The Geometry of Linear Equations
(MIT Gilbert Strang)

$2x - y = 0$
$-x + 2y = 3$

$\underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$x = 0$

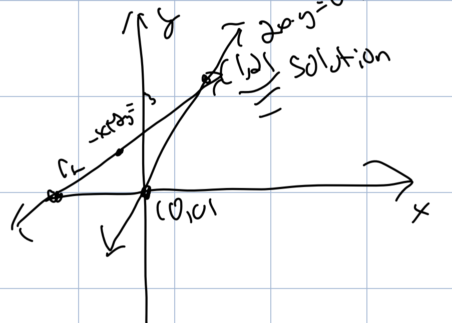
n linear equations, n unknowns

Goal: Describe
- > row picture
- *> column picture
- > Matrix form - algebra form

linear equation $\rightarrow Ax = b$

Row Picture

$2x - y = 0\ r_1$
$-x + 2y = 3\ r_2$

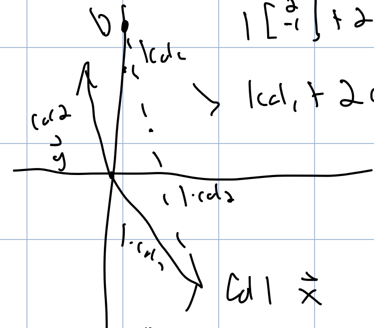


Column Picture

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Goal: Find the right linear combination to solve the system of equations.

$1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$



> $1col_1 + 2col_2 = (0,3)$

$b = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

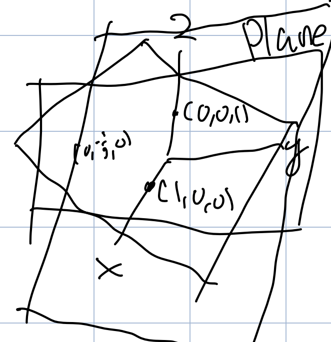
What would be all the linear combinations of $x\begin{bmatrix} 2 \\ -1 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ be?

There are infinite solutions to this system

$2x - y = 0$
$-x + 2y - z = -1$
$-3y + 4z = 4$

$\underbrace{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}}_{b}$

Row Picture



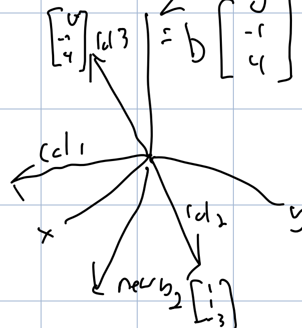
Each equation in a system of equations (solutions) can be mapped to a plane. Then a line can be drawn through the 3 or more planes to find the solution to the system of equations.

$$\underbrace{x \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}}_{\text{linear combination}} = b \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

$x = 0 \quad y = 0 \quad z = 1$

row good for 2

row 3+ dims, unclear



$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}1\\1\\-3\end{bmatrix}$$

$x=1,\ y=1,\ z=0$ $\quad (1,1,0)=b_2\begin{bmatrix}1\\1\\-3\end{bmatrix}$

Can I solve $Ax=b$ for every b? in other words, do the linear combos of the columns fill 3 dimensional space?

For this matrix A, yes! => invertible matrix

When could the linear combos not fill 3D space?

=> if all 3 vectors lie in the same plane: $Col_3 = Col_1 + Col_2$, noninvertible

Some matrix A × some vector x = b

$Ax=b$

$$\begin{bmatrix}2&5\\1&3\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

Two Ways: Column way: $\begin{bmatrix}1 \text{ of first column}\\2 \text{ of second column}\end{bmatrix}\begin{bmatrix}2\times1 & 5\times2\\1\times1 & 3\times2\end{bmatrix}=\begin{bmatrix}2+10\\1+6\end{bmatrix}$

$$1\begin{bmatrix}2\\1\end{bmatrix}+2\begin{bmatrix}5\\3\end{bmatrix}=\begin{bmatrix}12\\7\end{bmatrix}$$

Row Way: (dot products)

$$\begin{bmatrix}2^{(1)}+5^{(2)}\\1^{(1)}+3^{(2)}\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}12\\7\end{bmatrix}$$

Mathematics for Machine Learning Ch 2.1

Linear Algebra - the study of how vectors interact with eachother in relation to space.

in general, vectors are objects that can be added together or multiplied by scalars to produce a new object of the same dimension

↳ vectors can only perform mathematical operations with another vector of the same dimension.

Examples of Vectors:

1) geometric vectors - directed segments $\vec{x}$ that allow us to perform operations using mathematical intuition regarding direction and magnitude. addition and scalar multiplication are operations of a geometric vector
2) Polynomials - can be added together resulting in a new polynomial (property 1) can be multiplied by a scalar, resulting in a polynomial. (property 2) Thus, polynomials are an unusual form of a vector
3) Audio Signals - represented as a series of numbers. can be added together, resulting in a new series of numbers (p1) we can scale an audio signal into another one (p2)
4) Elements of $\mathbb{R}^n$ (tuples of n real numbers). $\mathbb{R}^n$ is more abstract than polynomials

$$a=\begin{bmatrix}1\\2\\3\end{bmatrix}\in\mathbb{R}^3$$ is saying a is a tuple of 3 real numbers, and is a member of $\mathbb{R}^3$

Adding two vectors $a,b\in\mathbb{R}^n$ will result in a vector $c\in\mathbb{R}^n$

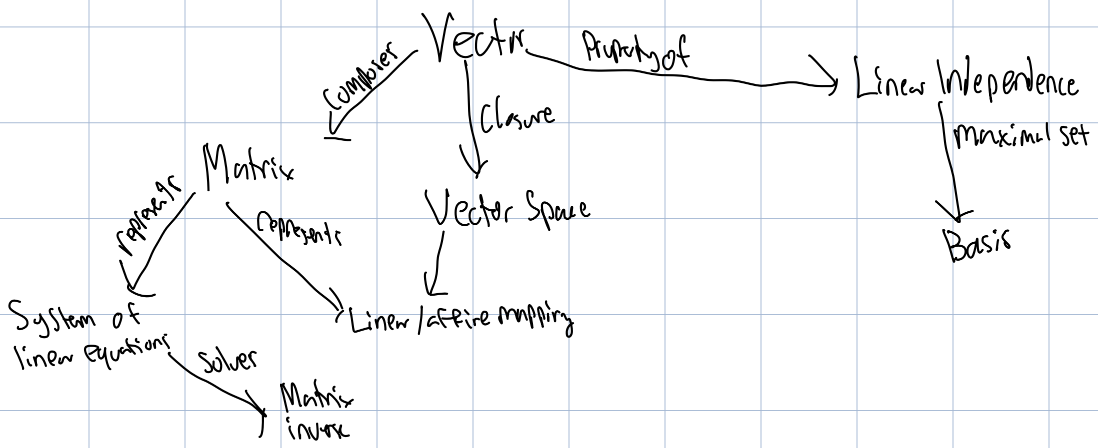
Multiplying $a\in\mathbb{R}^n$ by scalar $\lambda\in\mathbb{R}$ results in scaled vector $\lambda a\in\mathbb{R}^n$

Vectors $\in\mathbb{R}^n$ correspond loosely to an array of real numbers on a computer.

The idea of 'closure' - what is the set of all things that will result from my operation.

↳ in context of Vectors - from MIT lecture - Can I solve $Ax = b$ for every b? [vector space]

Vector Mind Map:



Geometric Interpretation in my own words (prompt)

The visual /graphed representation of vectors. By graphing, we can see how adding two vectors together alters the segments they were. Or alternatively, we can see that going from tip to tail of $\vec{x}$ and $\vec{y}$ would result in the same as adding them together and then graphing. With scaling, we see the length of the vector scales, while the direction stays the same.

How are vectors different than points?

A point is a static "point" in a plane. It has no direction or magnitude (length). A vector has both direction and magnitude. A vector can also be scaled while a point stays the same. We can view a point as a location on a map, while a vector can be viewed as the path to get to a point.

$\hat{i}$ - i-hat - unit vector x $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\hat{j}$ - j-hat - unit vector y $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Think of Vector coordinates as scalers of $\hat{i}$ and $\hat{j}$

$\hat{i}$ and $\hat{j}$ are the basis vectors of the xy coordinate system.

→ building blocks of Vector Space.

$a\vec{v} + b\vec{w}$ = linear combination of $\vec{v}$ and $\vec{w}$. Any vector can be expressed via scalers · unit vectors. The scalars are called coordinates.

Any 2D vector is a linear combination of $\hat{i}$ and $\hat{j}$.

Span - all possible vectors you can reach with linear combinations.
- Two 2D vectors span the entire 2D plane