The Geometry of Linear Equations

(MIT Gilbert Strang)

$2x - y = 0$
$-x + 2y = 3$

$\underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$x = 0$

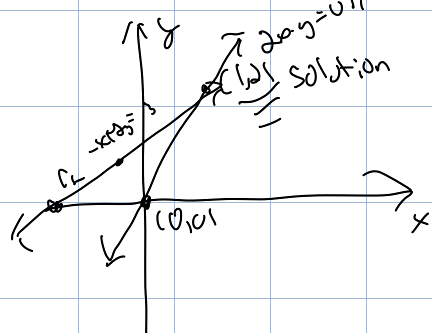
n linear equations, n unknowns

Goal: Describe
- > row picture
- *> column picture
- > Matrix form - algebra form

linear equation → $Ax = b$

Row Picture

$2x - y = 0 \; r_1$
$-x + 2y = 3 \; r_2$

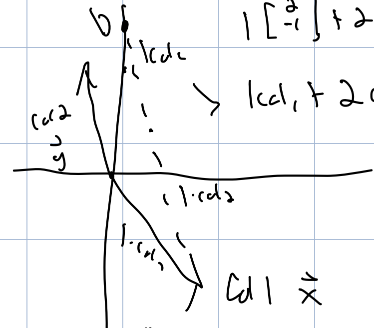


Column Picture

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Goal: Find the right linear combination to solve the system of equations.

$1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$



> $1 col_1 + 2 col_2 = (0,3)$

$b = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

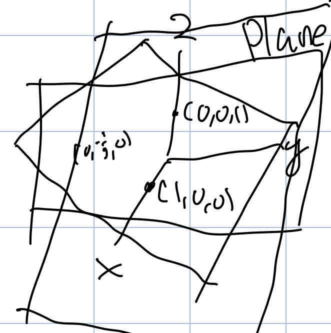
what would be all the linear combinations of $x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ be?

There are infinite solutions to this system

$2x - y = 0$
$-x + 2y - z = -1$
$-3y + 4z = 4$

$$\underbrace{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}}_{b}$$

Row Picture



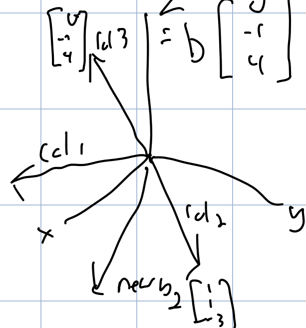
Each equation in a system of equations (solutions) can be mapped to a plane. Then a line can be drawn through the 3 or more planes to find the solution to the system of equations.

$$\underbrace{x \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}}_{\text{linear combination}} = b \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

$x = 0 \quad y = 0 \quad z = 1$

row good for 2

row 3+ dims, unclear



$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}1\\1\\-3\end{bmatrix}$$

Can I solve $Ax=b$ for every b? in other words, do the linear combos of the columns fill 3 dimensional space?

$x=1,\ y=1,\ z=0$ $\quad (1,1,0)=b_2\begin{bmatrix}1\\1\\-3\end{bmatrix}$

For this matrix A, yes! => invertible matrix

When could the linear combos not fill 3D space?

=> if all 3 vectors lie in the same plane: Col3 = Col1 + Col2, noninvertible

Some matrix A × some vector x = b

$Ax=b$

$$\begin{bmatrix}2&5\\1&3\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

Two Ways: Column way: $\begin{bmatrix}\text{1 of first column}\\\text{2 of second column}\end{bmatrix}$ $\begin{bmatrix}2\times1 & 5\times2\\1\times1 & 3\times2\end{bmatrix}=\begin{bmatrix}2+10\\1+6\end{bmatrix}$

$$1\begin{bmatrix}2\\1\end{bmatrix}+2\begin{bmatrix}5\\3\end{bmatrix}=\begin{bmatrix}12\\7\end{bmatrix}$$

Row Way: (dot products)

$$\begin{bmatrix}2^{(1)}+5^{(2)}\\1^{(1)}+3^{(2)}\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}12\\7\end{bmatrix}$$

Mathematics for Machine Learning Ch 2.1

Linear Algebra - the study of how vectors interact with eachother in relation to space.

in general, vectors are objects that can be added together or multiplied by scalars to produce a new object of the same dimension

↳ vectors can only perform mathematical operations with another vector of the same dimension.

Examples of Vectors:

1) geometric vectors - directed segments $\vec{x}$ that allow us to perform operations using mathematical intuition regarding direction and magnitude. addition and scalar multiplication are operations of a geometric vector
2) Polynomials - can be added together resulting in a new polynomial (property 1) can be multiplied by a scalar, resulting in a polynomial. (property 2) Thus, polynomials are an unusual form of a vector
3) Audio Signals - represented as a series of numbers. can be added together, resulting in a new series of numbers (p1) we can scale an audio signal into another one (p2)
4) Elements of $\mathbb{R}^n$ (tuples of n real numbers). $\mathbb{R}^n$ is more abstract than polynomials

$a=\begin{bmatrix}1\\2\\3\end{bmatrix}\in\mathbb{R}^3$ is saying a is a tuple of 3 real numbers, and is a member of $\mathbb{R}^3$

Adding two vectors $a, b \in \mathbb{R}^n$ will result in a vector $c \in \mathbb{R}^n$

Multiplying $a \in \mathbb{R}^n$ by scalar $\lambda \in \mathbb{R}$ results in scaled vector $\lambda a \in \mathbb{R}^n$

Vectors $\in \mathbb{R}^n$ correspond loosely to an array of real numbers on a computer.

The idea of 'closure' - what is the set of all things that will result from my operation.

↳ in context of vectors. from MIT lecture - Can I solve $Ax = b$ for every b? (vector space)

Vector Mind Map: