indicate the following:

- ΔC_{t-1} has a statistically significant negative effect at the 10th, 40th, 60th, 70th, 80th, and 90th percentiles. This suggests that the impact of ΔC_{t-1} becomes more pronounced in the higher quantiles of the dependent variable's distribution.
- ΔC_{t-2} shows a statistically significant negative effect at the 10th and 90th percentiles. This indicates that ΔC_{t-2} has a more pronounced negative impact in the lower and upper extremes of the distribution.
- ΔC_{t-3} exhibits a significant negative effect at the 20th, 70th, 80th, and 90th percentiles, indicating its uniformly adverse impact on the dependent variable, particularly in the upper part of the distribution.

- ΔZ_t shows a positive and significant effect at the 30th and 70th percentiles. This implies that as the dependent variable increases, the influence of ΔZ_t strengthens at these quantiles.
- Δr_t has a significant negative effect from the 20th to the 80th percentiles. This suggests that Δr_t has a strong negative impact on the dependent variable across a wide range of the distribution, particularly in the central quantiles. Thus it may account for small variations of the C-spread around it mean.
- ψ_{t-1} has a significant negative effect at the 10th, 20th, 30th, and 40th percentiles, and a significant positive effect at the 60th, 70th, and 80th percentiles. This indicates that ψ_{t-1} has varying impacts across the distribution, in particular its effect reverses for opposite quantiles.
- The WTI shows a significant positive effect at the 30th and 90th percentiles. This implies that the influence of WTI is more pronounced for extreme values of the C-spread differences.
- The SPX does not show any significant effects across the quantiles, indicating its limited impact on the dependent variable's distribution.
- The VIX shows a significant positive effect at the 20th and 80th percentiles. This suggests that VIX influences the dependent variable more at these points in the distribution, again influencing extreme values of the C-spread.
- Volatility (estimated using the GARCH) has a significant negative effect at the 10th, 20th, 30th, and 40th percentiles, and a significant positive effect at the 70th, 80th, and 90th percentiles. This indicates that Volatility has a varying impact, with a negative influence in the lower quantiles and a positive influence in the upper quantiles.

Furthermore, looking at the results for Model I, other than a few slight changes in significance and values, which can be attributed to the absence of other explanatory covariates, we can easily see that its results align with the ones commented above. As we expected from the OLS, these explanatory variables are the most significant for the model.

Overall we can conclude that the significance and sign of the coefficients for the three lags further supports our thesis that there is some sort of mean reversion mechanism at play in the values of the C-spread and it becomes more pronounced in the upper and lower quantiles. Furthermore we can see that the WTI and VIX influence more extreme values of the C-spread, while the Δr_t seems to account for small variations around the median.

2.9.1. Comparison of Quantile Regression and Linear Regression Results

The linear regression model is based on the assumption of normality of residuals. However, as we have seen, the Kolmogorov-Smirnov test indicates that this assumption is violated, which could compromise the validity of the statistical inferences derived from the model. In contrast, the quantile regression model does not require the assumption of normality of residuals, making it more robust to such violations.

In terms of the significance of variables, both the linear regression model and the quantile regression consistently find the three lags of ΔC_t and the cointegration term ψ_{t-1} to be significant.

Other variables, such as the first differences of the Z-index (ΔZ_t) and the risk-free rate (Δr_t), as well as control variables (SPX, VIX, WTI, and σ_t), are not significant for the OLS. On the other hand, the quantile regression models reveal that the significance of variables varies across different quantiles. For example, ΔC_{t-1}, ΔC_{t-2}, and ΔZ_t show significance at specific quantiles, indicating heterogeneous effects across the distribution.

The autoregressive behavior indicated by the linear regression model, where the negative coefficients for the lags of ΔC_t suggest a reverting mechanism towards equilibrium, is also captured in the quantile regression model. However, the quantile regression model shows that the impact of these lags is more pronounced at more extreme quantiles, providing a more nuanced understanding of this behavior.

Regarding the effects of external variables, the linear regression model suggests that short-term fluctuations in the spread are primarily driven by its own past values and deviation from the long-term equilibrium, with minimal influence from contemporaneous changes in financial benchmarks or the volatility of spot EUA returns. In contrast, the quantile regression model indicates that external variables, such as ΔZ_t, Δr_t, and control variables, have varying levels of significance at different quantiles, highlighting their non-uniform influences across the distribution and thus giving us a deeper look into their role in determining variations in the C-spread.

In summary, while the linear regression model provides a general overview of the mean effects of predictors, the quantile regression model offers a more detailed and robust analysis, capturing the heterogeneous effects of predictors across different points in the distribution of the dependent variable. This makes the quantile regression approach more versatile and informative, especially when the assumption of normality is not met.

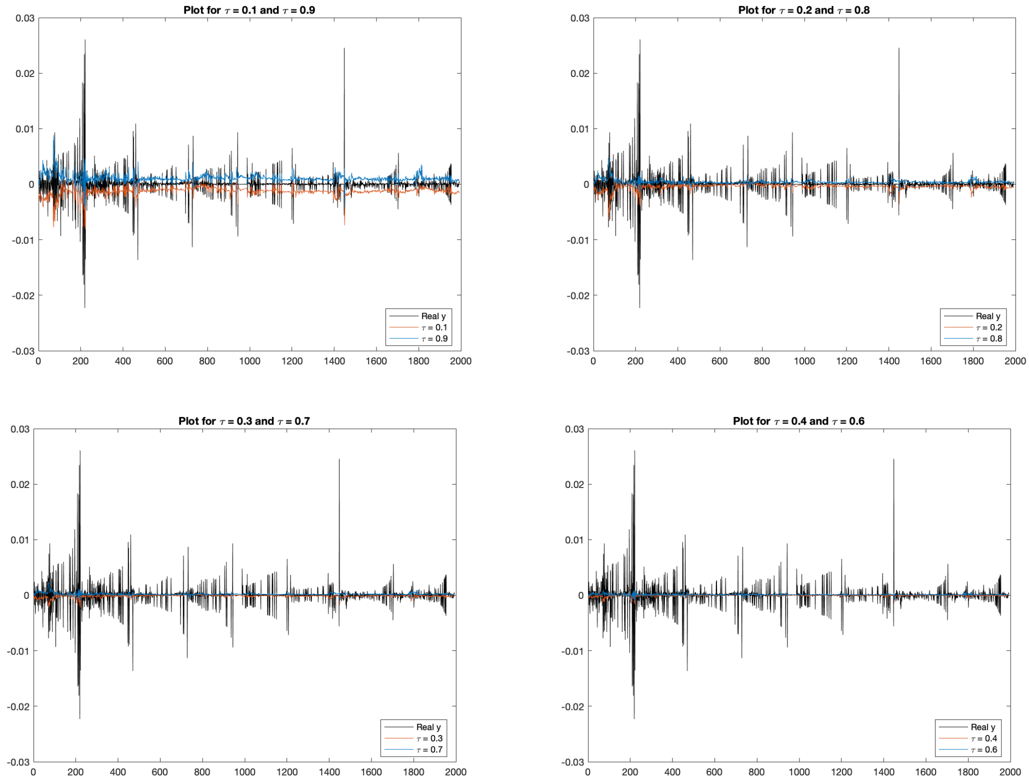
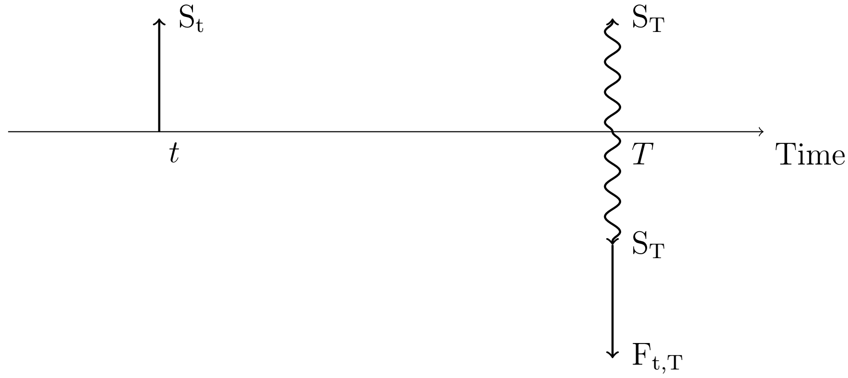


Figure 2.1: Plot of ΔC_t and the predicted quantiles for model VI

3 | Conclusions and future developments

3.1. Synthetic repo

A synthetic repo is an agreement where an investor simulates the exposure and cash flows of a traditional repo, but uses futures contracts instead of actual cash and securities. In particular, it is possible for European polluters among the EU-ETS compliant entities to get financed with a synthetic repo, and the strategy is explained in the figure below.



At the value date t, the market player shorts a spot EUA contract S_t and buys an equivalent future contract with price $F_{t,T}$ and expiring at time T. At maturity, the market player receives the EUA by paying the future contract and returns it back, having previously shorted it.

By analyzing the strategy, we observed that the player receives S_t at time t and must pay at time T the future price, which we know can be expressed as a function of the C-spread:

$$F_{t,T} = S_t \exp\{(r(t,T) + C(t,T))(T-t)\}$$

Therefore, the player finances itself at the rate $r(t,T) + C(t,T)$, this has led us to make two considerations:

- Putting the EUA in the list of European Central Bank eligible collateral for refinancing operations, the C-spread would effectively become zero, allowing European polluters to finance at a rate equivalent to the risk-free rate.
- A synthetic repo represents an alternative financing method for European polluters, in particular, if the C-spread is lower than the Z-index, it is more advantageous to finance using this method rather than a traditional financing method.

3.2. Conclusions and policy implications

To address the market inefficiency indicated by the positive C-spread, our study suggests a key policy change: including EU Allowances (EUAs) as eligible collateral for Eurosystem credit operations. This recommendation comes from the observed relationship between the C-spread, the credit spread of firms in the EU Emissions Trading Scheme (EU-ETS), i.e. the Z-index, and the risk free rate. By allowing EUAs to be used as reliable collateral, the European Central Bank (ECB) can help establish an active repurchase agreement (repo) market for these allowances, letting them serve as collateral in credit transactions at the ECB's policy rate.

Creating a repo market for EUAs would align the repo rate with the policy rate, eliminating the C-spread issue. This would improve the liquidity and efficiency of the EU-ETS market, helping to ensure that allowance prices reflect true future carbon constraints. As a result, companies would be more motivated to invest in long-term emission reduction projects and secure allowances in advance.

Furthermore, including EUAs as eligible collateral would strengthen the EU-ETS by showing a strong commitment from policy makers to support the system long-term. This approach could also reduce the need for more drastic measures, like a price floor, which has been proposed to prevent allowance prices from falling too low, as seen at the end of Phase I.

In summary, adding EUAs to the Eurosystem's collateral framework would address the current market inefficiency, improve market dynamics, and support the EU's transition to a sustainable and low-carbon economy.

3.3. Python

To further prove the correctness of our methodologies as well as to leverage the different practical advantages of the Python language, we chose to replicate our results and inquiries in Python.

To fully take advantage of Python's OOP syntax, we chose to represent different parts of our study as their own, stand alone classes with interface through which to interact with the other objects in our code.

In particular, we chose to represent the Preprocessing process as its own class and methods and attributes of this class represented the preprocessing methodologies for the different data sets we considered in our study. Similarly we chose to represent the C-spread and Z-index as their own classes with private attributes and whose public interfaces let the user compute and aggregate the data only in the manner described above. Finally, we created a plotter class to easily access all the plots we decided to provide in our report.

Overall, we found results that closely aligned with the ones found in MATLAB. In particular, the C-spread, Z-index, risk free rate and the linear regressions were nearly identical to the ones obtained using MATLAB.

On the other hand we found differing results when performing the ADF-GLS and Quantile Regressions. Let us highlight that neither of these statistical procedures have a built-in function in MATLAB.

In particular, the results for the ADF-GLS tests are the following:

	Test Statistic	P Value
C-Spread	-0.579	0.484
C-spread Diff	-8.004	0.000
Z-spread	-0.801	0.379
Z-spread Diff	-1.692	0.089
Risk Free rate	1.265	0.952
Risk Free Rate Diff	-40.459	0.000

Table 3.1: Critical Values: -2.58 (1%), -1.95 (5%), -1.63 (10%)

As highlighted in the Appendix B the ADF-GLS test performs better with roots close to one. In this case, while the normal ADF test gives us evidence to say that the C-spread itself is stationary, the ADF-GLS tells us that it is not.

Furthermore, let us present the table of the coefficients estimated for the Quantile Regressions:

	10th	**20th**	**30th**	**40th**	**60th**	**70th**	**80th**	**90th**
Diff C-spread Lag 1	-0.25 ***	-0.12 ***	-0.06 ***	-0.05 ***	-0.05 ***	-0.06 ***	-0.10 ***	-0.23 ***
Diff C-spread Lag 2	-0.17 ***	-0.07 ***	-0.03 ***	-0.04 ***	-0.03 ***	-0.04 ***	-0.07 ***	-0.16 ***
Diff C-spread Lag 3	-0.07	-0.03 **	-0.01	-0.01 ***	-0.01 ***	-0.02 ***	-0.04 ***	-0.06
Diff Z-spread	0.05	0.11	0.05	0.04 *	0.04 **	0.05 *	0.14 **	0.20
Diff Risk Free Rate	0.29	-0.55	-0.92 ***	-1.01 ***	-0.94 ***	-0.86 ***	-1.07 ***	-1.85
ECT Lag 1	-0.07 ***	-0.03 ***	-0.02 ***	-0.01 ***	-0.00	0.00	0.00	-0.04 ***
WTI	-0.00	-0.00	-0.00 ***	-0.00 **	-0.00 **	-0.00 ***	-0.00 **	-0.00 **
SPX	0.01	0.00	-0.00	-0.00 **	-0.00 *	0.00	0.00	0.00
VIX	0.00	-0.00 ***	-0.00 **	-0.00	0.00 *	0.00 **	0.00 **	-0.00
Volatility	-0.00 ***	-0.00 ***	-0.00 ***	-0.00 ***	0.00	0.00 ***	0.00 ***	0.00 ***
(Intercept)	-0.00 ***	0.00 *	0.00 ***	0.00 ***	0.00	-0.00	-0.00 **	0.00

Table 3.2: Summary table of Quantile Regression results for Model VI.

	10th	**20th**	**30th**	**40th**	**60th**	**70th**	**80th**	**90th**
Diff C-spread Lag 1	-0.28 **	-0.12 ***	-0.06 ***	-0.06 ***	-0.06 ***	-0.07 ***	-0.12 ***	-0.28 ***
Diff C-spread Lag 2	-0.16	-0.07 ***	-0.04 ***	-0.03 ***	-0.03 ***	-0.03 ***	-0.08 ***	-0.18 **
Diff C-spread Lag 3	-0.07	-0.02	-0.01	-0.01 ***	-0.01 ***	-0.01 **	-0.04 **	-0.10
ECT Lag 1	-0.07 ***	-0.04 ***	-0.02 ***	-0.01 ***	-0.00	0.00	0.01 *	0.00
(Intercept)	-0.00 ***	-0.00 ***	-0.00 ***	-0.00 ***	0.00 ***	0.00 ***	0.00 ***	0.00 ***

Table 3.3: Summary table of Quantile Regression results for Model I.

As we can see the results are a bit different from the ones obtained from MATLAB.

In particular we can immediately see that the Lags of the C-spread and the ECT are significant and have a consistent sign along all quantiles and are overall more consistent with the results obtained through the OLS. The coefficients and significance for the other control variables are quite similar to MATLAB.

These correspondences with the OLS and the fact that the Python function was pre implemented in a highly reputed library such as statsmodels, led us to trust these results more than the ones obtained from MATLAB.

Indeed, the python function also performs some useful test on the residuals of the regression and computes other useful metrics.

Nonetheless, what we observed for the Quantile regression still stands and our deductions are quite aligned with the results obtained from Python.

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A | Appendix A

A.1. GARCH vs EWMA

A.1.1. GARCH model

In this section, we describe the methodology employed to estimate the instantaneous variance of the EU-ETS spot price using a GARCH(1,1) model applied to the daily returns of the futures time series. The GARCH(1,1) model is an extension of the ARCH model and is particularly effective in capturing the time-varying volatility commonly observed in financial time series.

A.1.2. Model Specification

The GARCH(1,1) model can be expressed as follows:

$$r_t = \mu + \epsilon_t \tag{A.1}$$

$$\epsilon_t = \sigma_t z_t \tag{A.2}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{A.3}$$

Where:

- r_t represents the daily return at time t.
- μ is the mean return.
- ϵ_t is the residual return at time t.
- σ_t^2 is the conditional variance at time t.
- z_t is a sequence of i.i.d. standard normal random variables.

- α_0, α_1, and β_1 are parameters to be estimated.

A.1.3. Estimation of Parameters

The parameters μ, α_0, α_1, and β_1 of the GARCH(1,1) model are estimated using the Maximum Likelihood Estimation (MLE) method. The log-likelihood function for the GARCH(1,1) model is given by:

$$\mathcal{L}(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\left[\log(2\pi) + \log(\sigma_t^2) + \frac{\epsilon_t^2}{\sigma_t^2}\right] \tag{A.4}$$

where $\theta = (\mu, \alpha_0, \alpha_1, \beta_1)$ is the vector of parameters. The optimization of this function provides the parameter estimates.

A.1.4. Estimation of Instantaneous Variance

Using the estimated parameters, the conditional variance σ_t^2 at each time t is calculated. This conditional variance represents the instantaneous variance of the EU-ETS spot price.

Let us highlight some weakness and strengths of the GARCH model:

Pros:

- **Volatility Clustering:** The GARCH(1,1) model effectively captures volatility clustering, reflecting the persistence of volatility often observed in financial time series.
- **Mean Reversion:** The model can represent mean-reverting behavior of volatility, which is useful for long-term forecasting.
- **Flexibility:** The model's structure allows for the inclusion of both past variances and past squared returns, providing a detailed description of volatility dynamics.

Cons:

- **Computational Complexity:** Estimating the parameters of the GARCH(1,1) model can be computationally intensive and requires complex optimization algorithms.
- **Numerosity of Parameters:** The model requires the estimation of several parameters, which can be challenging and prone to overfitting if not handled correctly.
- **Assumptions:** The model makes strong assumptions about the distribution of residuals (normality), which may not be realistic for all financial time series. Indeed

often financial time series tend to be leptokurtic.

A.1.5. Conclusion

By applying the GARCH(1,1) model to the daily returns of the EU-ETS futures, we can effectively estimate the instantaneous variance of the spot price. This estimation is crucial for understanding the volatility dynamics of the EU-ETS market, aiding in risk management, pricing, and hedging strategies.

A.2. EWMA model

In this section, we describe the methodology employed to estimate the instantaneous variance of the EU-ETS spot price using an Exponentially Weighted Moving Average (EWMA) model applied to the daily returns of the futures time series. The EWMA model is a simpler alternative to the GARCH model and is particularly useful for capturing the time-varying volatility in financial time series.

A.2.1. Model Specification

The EWMA model estimates the conditional variance using a weighted average of past squared returns, with exponentially decreasing weights. The model can be expressed as follows:

$$\sigma_t^2 = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i \epsilon_{t-i}^2 \tag{A.5}$$

Where:

- σ_t^2 is the conditional variance at time t.
- ϵ_t is the residual return at time t, often calculated as $\epsilon_t = r_t - \mu$ where r_t is the return and μ is the mean return.
- λ is the smoothing parameter, set to 0.95 in this analysis.

A.2.2. Estimation of Instantaneous Variance

The conditional variance σ_t^2 computed using the EWMA model represents the instantaneous variance of the EU-ETS spot price. This measure of volatility is crucial for understanding the risk and uncertainty in the EU-ETS market.

Let us highlight some weaknesses and strengths of the EWMA model:

Pros:

- **Parsimony:** The EWMA model is simple to implement and requires only a single smoothing parameter (λ), making it easy to compute and understand.
- **Computational Efficiency:** It is computationally lightweight, allowing for quick updates of the volatility estimate, which is useful in high-frequency trading contexts.
- **Reactivity:** Provides real-time updates of volatility estimates, adapting quickly to changes in market data.

Cons:

- **Limited Dynamics Capture:** The EWMA model may not accurately capture complex volatility patterns present in some financial data, as it relies on an exponentially weighted moving average without a mean-reverting mechanism.
- **Choice of λ:** The selection of the smoothing parameter λ is somewhat arbitrary and can significantly impact the model's performance. There is no clear guidance on how to choose λ.
- **Absence of Volatility Clustering:** The model does not explicitly capture the phenomenon of volatility clustering, making it less accurate in the presence of persistent volatility changes.

A.2.3. Conclusion

By applying the EWMA model with a smoothing parameter of $\lambda = 0.95$ to the daily returns of the EU-ETS futures, we can effectively estimate the instantaneous variance of the spot price. The simplicity and responsiveness of the EWMA model make it a valuable tool for real-time volatility estimation, aiding in risk management, pricing, and hedging strategies in the EU-ETS market.

B | Appendix B

B.1. ADF test vs ADF-GLS test

The ADF-GLS test (Augmented Dickey-Fuller Generalized Least Squares test) is considered an improvement over the ADF test (Augmented Dickey-Fuller test) for several reasons, particularly its greater sensitivity to roots very close to one. This characteristic is crucial when analyzing economic and financial time series, which often exhibit such properties.

The traditional ADF test is used to check for the presence of a unit root in a time series, determining whether the series is stationary or follows a stochastic trend. The ADF test accounts for autocorrelation by including lags of the time series differences. However, when the unit root is very close to one, the ADF test can lose power, meaning it becomes less likely to correctly reject the null hypothesis of non-stationarity.

The ADF-GLS test addresses this issue by using a Generalized Least Squares (GLS) transformation. This transformation removes any deterministic trends from the time series, making the test statistics more robust and reliable. The main strengths of the ADF-GLS test are:

- **Trend Removal:** The GLS transformation helps eliminate trends that might obscure the true stationary nature of the series.
- **Increased Power:** The ADF-GLS test has a higher probability of rejecting the null hypothesis when it is false, especially in the presence of unit roots close to one. This makes the test more effective in identifying stationary series.
- **Better Autocorrelation Handling:** The GLS transformation improves the effectiveness of the lagged differences, further reducing the impact of autocorrelation on the test results.

In conclusion, the ADF-GLS test provides a more refined and powerful approach for unit root testing, making it particularly useful in economic analyses where distinguishing between stochastic trends and stationarity is critical.