HW # 1 Solutions

(all problems from Nolan/Speed Ch 1)

 # 6. $\frac{(61.5-64)}{2.5} = -1$ and $\frac{(64.5-64)}{2.5} = 0.2$ so the answer is $\Phi(0.2) - \Phi(-1) \approx 0.58 - 0.16 = 0.42$.

 # 7. 2180 grams, 2.64 SDs below average. About .4% of babies born to nonsmokers are below this weight.

 # 8. Let $z_{0.25} = \Phi^{-1}(0.25)$ and $z_{0.75} = \Phi^{-1}(0.75)$. Then $IQR = z_{0.75} - z_{0.25}$. The probability that a point lies outside the whiskers is the probability of going outside of the interval $[z_{0.25} - 1.5 \times IQR, z_{0.75} + 1.5 \times IQR]$. So we want $P(X \leq z_{0.25} - 1.5 \times IQR) + P(X \geq z_{0.75} + 1.5 \times IQR) = 2\Phi(z_{0.25} - 1.5 \times IQR)$. $z_{0.25} = -0.675$ and $IQR = 1.35$, so we want $2\Phi(-.675 - 1.5 \times 1.35) = 2\Phi(-2.7) \approx 0.0069$.

 # 12. Simply multiply by 0.035: $3180 \times 0.035 = 111.3$ and $500 \times 0.035 = 17.5$.

 # 19. We find the desired value of c by taking the derivative with respect to c, setting the expression equal to 0, and solving for c.

$$\begin{aligned}
\frac{d}{dc}\sum(x_i - c)^2 &= \frac{d}{dc}\sum(x_i^2 - 2x_i c + c^2) \\
&= \frac{d}{dc}\left[\sum(x_i^2) - 2c\sum x_i + \sum c^2)\right] \\
&= \frac{d}{dc}\left[\sum(x_i^2) - 2cn\bar{x} + n \times c^2)\right] \\
&= -2n\bar{x} + 2nc = 0
\end{aligned}$$

So $2nc = 2n\bar{x}$ and $c = \bar{x}$ is the critical value. To verify that this is a minimum, check that the 2nd derivative is $2n$, which is positive, so the original squared error expression is concave up. Thus $c = \bar{x}$ is the value of c which minimizes the squared error.