MIDTERM 1 Answer
Stat 135
March 20, 2018

1. **Problem #1**

(a) Using formula, we can get,

$$n_w = 36 \times \frac{2/3\sigma_w}{2/3\sigma_w + 1/3\sigma_m}$$

As $\sigma_w = 0.5\sigma_m$,

$$n_w = 36 \times 0.5 = 18, n_m = 18$$

(b) You can use either of the two formulas below:

$$Var(\frac{\bar{X}_1}{3} + \frac{2\bar{X}_2}{3}) = \frac{1}{9} \times \sigma_1^2 \times \frac{1}{18} \times \frac{32}{49} + \frac{4}{9} \times \sigma_1^2 \times \frac{1}{4} \times \frac{1}{18} \times \frac{82}{99}$$

or,

$$\frac{1}{36}(\frac{1}{3}\sigma_1 \times \frac{\sqrt{32}}{\sqrt{49}} + \frac{2}{3}\sigma_1 \times \frac{1}{2} \times \frac{\sqrt{82}}{\sqrt{99}})^2$$

(c) Using the correct weight and variances from two genders, you should get,

$$\frac{1}{36}(\frac{1}{3}\sigma_1^2 \times \frac{32}{49} + \frac{2}{3}\sigma_2^2 \times \frac{82}{99})$$

$$= \frac{1}{36}(\frac{1}{3}\sigma_1^2 \times \frac{32}{49} + \frac{1}{4} \times \frac{2}{3}\sigma_1^2 \times \frac{82}{99})$$

(d) Use the formula, simple variance = Proportional + difference for this question. So you will get, (from part c)
simple variance =

$$\frac{1}{36}(\frac{1}{3}\sigma_1^2 + \frac{2}{3}\sigma_2^2) + \frac{1}{36}(\frac{1}{3}(\mu_m - \mu)^2 + \frac{2}{3}(\mu_w - \mu)^2)$$

Now you need to solve for difference between averages.

$$\mu_m - \mu_w = 2\sigma_1$$

$$\mu = \frac{1}{3}\mu_m + \frac{2}{3}\mu_w$$

$$\sigma_2^2 = \frac{1}{4}\sigma_1^2$$

So we have,

$$\mu_w - \mu = \frac{1}{3}(\mu_w - \mu_m)$$

$$\mu_m - \mu = \frac{2}{3}(\mu_m - \mu_w)$$

Sub into equations above,

$$= \frac{1}{36}(\frac{1}{3}\sigma_1^2 + \frac{1}{6}\sigma_1^2) + \frac{1}{36}(\frac{1}{3} \times \frac{4}{9}(\mu_m - \mu_w)^2 + \frac{2}{3} \times \frac{1}{9}(\mu_w - \mu_m)^2)$$

$$= \frac{1}{72}\sigma_1^2 + \frac{1}{36}(\frac{4}{27} \times 4\sigma_1^2 + \frac{2}{27} \times 4\sigma_1^2)$$

$$= \frac{1}{72}\sigma_1^2 + \frac{2}{81}\sigma_1^2$$

Now include FSC for popultation,

$$= (\frac{1}{72}\sigma_1^2 + \frac{2}{81}\sigma_1^2) \times \frac{150 - 36}{149}$$

2. **Problem #3**
$H_0$ : die is fair or data is due to chance or something that indicates that die is fair.
$H_1$ : die is biased or data not based on fair die rolls or something similar.
The number of six follows binomial distribution $Binon(2, \frac{1}{6})$

$$P(nosixes) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

$$P(1six) = 2 \times \frac{5}{6} \times \frac{1}{6} = \frac{10}{36}$$

$$P(2sixes) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

So the expected value is,
E(no sixes in 72 rolls)=$72 \times \frac{25}{36} = 50$, E(1 six in 72 rolls)=$72 \times \frac{10}{36} = 20$, E(2 sixes in 72 rolls)=$72 \times \frac{1}{36} = 2$
Now to calculate Chi-square test statistics,

$$\chi^2_{obs} = \frac{(55-50)^2}{50} + \frac{(14-20)^2}{20} + \frac{(1)^2}{2} = 2.8$$

Now to calculate P-value, using $df = 3 - 1 = 2$ in Chi-square test,

$$P(\chi^2_2 > 2.8) \approx 0.25$$

(anything between 0.9 and 0.1 is acceptable).
So P-value is greater than 10%. So we do not reject $H_0$.
Conclusion: The test suggests (strongly) the die could be fair.