Name:

Student ID:

Quiz 4

Question 1 (linear regression)

(a) Prove the average of the fitted values satisfying $\frac{1}{n}\sum_{i=1}^{n}\hat{y}_i = \bar{y}$, where $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$. (3 points)

Solution:

$$\begin{aligned}\frac{1}{n}\sum_{i=1}^{n}\hat{y}_i &= \frac{1}{n}\sum_{i=1}^{n}(\bar{y} - \hat{\beta}_1\bar{x} + \hat{\beta}_1 x_i)\\ &= \bar{y} - \hat{\beta}_1\bar{x} + \hat{\beta}_1\bar{x}\\ &= \bar{y}.\end{aligned}$$

$\square$

(b) Given the results in part (a) and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$, $\hat{\beta}_1 = r_{xy}\sqrt{\frac{SS_y}{SS_x}}$, where $SS_x = \sum_{i=1}^{n}(x_i - \bar{x})^2$ and $SS_y = \sum_{i=1}^{n}(y_i - \bar{y})^2$, prove

$$\sqrt{SS_{\hat{y}}} = r_{xy}\sqrt{SS_y}.$$

Solution: $\hat{y} = \bar{y} - \hat{\beta}_1\bar{x} + \hat{\beta}_1 x$.

$$\begin{aligned}SS_{\hat{y}} &= \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2\\ &= \sum_{i=1}^{n}\hat{\beta}_1^2(x_i - \bar{x})^2\\ &= r_{xy}^2\frac{SS_y}{SS_x}\times SS_x\\ &= r_{xy}^2 SS_y.\end{aligned}$$

(3 points)

Question 2

Suppose we took a poll of which candidate the participates were planning on voting for (Democrat or Republican) before the debate and again after the debate. Below is the summary table.

Table 1: Table to Question 2

				after		
			D		R	
	D		60		24	84
before						
	R		4		12	16
			64		36	100

Consider the following hypothesis:

H_0 : Population voting alignment was not altered by debate.
H_a : The voting alignment was altered.

What conclusion can you make based on current information, given $\alpha = 0.05$? (4 points)

Solution:

$$T = \frac{(24-4)^2}{24+4} \approx 14.28571 > 3.84 = \chi^2(1).$$

we would reject the null hypothesis and conclude that the debate did alter the voting alignment.

1. Correct test statistic T: 1 point.
2. Correct critical value: 2 points.
3. Conclusion: 1 point.

Question 3

In order to investigate whether adults report verbally presented material more accurately from their right than from their left ear, a dichotic listening task was carried out. The data were found to be positively skewed.

Table 2: Table to question 3

Participant	Left ear	Right ear
1	25	32
2	29	30
3	10	8
4	31	32
5	27	20
6	24	32
7	26	27
8	29	30
9	30	32
10	32	32
11	20	30
12	5	32

We'd like to test whether there is a difference between the number of words recalled from the right ear and the number of words recalled from the left ear. What kind of test should we use for hypothesis testing? Do the testing and draw a conclusion (**significant level** $\boldsymbol{\alpha} = 0.05$). (10 points)
(Some additional information may or may not be useful: $\mathbb{E}(W_+) = \frac{n(n+1)}{4}$, $\mathrm{Var}(W_+) = \frac{n(n+1)(2n+1)}{24}$)

Solution:

0. H_0 : There is a difference. vs. There is not a difference. (1 point)
1. Find the differences between each pair of scores. (1 point)
2. Rank these differences, ignoring any "0" differences and ignoring the sign of the difference. (3 points: ignoring "0"-*1 point*; average ranks for those differences with the same value-*1 point*; remaining-*1 point*)

   Note: Since there are 4 instances of "1", students should add up the ranks they would take e.g., 1 + 2 + 3 + 4 = 10, and then divide this by the number of ranks, so rank=10 / 4 = 2.5. Similarly, also add together ranks corresponding to "2" 5 + 6 = 11. The ranks assigned would therefore actually be 11 / 2 = 5.5.

Participant	Left ear	Right ear	Difference(d)	$\lvert d \rvert$	Rank	Signed rank
1	25	32	- 7	7	8	-8
2	30	29	1	1	2.5	2.5
3	8	10	-2	2	5.5	-5.5
4	31	32	- 1	1	2.5	-2.5
5	26	20	6	6	7	7
6	24	32	- 8	8	9	-9
7	26	27	- 1	1	2.5	-2.5
8	29	30	- 1	1	2.5	-2.5
9	30	32	- 2	2	5.5	-5.5
10	32	32	0	0	discard	discard
11	20	30	- 10	10	10	-10
12	5	32	- 27	27	11	-11

3. Add together the ranks belonging to scores with a positive sign:

$$2.5 + 7 = 9.5.$$

4. Add together the ranks belonging to scores with a negative sign:

$$8 + 5.5 + 2.5 + 9 + 2.5 + 2.5 + 5.5 + 10 + 11 = 56.5.$$

   (1 point, together with step 3)

5. Test statistics W = 13. (1 point)

6. Number of differences $n = 12 - 1 = 11$, since we've discarded the zero difference. (1 point)

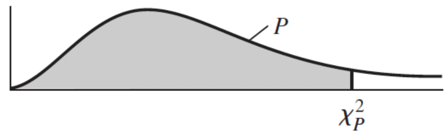
7. Check the critical value $W_\alpha(n)$ for the Wilcoxon Signed-Ranks Test and compare with $W = 9.5$ obtained in step 5. Since

$$W = 9.5 < W_\alpha(11) = 11,$$

   we should reject H_0. (2 points)

(Partial credits might be accounted if they apply the normal approximation.)

TABLE 3 Percentiles of the χ^2 Distribution—Values of χ^2_P Corresponding to P



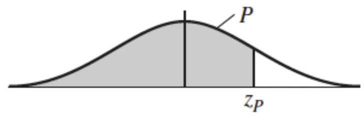
df	$\chi^2_{.005}$	$\chi^2_{.01}$	$\chi^2_{.025}$	$\chi^2_{.05}$	$\chi^2_{.10}$	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$
1	.000039	.00016	.00098	.0039	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.1026	.2107	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.064	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
120	83.85	86.92	91.58	95.70	100.62	140.23	146.57	152.21	158.95	163.64

TABLE 9 Critical Values of $W_\alpha(n)$ for the Wilcoxon Signed-Ranks Test

W_α is the integer such that the probability that $W \le W_\alpha$ is closest to α. For example, for $n = 8$, $P(W \le 3) = .020$ and $P(W \le 4) = .027$; therefore, $W_{.025}(8) = 4$.

	α for One-Sided Test		
	.025	.01	.005
	α for Two-Sided Test		
n	.05	.02	.01
6	0	—	—
7	2	0	—
8	4	2	0
9	6	3	2
10	8	5	3
11	11	7	5
12	14	10	7
13	17	13	10
14	21	16	13
15	25	20	16
16	30	24	20
17	35	28	23
18	40	33	28
19	46	38	32
20	52	43	38
21	59	49	43
22	66	56	49
23	73	62	55
24	81	69	61
25	89	77	68

TABLE 2 Cumulative Normal Distribution—Values of P Corresponding to z_p for the Normal Curve



z is the standard normal variable. The value of P for $-z_p$ equals 1 minus the value of P for $+z_p$; for example, the P for –1.62 equals 1 – .9474 = .0526.

z_p	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998