

### Formulas

The asymptotic variance of an MLE estimate  $\hat{\theta}$  is  $\frac{1}{(nI(\theta_0))}$  where  $I(\theta)$  is the Fisher information which can be solved using either of the following:

Define  $I(\theta)$  by

$$I(\theta) = E \left[ \frac{\partial}{\partial \theta} \log f(X|\theta) \right]^2$$

Under appropriate smoothness conditions on  $f$ ,  $I(\theta)$  may also be expressed as

$$I(\theta) = -E \left[ \frac{\partial^2}{\partial \theta^2} \log f(X|\theta) \right]$$

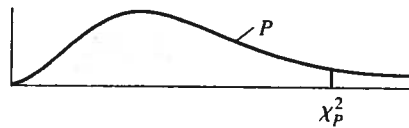
For a simple random sample,

Population Parameter	Estimate	Variance of Estimate	Estimated Variance
$\mu$	$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$	$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} \left( \frac{N-n}{N-1} \right)$	$s_{\bar{X}}^2 = \frac{s^2}{n} \left( 1 - \frac{n}{N} \right)$
$p$	$\hat{p} = \text{sample proportion}$	$\sigma_{\hat{p}}^2 = \frac{p(1-p)}{n} \left( \frac{N-n}{N-1} \right)$	$s_{\hat{p}}^2 = \frac{\hat{p}(1-\hat{p})}{n-1} \left( 1 - \frac{n}{N} \right)$
$\tau$	$T = N\bar{X}$	$\sigma_T^2 = N^2 \sigma_{\bar{X}}^2$	$s_T^2 = N^2 s_{\bar{X}}^2$
$\sigma^2$	$\left( 1 - \frac{1}{N} \right) s^2$		

TABLE 3.4. Summary of variances for different sampling techniques (ignoring the finite population correction factor).

Sampling method	Variance	Difference
Simple	$\frac{1}{n} \sigma^2$	Simple-Proportional $\frac{1}{n} \sum_{j=1}^J w_j (\mu_j - \mu)^2$
Proportional	$\frac{1}{n} \sum_{j=1}^J w_j \sigma_j^2$	
Optimal	$\frac{1}{n} \left( \sum_{j=1}^J w_j \sigma_j \right)^2$	Proportional-Optimal $\frac{1}{n} \sum_{j=1}^J w_j (\sigma_j - \bar{\sigma})^2$

TABLE 3 Percentiles of the  $\chi^2$  Distribution—Values of  $\chi_P^2$  Corresponding to  $P$



$df$	$\chi_{.005}^2$	$\chi_{.01}^2$	$\chi_{.025}^2$	$\chi_{.05}^2$	$\chi_{.10}^2$	$\chi_{.90}^2$	$\chi_{.95}^2$	$\chi_{.975}^2$	$\chi_{.99}^2$	$\chi_{.995}^2$
1	.000039	.00016	.00098	.0039	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.1026	.2107	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.064	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19