

Complexity and Modularity: A Note on Roger Sessions' *The Mathematics of IT Simplification*

J. Subramanyam*

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1 Introduction and Purpose

In his article *The Mathematics of IT Simplification*¹, Roger Sessions tackles the problem of minimizing the complexity of an application built from combining elements of business functionality. If we think of software applications as collections of business functionality that are then implemented by IT architects and programmers, is there a recipe we can provide them for grouping this functionality to minimize complexity? To answer this question we must first define complexity and we'll get to that in the next section.

Having a architecture complexity-minimizer recipe, while extremely useful, seems difficult to pull off. After all, creating an IT architecture is a complex task requiring multiple inputs that are often not always available, especially during the early stages of requirements gathering. Moreover, the one certainty in application development is that requirements change throughout the development life cycle keeping business analysts, architects, and developers busy making changes and struggling to manage the ripple effects. So it would be really nice if the recipe for minimizing the complexity of a software application can be used early in the requirements gathering period, ideally when we have a relatively poor understanding of exactly what the application is supposed to do. Is this possible or is it too good to be true?

Sessions has developed a recipe that seems up to the task – it reliably minimizes architectural complexity *and* can be used early in the development life cycle when details about the application are sparse. Sessions' recipe relies on a claim which I'll state as follows:

Claim 1 *The method of Simple Iterative Partitions (SIP) – i.e. a method based on partitioning (in the mathematical sense) a set of business functionality items using a particular kind of equivalence relation (“business synergy”) – always minimizes the architectural complexity of an application.*

*jitendra.subramanyam@gmail.com

¹www.objectwatch.com/white-papers.htm/#Math. All page numbers in the text refer to pages in Sessions' article.

It's worth noting right up front that this claim may indeed be a theorem – I simply don't know and am not aware of a proof. Even if it is a theorem, it is still worth investigating the grouping patterns that increase or decrease complexity. There are two reasons for this. First, this investigation will reveal how dependent we are on this claim and hence if it a unique route to minimizing complexity or if there are other ways to accomplish minimal complexity without using SIP. [Having a non-semantic way of accomplishing the objective – important when we don't know any details about the application]. Second, while minimizing complexity is a good thing, it might not always be the optimal thing. Sometimes business constraints might not permit it; sometimes technical constraints might not permit it. And sometimes, designing for particular performance, security, extensibility, or modifiability goals might require injecting complexity into the system. In these situations it's useful for architects to have a way to understand how far from the minimum the design considerations are taking them and the design decisions available to keep complexity, if not at a minimum, then at least in check.

Therefore, my aim in this note is to supplement Sessions' recipe using mathematical simulations that provide IT architects insight into the types of arrangements that reduce (or increase) complexity. In this spirit of practical advice and additional illumination, the main purpose of this note is to answer the following questions.

1. Given N functionality elements, how should they be grouped to minimize system complexity? How many groups appear in the minimal complexity arrangement? How do these results vary as architectural techniques are used to eliminate or reduce certain kinds of complexity?
2. When making architectural design choices of how many groups to use and how many elements to put into each group, are there rules of thumb that can guide these choices? How do these rules of thumb change as architectural rules are used to eliminate or reduce certain kinds of complexity?

To answer these questions we need a clear definition of application complexity and that is what we turn to in the next section.

2 Defining Application Complexity

To begin, I'm going to make three assumptions that will simplify the calculation of application complexity. This will result in a definition of overall system complexity that is different from Sessions' definition, but one that hews closely to his conceptual model.

First, we assume that we have a "periodic table" of functionality elements from which any desired business functionality can be built. This assumption is neither trivial nor something I can show by providing examples, but it certainly makes modeling the complexity of an IT application easier (I'll let you decide if it's also useful).

The second assumption is to give each of these functionality elements a functional size of 1 unit, and hence a functional complexity of 1 unit. This assumption sidesteps exactly how functionality is measured. Use your favorite measure of functional size and make sure the periodic table of functionality elements are such that each ends up having a functional size of 1 unit. This way, each of the functionality elements also has a functional complexity of 1 unit. This assumption, which treats each functionality element as just the same as any other functionality element in terms of its functional size, allows us to generate groupings of functionality elements based on the mathematics of integer partitions rather than the more complicated (and computationally intensive) mathematics of set partitions.

Now that we have our functionality elements and their unit measures of size and complexity, the third assumption tackles how complexity is calculated when these elements are combined. Sessions distinguishes two forms of complexity – *functional* and *coordination* complexity (pp. 8–12). To understand these concepts, let's think about an application as being built from groups of functionality elements. The application as a whole comprises these groups of functionality and the connections between the groups as well as the connections within the functional elements of each group. This notion of how an application is put together is a familiar concept in IT architecture. The devil is of course in the details – how exactly to translate the requirements into functionality elements (or services) and how best to group and connect the functional elements to create a robust application architecture.

2.1 Diagnostic Complexity

Suppose you have 4 elements of functionality grouped together. Sessions' idea is that the complexity of this grouping arises from the number of subsets of this group you would have to check in order to diagnose or fix a bug in this set (p.9). He terms this functional complexity, but to keep it distinct from measures of functionality such as function points, I will call this type of complexity *diagnostic complexity* instead. Hence, the diagnostic complexity of a set A of n functional elements is the number of elements in the power set of A minus 1 (because the empty set, which is always an element of the power set is ignored). The number of elements in the power set of a set with n elements is 2^n . Hence the diagnostic complexity of a group of n functional elements is:

$$d(n) = 2^n - 1 \quad (1)$$

Rather than the power set function, Sessions chooses to define the diagnostic complexity of a set A of n elements as $n^{3.11}$ where 3.11 is Glass' constant. The functional complexity generated by Glass' constant is much lower than that generated by the power set in equation 1. I'll stick with the power set definition because it's more in line with Sessions' concept of functional complexity as he defines it on p.9 of his article. We can compare the rates of growth of diagnostic complexity under equation 1 and Glass' constant. We see that a set of 10 or fewer

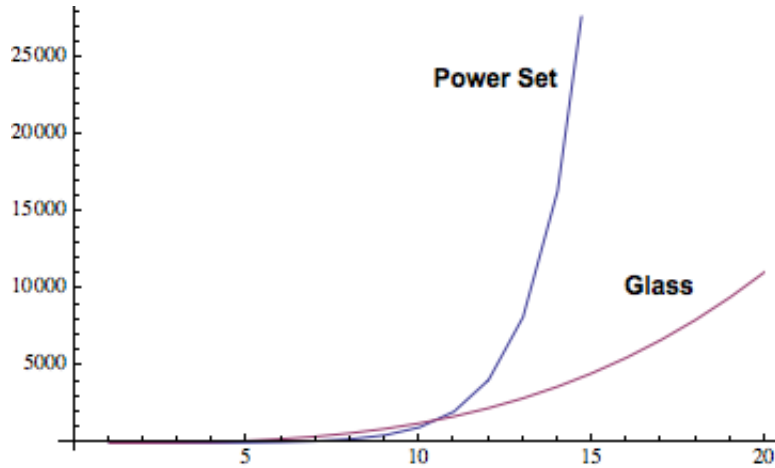


Figure 1: Rate of Growth of Diagnostic Complexity

functionality elements has a greater Glass diagnostic complexity but once the number of elements goes past 11, the diagnostic complexity as defined the size of the power set in equation 1 radically outpaces Glass functional complexity. As I mentioned, I'll use the power set equation 1 to calculate diagnostic complexity because it fits better with the concept of the number of groups of things you have to investigate in order to diagnose and solve a problem found in a set of n things.

2.2 Functional Complexity

The intuition behind functional complexity is that it is related to the functional size of an element – the greater the functional size, the more functionality it has and hence the more complex it must be to create the element. Measuring the amount of functionality in a software system is not a trivial affair. A traditional measure of it is in terms of function points which come in many varieties. And there are a number of other approaches to measuring software size as well. In this note, I'll stick with the second assumption above and pretend that all functional elements have a unit functional size. Under this assumption, a group of n functionality elements has functional size of n size units and a functional complexity of n functional complexity units. Notice that this is quite different from Sessions' functional complexity which I denote as diagnostic complexity instead.

2.3 Coordination Complexity

Coordination complexity arises due to the dependencies between functional elements (p.11). If we think of a dependency as a connection between any two elements, the maximal coordination complexity of a set of n functional elements,

assuming two-way connections between all pairs of elements is

$$c(n) = (n - 1)^n \quad (2)$$

2.4 System Complexity

A system is a collection of one or more groups of functionality elements. Each group of functionality elements would have a diagnostic complexity, a functional complexity, and a coordination complexity; there would be dependencies between the groups which would add to the coordination complexity, and finally, there would also be diagnostic complexity at the group level. We're assuming that when it comes to connections from one group of elements to another group of elements, those connections are made at the group level. In other words, connections at the group level don't penetrate the "cell wall" of the group.

As an example, consider a simple system that has 3 groups of functional elements; the first group has 4 elements, the second group has 5 elements, and the third group has 6 elements. Let's label the groups g_1 , g_2 , and g_3 . To calculate the complexity of this system, we have to add up the diagnostic and coordination complexity of each group and then add the diagnostic and coordination complexity of the three groups at the group level. Within a group, the diagnostic and coordination complexity can be termed "intra-group complexity"; at the group level, these become "inter-group complexity". At the group level there is no functional complexity because it has already been taken care of at the intra-group level.

$$\begin{aligned} S = [d_{g1} + c_{g1} + f_{g1}] &+ [d_{g2} + c_{g2} + f_{g2}] + [d_{g3} + c_{g3} + f_{g3}] \\ &+ [d_{g1,g2,g3} + c_{g1,g2,g3}] \end{aligned} \quad (3)$$

Proceeding step by step for the example system we have at hand,

$$d_{g1} = 2^4 - 1 \quad (4)$$

$$c_{g1} = (4 - 1)^4 \quad (5)$$

$$f_{g1} = 4 \quad (6)$$

$$d_{g2} = 2^5 - 1 \quad (7)$$

$$c_{g2} = (5 - 1)^5 \quad (8)$$

$$f_{g2} = 5 \quad (9)$$

$$d_{g3} = 2^6 - 1 \quad (10)$$

$$c_{g3} = (6 - 1)^6 \quad (11)$$

$$f_{g3} = 6 \quad (12)$$

$$f_{g1,g2,g3} = 2^3 - 1 \quad (13)$$

$$c_{g1,g2,g3} = (3 - 1)^3 \quad (14)$$

which gives us a system complexity total of $S = 1148$. You may be wondering about the units for measuring complexity. Think of these numerical values and the subsequent complexity values in this note as values expressed in *standard complexity units*.

Configuration	Complexity
{2, 1}	12
{3}	19
{1, 1, 1}	21

,

Configuration	Complexity
{2, 2}	16
{3, 1}	24
{2, 1, 1}	25
{4}	101
{1, 1, 1, 1}	104

,

Configuration	Complexity
{3, 2}	28
{2, 2, 1}	29
{3, 1, 1}	37
{4, 1}	106
{2, 1, 1, 1}	108
{5}	1061
{1, 1, 1, 1, 1}	1065

Figure 2: System Complexity for Modular Arrangements of 3, 4, and 5 Functional Elements

3 Modularity and Complexity

Let's start by answering question 1 above: Given N functionality elements, how should they be grouped to minimize system complexity? How many groups appear in the minimal complexity arrangement?

The complexity of a system depends on how it is modularized – that is, how the functional elements are grouped. Suppose we have N functional elements. To answer question 1 we need to find out how to split the system into k groups of n_1, n_2, \dots, n_k elements in each group such that we minimize the following expression for system complexity:

$$s = \sum_{i=1}^k [(2^{n_i} - 1) + (n_i - 1)^{n_i} + n_i] + (2^k - 1) + (k - 1)^k \quad (15)$$

The items under the summation in equation 15 stand for the diagnostic, coordination, and functional complexity of the elements within each of the groups and the items outside the summation stand for the diagnostic and coordination complexity of the k groups themselves. Since we've assumed all possible connections, s expresses the maximum amount of complexity in a system with any given modularization. If there is a way to reduce the connections, the total complexity will be smaller. We'll later consider some of these possibilities for reducing architectural complexity.

Let's investigate how to minimize s . Initially, we place no restrictions on how functionality elements can be grouped – i.e. we remain semantically agnostic about how functionality elements are related to one another and try out all possible groupings. The task is the following: Given N elements, how many ways can we distribute these elements into a set of containers?

Figure 2 shows how system complexity grows as a function of both the number of elements of functionality and the way in which these elements are grouped.

It is clear from figure 2 how system complexity is dependent (and quite sensitive) to the way in which the elements of functionality are grouped. In the rightmost table where the various modular arrangements of 5 functional elements is shown, the lowest system complexity of 28 for the grouping $\{3, 2\}$

Configuration	Complexity
{2, 2, 2}	33
{3, 3}	40
{3, 2, 1}	41
{4, 2}	110
{2, 2, 1, 1}	112
{4, 1, 1}	119
{3, 1, 1, 1}	120
{5, 1}	1066
{2, 1, 1, 1, 1}	1069
{6}	15695
{1, 1, 1, 1, 1, 1}	15700

Configuration	Complexity
{3, 2, 2}	45
{3, 3, 1}	53
{2, 2, 2, 1}	116
{4, 3}	122
{4, 2, 1}	123
{3, 2, 1, 1}	124
{4, 1, 1, 1}	202
{5, 2}	1070
{2, 2, 1, 1, 1}	1073
{5, 1, 1}	1079
{3, 1, 1, 1, 1}	1081
{6, 1}	15700
{2, 1, 1, 1, 1, 1}	15704
{7}	280071
{1, 1, 1, 1, 1, 1, 1}	280077

Configuration	Complexity
{3, 3, 2}	57
{2, 2, 2, 2}	120
{4, 2, 2}	127
{3, 2, 2, 1}	128
{4, 3, 1}	135
{3, 3, 1, 1}	136
{4, 4}	204
{4, 2, 1, 1}	206
{2, 2, 2, 1, 1}	1077
{5, 3}	1082
{5, 2, 1}	1083
{3, 2, 1, 1, 1}	1085
{5, 1, 1, 1}	1162
{4, 1, 1, 1, 1}	1163
{6, 2}	15704
{2, 2, 1, 1, 1, 1}	15708
{6, 1, 1}	15713
{3, 1, 1, 1, 1, 1}	15716
{7, 1}	280076
{2, 1, 1, 1, 1, 1, 1}	280081
{8}	5765065
{1, 1, 1, 1, 1, 1, 1, 1}	5765072

Figure 3: System Complexity for Modular Arrangements of 6, 7, and 8 Functional Elements

which denotes two groups with one group having 3 elements and the other having two elements is about 40 times lower than the highest complexity 1065 obtained with the $\{1, 1, 1, 1, 1\}$ grouping which denotes 5 groups each with 1 element.

Further investigation of system complexity values for 6 to 8 functional elements continues to build on the pattern we see in figure 2. Using too few or too many groups result in high system complexity; somewhere in the middle seems to be where system complexity is at a minimum. For the case of 8 functionality elements, the difference between the minimum and maximum system complexity is a factor of 100,000 (57 to 5,765,072).

As the number of functionality elements grows larger, the complexity of sub-optimal groupings grows very rapidly. Figures 4, 5, 6, and 7 show the groupings that result in the lowest and highest 10 complexity values respectively for functionality elements that vary from 10 to 12 and then from 30 to 32.

With just this little bit of simulation we can see a clear pattern for reducing system complexity. To keep system complexity to a minimum, it's best to group functionality elements into groups that are roughly equal in number. This is the lesson that we can take away from our brief initial investigation. Thus far we have not made any assumptions about how the functionality elements are related. We can reduce complexity further when we have information about how the functionality elements are semantically related.

In the following two sections we will continue to address question 1 but alter the system complexity function in equation 15 to reflect architectural designs

Configuration	Complexity	Configuration	Complexity	Configuration	Complexity
{3, 3, 2, 2}	144	{3, 3, 3, 2}	156	{3, 3, 3, 3}	168
{4, 3, 3}	151	{4, 3, 2, 2}	226	{4, 3, 3, 2}	238
{3, 3, 3, 1}	152	{4, 4, 3}	233	{4, 4, 2, 2}	308
{4, 2, 2, 2}	214	{4, 3, 3, 1}	234	{4, 4, 4}	315
{4, 4, 2}	221	{4, 4, 2, 1}	304	{4, 4, 3, 1}	316
{4, 3, 2, 1}	222	{3, 2, 2, 2, 2}	1097	{3, 3, 2, 2, 2}	1109
{4, 4, 1, 1}	300	{3, 3, 2, 2, 1}	1105	{3, 3, 3, 2, 1}	1117
{2, 2, 2, 2, 2}	1085	{5, 3, 3}	1111	{4, 2, 2, 2, 2}	1179
{3, 2, 2, 2, 1}	1093	{3, 3, 3, 1, 1}	1113	{5, 3, 2, 2}	1186
{5, 3, 2}	1099	{5, 2, 2, 2}	1174	{4, 3, 2, 2, 1}	1187

Figure 4: Ten Lowest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements

Configuration	Complexity	Configuration	Complexity	Configuration	Complexity
{7, 1, 1, 1}	280172	{8, 1, 1, 1}	5765166	{9, 1, 1, 1}	134218350
{4, 1, 1, 1, 1, 1, 1}	280175	{4, 1, 1, 1, 1, 1, 1, 1}	5765170	{4, 1, 1, 1, 1, 1, 1, 1, 1}	134218355
{8, 2}	5765074	{9, 2}	134218258	{10, 2}	3486785444
{2, 2, 1, 1, 1, 1, 1, 1}	5765080	{2, 2, 1, 1, 1, 1, 1, 1, 1}	134218265	{2, 2, 1, 1, 1, 1, 1, 1, 1, 1}	3486785452
{8, 1, 1}	5765083	{9, 1, 1}	134218267	{10, 1, 1}	3486785453
{3, 1, 1, 1, 1, 1, 1, 1}	5765088	{3, 1, 1, 1, 1, 1, 1, 1, 1}	134218273	{3, 1, 1, 1, 1, 1, 1, 1, 1, 1}	3486785460
{9, 1}	134218254	{10, 1}	3486785440	{11, 1}	100000002064
{2, 1, 1, 1, 1, 1, 1, 1, 1}	134218261	{2, 1, 1, 1, 1, 1, 1, 1, 1, 1}	3486785448	{2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	100000002073
{10}	3486785435	{11}	100000002059	{12}	3138428380829
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	3486785444	{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	100000002069	{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	3138428380840

Figure 5: Ten Highest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements

Configuration	Complexity	Configuration	Complexity	Configuration	Complexity
{5, 5, 5, 5, 5, 5}	22048	{6, 5, 5, 5, 5, 5}	36682	{6, 6, 5, 5, 5, 5}	51316
{6, 5, 5, 5, 5, 4}	35722	{6, 6, 5, 5, 5, 4}	50356	{6, 6, 6, 5, 5, 4}	64990
{6, 6, 5, 5, 4, 4}	49396	{6, 6, 6, 5, 4, 4}	64030	{6, 6, 6, 6, 4, 4}	78664
{6, 6, 5, 5, 5, 3}	50274	{6, 6, 6, 5, 5, 3}	64908	{6, 6, 6, 6, 5, 3}	79542
{6, 6, 6, 4, 4, 4}	63070	{6, 6, 6, 6, 4, 3}	78582	{6, 6, 6, 6, 6, 2}	94164
{6, 6, 6, 5, 4, 3}	63948	{6, 6, 6, 6, 5, 2}	79530	{5, 5, 5, 5, 4, 4, 4}	284603
{6, 6, 6, 5, 5, 2}	64896	{6, 6, 6, 6, 6, 1}	94160	{5, 5, 5, 5, 5, 4, 3}	285481
{6, 6, 6, 6, 3, 3}	78500	{5, 5, 5, 4, 4, 4, 4}	283643	{5, 5, 5, 5, 5, 5, 2}	286429
{6, 6, 6, 6, 4, 2}	78570	{5, 5, 5, 5, 4, 4, 3}	284521	{6, 5, 5, 4, 4, 4, 4}	298277
{6, 6, 6, 6, 6}	79525	{5, 5, 5, 5, 5, 3, 3}	285399	{6, 5, 5, 5, 4, 4, 3}	299155

Figure 6: Ten Lowest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements

[illegible]

Figure 7: Ten Highest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements

Arrangement	System Complx		Arrangement	System Complx		Arrangement	System Complx
{2, 1}	11	{	{2, 2}	15	{	{2, 2, 1}	21
{1, 1, 1}	13		{2, 1, 1}	17		{3, 2}	27
{3}	19		{3, 1}	23		{2, 1, 1, 1}	27
			{1, 1, 1, 1}	23		{3, 1, 1}	29
			{4}	101		{1, 1, 1, 1, 1}	41
						{4, 1}	105
						{5}	1061

Figure 8: System Complexity Values for Modular Arrangements of 3, 4, and 5 Functional Elements When Inter-Group Coordination Complexity = 0

that eliminate or reduce components of complexity. This brings us closer to the way Sessions sets up the partition under the equivalence relation of business synergy and allows us to see how the different component complexities impact system complexity. As long as architects can control the complexities of these individual components via their design, they will be able to fine tune their designs to keep system complexity in check.

4 System Complexity as Inter-Group Coordination Complexity is Eliminated or Reduced

What if the grouping of functionality elements are such that there need be no dependencies between the elements in each group? In other words, what if each group is completely self contained? Let's investigate how system complexity behaves under this scenario.

4.1 Inter-Group Coordination Complexity is Zero

In this scenario, we eliminate inter-group coordination complexity and see which arrangements minimize and maximize system complexity. Figures 8 to 13 capture the results.

4.2 Inter-Group Coordination Complexity is 0.6

When inter-group coordination complexity is not completely eliminated but attenuated to 0.6, we get similar results as we did in the previous subsection. The results for inter-group coordination complexity at 0.6 are captured in Figures 14 to 19.

When the groups can be made independent by designing them with no inter-group coordination dependencies, we see that having as close to equal number of elements in each group is key for keeping complexity in check. This is not too different from the results we saw in section 3 above.

Arrangement	System Complx
{2, 2, 2}	25
{2, 2, 1, 1}	31
{3, 2, 1}	33
{3, 3}	39
{3, 1, 1, 1}	39
{2, 1, 1, 1, 1}	45
{1, 1, 1, 1, 1, 1}	75
{4, 2}	109
{4, 1, 1}	111
{5, 1}	1065
{6}	15 695

Arrangement	System Complx
{2, 2, 2, 1}	35
{3, 2, 2}	37
{3, 2, 1, 1}	43
{3, 3, 1}	45
{2, 2, 1, 1, 1}	49
{3, 1, 1, 1, 1}	57
{2, 1, 1, 1, 1, 1}	79
{4, 2, 1}	115
{4, 3}	121
{4, 1, 1, 1}	121
{1, 1, 1, 1, 1, 1, 1}	141
{5, 2}	1069
{5, 1, 1}	1071
{6, 1}	15 699
{7}	280 071

Arrangement	System Complx
{2, 2, 2, 2}	39
{3, 2, 2, 1}	47
{3, 3, 2}	49
{2, 2, 2, 1, 1}	53
{3, 3, 1, 1}	55
{3, 2, 1, 1, 1}	61
{2, 2, 1, 1, 1, 1}	83
{3, 1, 1, 1, 1, 1}	91
{4, 2, 2}	119
{4, 2, 1, 1}	125
{4, 3, 1}	127
{4, 1, 1, 1, 1}	139
{2, 1, 1, 1, 1, 1, 1}	145
{4, 4}	203
{1, 1, 1, 1, 1, 1, 1, 1}	271
{5, 2, 1}	1075
{5, 3}	1081
{5, 1, 1, 1}	1081
{6, 2}	15 703
{6, 1, 1}	15 705
{7, 1}	280 075
{8}	5 765 065

Figure 9: System Complexity Values for Modular Arrangements of 6, 7, and 8 Functional Elements When Inter-Group Coordination Complexity = 0

Arrangement	System Complx
{2, 2, 2, 2, 2}	61
{3, 3, 2, 2}	63
{3, 2, 2, 2, 1}	69
{3, 3, 3, 1}	71
{3, 3, 2, 1, 1}	77
{2, 2, 2, 2, 1, 1}	91
{3, 2, 2, 1, 1, 1}	99
{3, 3, 1, 1, 1, 1}	107
{4, 2, 2, 2}	133
{4, 3, 2, 1}	141

Arrangement	System Complx
{3, 2, 2, 2, 2}	73
{3, 3, 3, 2}	75
{3, 3, 2, 2, 1}	81
{3, 3, 3, 1, 1}	89
{2, 2, 2, 2, 2, 1}	95
{3, 2, 2, 2, 1, 1}	103
{3, 3, 2, 1, 1, 1}	111
{4, 3, 2, 2}	145
{4, 2, 2, 2, 1}	151
{4, 3, 3, 1}	153

Arrangement	System Complx
{3, 3, 2, 2, 2}	85
{3, 3, 3, 3}	87
{3, 3, 3, 2, 1}	93
{2, 2, 2, 2, 2, 2}	99
{3, 2, 2, 2, 2, 1}	107
{3, 3, 2, 2, 1, 1}	115
{3, 3, 3, 1, 1, 1}	123
{4, 2, 2, 2, 2}	155
{4, 3, 3, 2}	157
{2, 2, 2, 2, 2, 1, 1}	161

Figure 10: 10 Lowest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements When Inter-Group Coordination Complexity = 0

Arrangement	System Complx	Arrangement	System Complx	Arrangement	System Complx
{6, 3, 1}	15 721	{7, 3, 1}	280 097	{8, 3, 1}	5 765 091
{6, 1, 1, 1, 1}	15 733	{7, 1, 1, 1, 1}	280 109	{8, 1, 1, 1, 1}	5 765 103
{6, 4}	15 797	{7, 4}	280 173	{8, 4}	5 765 167
{7, 2, 1}	280 085	{8, 2, 1}	5 765 079	{9, 2, 1}	134 218 263
{7, 3}	280 091	{8, 3}	5 765 085	{9, 3}	134 218 269
{7, 1, 1, 1}	280 091	{8, 1, 1, 1}	5 765 085	{9, 1, 1, 1}	134 218 269
{8, 2}	5 765 073	{9, 2}	134 218 257	{10, 2}	3 486 785 443
{8, 1, 1}	5 765 075	{9, 1, 1}	134 218 259	{10, 1, 1}	3 486 785 445
{9, 1}	134 218 253	{10, 1}	3 486 785 439	{11, 1}	100 000 002 063
{10}	3 486 785 435	{11}	100 000 002 059	{12}	3 138 428 380 829

Figure 11: 10 Highest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements When Inter-Group Coordination Complexity = 0

Arrangement	System Complx	Arrangement	System Complx	Arrangement	System Complx
{4, 4, 4, 4, 4, 4, 3, 3}	891	{4, 4, 4, 4, 4, 4, 4, 3}	973	{4, 4, 4, 4, 4, 4, 4, 4}	1055
{4, 4, 4, 3, 3, 3, 3, 3, 3}	919	{4, 4, 4, 4, 3, 3, 3, 3, 3}	1001	{4, 4, 4, 4, 4, 3, 3, 3, 3}	1083
{4, 4, 4, 4, 4, 4, 4, 2}	961	{4, 4, 4, 4, 4, 3, 3, 3, 2}	1071	{4, 4, 4, 4, 4, 4, 3, 3, 2}	1153
{4, 4, 4, 4, 3, 3, 3, 3, 2}	989	{4, 4, 4, 4, 4, 4, 3, 2, 2}	1141	{4, 4, 4, 4, 4, 4, 4, 2, 2}	1223
{4, 4, 4, 4, 4, 3, 3, 2, 2}	1059	{4, 4, 4, 4, 4, 4, 3, 3, 1}	1149	{4, 4, 4, 4, 4, 4, 4, 3, 1}	1231
{4, 4, 4, 4, 4, 3, 3, 3, 1}	1067	{4, 4, 4, 4, 4, 4, 4, 2, 1}	1219	{4, 4, 3, 3, 3, 3, 3, 3, 3, 3}	1367
{4, 4, 4, 4, 4, 4, 2, 2, 2}	1129	{4, 3, 3, 3, 3, 3, 3, 3, 3, 3}	1285	{4, 4, 4, 3, 3, 3, 3, 3, 3, 2}	1437
{4, 4, 4, 4, 4, 4, 3, 2, 1}	1137	{4, 4, 3, 3, 3, 3, 3, 3, 3, 2}	1355	{4, 4, 4, 4, 3, 3, 3, 3, 2, 2}	1507
{3, 3, 3, 3, 3, 3, 3, 3, 3}	1203	{4, 4, 4, 3, 3, 3, 3, 3, 2, 2}	1425	{4, 4, 4, 4, 3, 3, 3, 3, 3, 1}	1515
{4, 4, 4, 4, 4, 4, 4, 1, 1}	1215	{4, 4, 4, 3, 3, 3, 3, 3, 3, 1}	1433	{4, 4, 4, 4, 4, 3, 3, 2, 2, 2}	1577

Figure 12: 10 Lowest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements When Inter-Group Coordination Complexity = 0

Arrangement	System Complx
{26, 3, 1}	2 220 446 049 250 313 080 847 263 336 248 749 541
{26, 1, 1, 1, 1}	2 220 446 049 250 313 080 847 263 336 248 749 553
{26, 4}	2 220 446 049 250 313 080 847 263 336 248 749 617
{27, 2, 1}	160 059 109 085 386 090 080 713 531 498 539 515 945
{27, 3}	160 059 109 085 386 090 080 713 531 498 539 515 951
{27, 1, 1, 1}	160 059 109 085 386 090 080 713 531 498 539 515 951
{28, 2}	11 972 515 182 562 019 788 602 740 026 717 315 541 173
{28, 1, 1}	11 972 515 182 562 019 788 602 740 026 717 315 541 175
{29, 1}	928 074 647 171 094 496 152 036 391 094 209 499 627 553
{30}	74 462 898 441 675 122 902 293 018 227 199 468 741 762 455

Arrangement	System Complx
{27, 3, 1}	160 059 109 085 386 090 080 713 531 498 539 515 957
{27, 1, 1, 1, 1}	160 059 109 085 386 090 080 713 531 498 539 515 969
{27, 4}	160 059 109 085 386 090 080 713 531 498 539 516 033
{28, 2, 1}	11 972 515 182 562 019 788 602 740 026 717 315 541 179
{28, 3}	11 972 515 182 562 019 788 602 740 026 717 315 541 185
{28, 1, 1, 1}	11 972 515 182 562 019 788 602 740 026 717 315 541 185
{29, 2}	928 074 647 171 094 496 152 036 391 094 209 499 627 557
{29, 1, 1}	928 074 647 171 094 496 152 036 391 094 209 499 627 559
{30, 1}	74 462 898 441 675 122 902 293 018 227 199 468 741 762 459
{31}	6 176 733 962 839 470 000 000 000 000 000 000 002 147 483 679

Arrangement	System Complx
{28, 3, 1}	11 972 515 182 562 019 788 602 740 026 717 315 541 191
{28, 1, 1, 1, 1}	11 972 515 182 562 019 788 602 740 026 717 315 541 203
{28, 4}	11 972 515 182 562 019 788 602 740 026 717 315 541 267
{29, 2, 1}	928 074 647 171 094 496 152 036 391 094 209 499 627 563
{29, 3}	928 074 647 171 094 496 152 036 391 094 209 499 627 569
{29, 1, 1, 1}	928 074 647 171 094 496 152 036 391 094 209 499 627 569
{30, 2}	74 462 898 441 675 122 902 293 018 227 199 468 741 762 463
{30, 1, 1}	74 462 898 441 675 122 902 293 018 227 199 468 741 762 465
{31, 1}	6 176 733 962 839 470 000 000 000 000 000 000 002 147 483 683
{32}	529 144 398 052 420 314 716 929 933 900 838 757 441 681 734 689

Figure 13: 10 Highest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements When Inter-Group Coordination Complexity = 0

Arrangement	System Complx
{2, 1}	11.6
{1, 1, 1}	17.8
{3}	19.

Arrangement	System Complx
{2, 2}	15.6
{2, 1, 1}	21.8
{3, 1}	23.6
{1, 1, 1, 1}	71.6
{4}	101.

Arrangement	System Complx
{2, 2, 1}	25.8
{3, 2}	27.6
{3, 1, 1}	33.8
{2, 1, 1, 1}	75.6
{4, 1}	105.6
{1, 1, 1, 1, 1}	655.4
{5}	1061.

Figure 14: System Complexity Values for Modular Arrangements of 3, 4, and 5 Functional Elements When Inter-Group Coordination Complexity = 0.6

Arrangement	System Complx
{2, 2, 2}	29.8
{3, 2, 1}	37.8
{3, 3}	39.6
{2, 2, 1, 1}	79.6
{3, 1, 1, 1}	87.6
{4, 2}	109.6
{4, 1, 1}	115.8
{2, 1, 1, 1, 1}	659.4
{5, 1}	1065.6
{1, 1, 1, 1, 1, 1}	9450.
{6}	15 695.

Arrangement	System Complx
{3, 2, 2}	41.8
{3, 3, 1}	49.8
{2, 2, 2, 1}	83.6
{3, 2, 1, 1}	91.6
{4, 2, 1}	119.8
{4, 3}	121.6
{4, 1, 1, 1}	169.6
{2, 2, 1, 1, 1}	663.4
{3, 1, 1, 1, 1}	671.4
{5, 2}	1069.6
{5, 1, 1}	1075.8
{2, 1, 1, 1, 1, 1}	9454.
{6, 1}	15 699.6
{1, 1, 1, 1, 1, 1, 1}	168 103.
{7}	280 071.

Arrangement	System Complx
{3, 3, 2}	53.8
{2, 2, 2, 2}	87.6
{3, 2, 2, 1}	95.6
{3, 3, 1, 1}	103.6
{4, 2, 2}	123.8
{4, 3, 1}	131.8
{4, 2, 1, 1}	173.6
{4, 4}	203.6
{2, 2, 2, 1, 1}	667.4
{3, 2, 1, 1, 1}	675.4
{4, 1, 1, 1, 1}	753.4
{5, 2, 1}	1079.8
{5, 3}	1081.6
{5, 1, 1, 1}	1129.6
{2, 2, 1, 1, 1, 1}	9458.
{3, 1, 1, 1, 1, 1}	9466.
{6, 2}	15 703.6
{6, 1, 1}	15 709.8
{2, 1, 1, 1, 1, 1, 1}	168 107.
{7, 1}	280 076.
{1, 1, 1, 1, 1, 1, 1, 1}	3.45915×10^6
{8}	5.76507×10^6

Figure 15: System Complexity Values for Modular Arrangements of 6, 7, and 8 Functional Elements When Inter-Group Coordination Complexity = 0.6

Arrangement	System Complx
{3, 3, 2, 2}	111.6
{3, 3, 3, 1}	119.6
{4, 3, 3}	147.8
{4, 2, 2, 2}	181.6
{4, 3, 2, 1}	189.6
{4, 4, 2}	217.8
{4, 4, 1, 1}	267.6
{2, 2, 2, 2, 2}	675.4
{3, 2, 2, 2, 1}	683.4
{3, 3, 2, 1, 1}	691.4

Arrangement	System Complx
{3, 3, 3, 2}	123.6
{4, 3, 2, 2}	193.6
{4, 3, 3, 1}	201.6
{4, 4, 3}	229.8
{4, 4, 2, 1}	271.6
{3, 2, 2, 2, 2}	687.4
{3, 3, 2, 2, 1}	695.4
{3, 3, 3, 1, 1}	703.4
{4, 2, 2, 2, 1}	765.4
{4, 3, 2, 1, 1}	773.4

Arrangement	System Complx
{3, 3, 3, 3}	135.6
{4, 3, 3, 2}	205.6
{4, 4, 2, 2}	275.6
{4, 4, 3, 1}	283.6
{4, 4, 4}	311.8
{3, 3, 2, 2, 2}	699.4
{3, 3, 3, 2, 1}	707.4
{4, 2, 2, 2, 2}	769.4
{4, 3, 2, 2, 1}	777.4
{4, 3, 3, 1, 1}	785.4

Figure 16: 10 Lowest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements When Inter-Group Coordination Complexity = 0.6

Arrangement	System Complx
{7, 3}	280 092.
{7, 1, 1, 1}	280 140.
{2, 2, 1, 1, 1, 1, 1}	3.45916×10^6
{3, 1, 1, 1, 1, 1, 1}	3.45917×10^6
{8, 2}	5.76507×10^6
{8, 1, 1}	5.76508×10^6
{2, 1, 1, 1, 1, 1, 1, 1}	8.05312×10^7
{9, 1}	1.34218×10^8
{1, 1, 1, 1, 1, 1, 1, 1, 1}	2.09207×10^9
{10}	3.48679×10^9

Arrangement	System Complx
{8, 3}	5.76509×10^6
{8, 1, 1, 1}	5.76513×10^6
{2, 2, 1, 1, 1, 1, 1, 1}	8.05312×10^7
{3, 1, 1, 1, 1, 1, 1, 1}	8.05312×10^7
{9, 2}	1.34218×10^8
{9, 1, 1}	1.34218×10^8
{2, 1, 1, 1, 1, 1, 1, 1, 1}	2.09207×10^9
{10, 1}	3.48679×10^9
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	$6. \times 10^{10}$
{11}	$1. \times 10^{11}$

Arrangement	System Complx
{9, 3}	1.34218×10^8
{9, 1, 1, 1}	1.34218×10^8
{2, 2, 1, 1, 1, 1, 1, 1, 1}	2.09207×10^9
{3, 1, 1, 1, 1, 1, 1, 1, 1}	2.09207×10^9
{10, 2}	3.48679×10^9
{10, 1, 1}	3.48679×10^9
{2, 1, 1, 1, 1, 1, 1, 1, 1, 1}	$6. \times 10^{10}$
{11, 1}	$1. \times 10^{11}$
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	1.88306×10^{12}
{12}	3.13843×10^{12}

Figure 17: 10 Highest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements When Inter-Group Coordination Complexity = 0.6

Arrangement	System Complx
{5, 5, 5, 5, 5, 5}	15 798.
{6, 5, 5, 5, 5, 4}	29 472.
{6, 6, 5, 5, 4, 4}	43 146.
{6, 6, 5, 5, 5, 3}	44 024.
{6, 6, 6, 4, 4, 4}	56 820.
{6, 6, 6, 5, 4, 3}	57 698.
{6, 6, 6, 5, 5, 2}	58 646.
{6, 6, 6, 6, 3, 3}	72 250.
{6, 6, 6, 6, 4, 2}	72 320.
{6, 6, 6, 6, 5, 1}	73 276.

Arrangement	System Complx
{6, 5, 5, 5, 5, 5}	30 432.
{6, 6, 5, 5, 5, 4}	44 106.
{6, 6, 6, 5, 4, 4}	57 780.
{6, 6, 6, 5, 5, 3}	58 658.
{6, 6, 6, 6, 4, 3}	72 332.
{6, 6, 6, 6, 5, 2}	73 280.
{6, 6, 6, 6, 6, 1}	87 910.
{5, 5, 5, 4, 4, 4, 4}	171 669.
{5, 5, 5, 5, 4, 4, 3}	172 547.
{5, 5, 5, 5, 5, 3, 3}	173 425.

Arrangement	System Complx
{6, 6, 5, 5, 5, 5}	45 066.
{6, 6, 6, 5, 5, 4}	58 740.
{6, 6, 6, 6, 4, 4}	72 414.
{6, 6, 6, 6, 5, 3}	73 292.
{6, 6, 6, 6, 6, 2}	87 914.
{5, 5, 5, 5, 4, 4, 4}	172 629.
{5, 5, 5, 5, 5, 4, 3}	173 507.
{5, 5, 5, 5, 5, 5, 2}	174 455.
{6, 5, 5, 4, 4, 4, 4}	186 303.
{6, 5, 5, 5, 4, 4, 3}	187 181.

Figure 18: 10 Lowest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements When Inter-Group Coordination Complexity = 0.6

Arrangement	System Complex
{29, 2, 1}	9.28075×10^{41}
{29, 1, 1, 1}	9.28075×10^{41}
{3, 1}	4.46777×10^{43}
{2, 2, 1}	4.46777×10^{43}
{30, 2}	7.44629×10^{43}
{30, 1, 1}	7.44629×10^{43}
{2, 1}	3.70604×10^{45}
{31, 1}	6.17673×10^{45}
{1, 1}	3.17487×10^{47}
{32}	5.29144×10^{47}

Figure 19: 10 Highest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements When Inter-Group Coordination Complexity = 0.6

Arrangement	System Complx		Arrangement	System Complx		Arrangement	System Complx
{2, 1}	10	{	{2, 2}	13	{	{3, 2}	18
{3}	11		{3, 1}	15		{2, 2, 1}	19
{1, 1, 1}	13		{2, 1, 1}	16		{3, 1, 1}	21
			{4}	20		{4, 1}	24
			{1, 1, 1, 1}	23		{2, 1, 1, 1}	26
						{5}	37
						{1, 1, 1, 1, 1}	41

Figure 20: System Complexity Values for Modular Arrangements of 3, 4, and 5 Functional Elements When Inter- and Intra-Group Coordination Complexity = 0

5 System Complexity as Inter- and Intra-Group Coordination is Eliminated or Reduced

From Sessions' Figure 14 on p.27 of his paper I take it that this is the partition driven by the equivalence relation of business synergy. What types of configuration result in minimal system complexity? Figures 20 to 25 capture the results.

The pattern that continues to emerge from the simulations is the following: keeping the number of functionality elements balanced (i.e roughly equal) between the groups is the way to keep complexity in check. And this holds true even as intra- and inter-group coordination complexity is eliminated to mimic Sessions' case of a partition driven by the equivalence relation of business synergy.

So far we've dealt with coordination complexity. What happens when the system complexity function is altered by eliminating inter-group diagnostic complexity and then both inter- and intra-group diagnostic complexity? We turn to those scenarios in the next section.

6 System Complexity as Inter-Group Diagnostic Complexity is Eliminated

Figures 26 to 31 show the results of eliminating inter-group diagnostic complexity.

7 System Complexity as Inter- and Intra-Group Diagnostic Complexity are Eliminated

Figures 32 to 37 show the results of eliminating both inter- and intra-group diagnostic complexity.

Arrangement	System Complx
{2, 2, 2}	22
{3, 3}	23
{3, 2, 1}	24
{4, 2}	27
{2, 2, 1, 1}	29
{4, 1, 1}	30
{3, 1, 1, 1}	31
{5, 1}	41
{2, 1, 1, 1, 1}	44
{6}	70
{1, 1, 1, 1, 1, 1}	75

Arrangement	System Complx
{3, 2, 2}	27
{3, 3, 1}	29
{4, 3}	32
{2, 2, 2, 1}	32
{4, 2, 1}	33
{3, 2, 1, 1}	34
{4, 1, 1, 1}	40
{5, 2}	44
{5, 1, 1}	47
{2, 2, 1, 1, 1}	47
{3, 1, 1, 1, 1}	49
{6, 1}	74
{2, 1, 1, 1, 1, 1}	78
{7}	135
{1, 1, 1, 1, 1, 1, 1}	141

Arrangement	System Complx
{3, 3, 2}	32
{2, 2, 2, 2}	35
{4, 2, 2}	36
{3, 2, 2, 1}	37
{4, 3, 1}	38
{3, 3, 1, 1}	39
{4, 4}	41
{4, 2, 1, 1}	43
{5, 3}	49
{5, 2, 1}	50
{2, 2, 2, 1, 1}	50
{3, 2, 1, 1, 1}	52
{5, 1, 1, 1}	57
{4, 1, 1, 1, 1}	58
{6, 2}	77
{6, 1, 1}	80
{2, 2, 1, 1, 1, 1}	81
{3, 1, 1, 1, 1, 1}	83
{7, 1}	139
{2, 1, 1, 1, 1, 1, 1}	144
{8}	264
{1, 1, 1, 1, 1, 1, 1, 1}	271

Figure 21: System Complexity Values for Modular Arrangements of 6, 7, and 8 Functional Elements When Inter- and Intra-Group Coordination Complexity = 0

Arrangement	System Complx
{3, 3, 2, 2}	45
{4, 3, 3}	46
{3, 3, 3, 1}	47
{4, 2, 2, 2}	49
{4, 4, 2}	50
{4, 3, 2, 1}	51
{2, 2, 2, 2, 2}	56
{4, 4, 1, 1}	57
{5, 3, 2}	58
{3, 2, 2, 2, 1}	58

Arrangement	System Complx
{3, 3, 3, 2}	50
{4, 3, 2, 2}	54
{4, 4, 3}	55
{4, 3, 3, 1}	56
{4, 4, 2, 1}	60
{3, 2, 2, 2, 2}	61
{5, 3, 3}	63
{3, 3, 2, 2, 1}	63
{3, 3, 3, 1, 1}	65
{5, 2, 2, 2}	66

Arrangement	System Complx
{3, 3, 3, 3}	55
{4, 3, 3, 2}	59
{4, 4, 2, 2}	63
{4, 4, 4}	64
{4, 4, 3, 1}	65
{3, 3, 2, 2, 2}	66
{3, 3, 3, 2, 1}	68
{4, 2, 2, 2, 2}	70
{5, 3, 2, 2}	71
{5, 4, 3}	72

Figure 22: 10 Lowest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements When Inter- and Intra-Group Coordination Complexity = 0

Arrangement	System Complx
{7, 1, 1, 1}	155
{4, 1, 1, 1, 1, 1, 1}	158
{8, 2}	271
{8, 1, 1}	274
{2, 2, 1, 1, 1, 1, 1, 1}	277
{3, 1, 1, 1, 1, 1, 1, 1}	279
{9, 1}	525
{2, 1, 1, 1, 1, 1, 1, 1, 1}	532
{10}	1034
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	1043

Arrangement	System Complx
{8, 1, 1, 1}	284
{4, 1, 1, 1, 1, 1, 1, 1}	288
{9, 2}	528
{9, 1, 1}	531
{2, 2, 1, 1, 1, 1, 1, 1, 1}	535
{3, 1, 1, 1, 1, 1, 1, 1, 1}	537
{10, 1}	1038
{2, 1, 1, 1, 1, 1, 1, 1, 1, 1}	1046
{11}	2059
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	2069

Arrangement	System Complx
{9, 1, 1, 1}	541
{4, 1, 1, 1, 1, 1, 1, 1, 1}	546
{10, 2}	1041
{10, 1, 1}	1044
{2, 2, 1, 1, 1, 1, 1, 1, 1, 1}	1049
{3, 1, 1, 1, 1, 1, 1, 1, 1, 1}	1051
{11, 1}	2063
{2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	2072
{12}	4108
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	4119

Figure 23: 10 Highest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements When Inter- and Intra-Group Coordination Complexity = 0

Arrangement	System Complx
{5, 5, 5, 5, 5, 5}	279
{5, 5, 4, 4, 4, 4, 4}	294
{6, 5, 5, 5, 5, 4}	295
{5, 5, 5, 4, 4, 4, 3}	302
{6, 4, 4, 4, 4, 4, 4}	310
{5, 5, 5, 5, 4, 3, 3}	310
{6, 6, 5, 5, 4, 4}	311
{5, 5, 5, 5, 4, 4, 2}	314
{6, 5, 4, 4, 4, 4, 3}	318
{6, 6, 5, 5, 5, 3}	319

Arrangement	System Complx
{5, 5, 5, 4, 4, 4, 4}	311
{6, 5, 5, 5, 5, 5}	312
{5, 5, 5, 5, 4, 4, 3}	319
{6, 5, 4, 4, 4, 4, 4}	327
{5, 5, 5, 5, 5, 3, 3}	327
{6, 6, 5, 5, 5, 4}	328
{5, 5, 5, 5, 5, 4, 2}	331
{6, 5, 5, 4, 4, 4, 3}	335
{6, 5, 5, 5, 4, 3, 3}	343
{6, 6, 6, 5, 4, 4}	344

Arrangement	System Complx
{5, 5, 5, 5, 4, 4, 4}	328
{5, 5, 5, 5, 5, 4, 3}	336
{6, 5, 5, 4, 4, 4, 4}	344
{6, 6, 5, 5, 5, 5}	345
{5, 5, 5, 5, 5, 5, 2}	348
{6, 5, 5, 5, 4, 4, 3}	352
{6, 6, 4, 4, 4, 4, 4}	360
{6, 5, 5, 5, 5, 3, 3}	360
{6, 6, 6, 5, 5, 4}	361
{6, 5, 5, 5, 5, 4, 2}	364

Figure 24: 10 Lowest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements When Inter- and Intra-Group Coordination Complexity = 0

Arrangement	System Complx
{3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	134 217 792
{4, 1}	134 217 798
{28, 2}	268 435 491
{28, 1, 1}	268 435 494
{2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	268 435 517
{3, 1}	268 435 519
{29, 1}	536 870 945
{2, 1}	536 870 972
{30}	1 073 741 854
{1, 1}	1 073 741 883

Arrangement	System Complx
{3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	268 435 522
{4, 1}	268 435 528
{29, 2}	536 870 948
{29, 1, 1}	536 870 951
{2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	536 870 975
{3, 1}	536 870 977
{30, 1}	1 073 741 858
{2, 1}	1 073 741 886
{31}	2 147 483 679
{1, 1}	2 147 483 709

Arrangement	System Complx
{3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	536 870 980
{4, 1}	536 870 986
{30, 2}	1 073 741 861
{30, 1, 1}	1 073 741 864
{2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	1 073 741 889
{3, 1}	1 073 741 891
{31, 1}	2 147 483 683
{2, 1}	2 147 483 712
{32}	4 294 967 328
{1, 1}	4 294 967 359

Figure 25: 10 Highest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements When Inter- and Intra-Group Coordination Complexity = 0

Arrangement	System Complx
{2, 1}	9
{1, 1, 1}	14
{3}	18

Arrangement	System Complx
{2, 2}	13
{2, 1, 1}	18
{3, 1}	21
{1, 1, 1, 1}	89
{4}	100

Arrangement	System Complx
{2, 2, 1}	22
{3, 2}	25
{3, 1, 1}	30
{2, 1, 1, 1}	93
{4, 1}	103
{1, 1, 1, 1, 1}	1034
{5}	1060

Figure 26: System Complexity Values for Modular Arrangements of 3, 4, and 5 Functional Elements When Inter-Group Diagnostic Complexity = 0

Arrangement	System Complx
{2, 2, 2}	26
{3, 2, 1}	34
{3, 3}	37
{2, 2, 1, 1}	97
{3, 1, 1, 1}	105
{4, 2}	107
{4, 1, 1}	112
{2, 1, 1, 1, 1}	1038
{5, 1}	1063
{1, 1, 1, 1, 1, 1}	15 637
{6}	15 694

Arrangement	System Complx
{3, 2, 2}	38
{3, 3, 1}	46
{2, 2, 2, 1}	101
{3, 2, 1, 1}	109
{4, 2, 1}	116
{4, 3}	119
{4, 1, 1, 1}	187
{2, 2, 1, 1, 1}	1042
{3, 1, 1, 1, 1}	1050
{5, 2}	1067
{5, 1, 1}	1072
{2, 1, 1, 1, 1, 1}	15 641
{6, 1}	15 697
{1, 1, 1, 1, 1, 1, 1}	279 950
{7}	280 070

Arrangement	System Complx
{3, 3, 2}	50
{2, 2, 2, 2}	105
{3, 2, 2, 1}	113
{4, 2, 2}	120
{3, 3, 1, 1}	121
{4, 3, 1}	128
{4, 2, 1, 1}	191
{4, 4}	201
{2, 2, 2, 1, 1}	1046
{3, 2, 1, 1, 1}	1054
{5, 2, 1}	1076
{5, 3}	1079
{4, 1, 1, 1, 1}	1132
{5, 1, 1, 1}	1147
{2, 2, 1, 1, 1, 1}	15 645
{3, 1, 1, 1, 1, 1}	15 653
{6, 2}	15 701
{6, 1, 1}	15 706
{2, 1, 1, 1, 1, 1, 1}	279 954
{7, 1}	280 073
{1, 1, 1, 1, 1, 1, 1, 1}	5764 817
{8}	5765 064

Figure 27: System Complexity Values for Modular Arrangements of 6, 7, and 8 Functional Elements When Inter-Group Diagnostic Complexity = 0

Arrangement	System Complx
{3, 3, 2, 2}	129
{3, 3, 3, 1}	137
{4, 3, 3}	144
{4, 2, 2, 2}	199
{4, 3, 2, 1}	207
{4, 4, 2}	214
{4, 4, 1, 1}	285
{2, 2, 2, 2, 2}	1054
{3, 2, 2, 2, 1}	1062
{3, 3, 2, 1, 1}	1070

Arrangement	System Complx
{3, 3, 3, 2}	141
{4, 3, 2, 2}	211
{4, 3, 3, 1}	219
{4, 4, 3}	226
{4, 4, 2, 1}	289
{3, 2, 2, 2, 2}	1066
{3, 3, 2, 2, 1}	1074
{3, 3, 3, 1, 1}	1082
{5, 3, 3}	1104
{4, 2, 2, 2, 1}	1144

Arrangement	System Complx
{3, 3, 3, 3}	153
{4, 3, 3, 2}	223
{4, 4, 2, 2}	293
{4, 4, 3, 1}	301
{4, 4, 4}	308
{3, 3, 2, 2, 2}	1078
{3, 3, 3, 2, 1}	1086
{4, 2, 2, 2, 2}	1148
{4, 3, 2, 2, 1}	1156
{4, 3, 3, 1, 1}	1164

Figure 28: 10 Lowest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements When Inter-Group Diagnostic Complexity = 0

Arrangement	System Complx	Arrangement	System Complx	Arrangement
{7, 3}	280 089	{8, 3}	5 765 083	{9, 3}
{7, 1, 1, 1}	280 157	{8, 1, 1, 1}	5 765 151	{9, 1, 1, 1}
{2, 2, 1, 1, 1, 1, 1, 1}	5 764 825	{2, 2, 1, 1, 1, 1, 1, 1, 1}	134 217 754	{2, 2, 1, 1, 1, 1, 1, 1, 1}
{3, 1, 1, 1, 1, 1, 1, 1}	5 764 833	{3, 1, 1, 1, 1, 1, 1, 1, 1}	134 217 762	{3, 1, 1, 1, 1, 1, 1, 1, 1}
{8, 2}	5 765 071	{9, 2}	134 218 255	{10, 2}
{8, 1, 1}	5 765 076	{9, 1, 1}	134 218 260	{10, 1, 1}
{2, 1, 1, 1, 1, 1, 1, 1, 1}	134 217 750	{2, 1, 1, 1, 1, 1, 1, 1, 1, 1}	3 486 784 425	{2, 1, 1, 1, 1, 1, 1, 1, 1, 1}
{9, 1}	134 218 251	{10, 1}	3 486 785 437	{11, 1}
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	3 486 784 421	{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	100 000 000 022	{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
{10}	3 486 785 434	{11}	100 000 002 058	{12}

Figure 29: 10 Highest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements When Inter-Group Diagnostic Complexity = 0

Arrangement	System Complx	Arrangement	System Complx	Arrangement	System Complx
{5, 5, 5, 5, 5, 5}	21 985	{6, 5, 5, 5, 5, 5}	36 619	{6, 6, 5, 5, 5, 5}	51 253
{6, 5, 5, 5, 5, 4}	35 659	{6, 6, 5, 5, 5, 4}	50 293	{6, 6, 6, 5, 5, 4}	64 927
{6, 6, 5, 5, 4, 4}	49 333	{6, 6, 6, 5, 4, 4}	63 967	{6, 6, 6, 6, 4, 4}	78 601
{6, 6, 5, 5, 5, 3}	50 211	{6, 6, 6, 5, 5, 3}	64 845	{6, 6, 6, 6, 5, 3}	79 479
{6, 6, 6, 4, 4, 4}	63 007	{6, 6, 6, 6, 4, 3}	78 519	{6, 6, 6, 6, 6, 2}	94 101
{6, 6, 6, 5, 4, 3}	63 885	{6, 6, 6, 6, 5, 2}	79 467	{5, 5, 5, 5, 4, 4, 4}	284 476
{6, 6, 6, 5, 5, 2}	64 833	{6, 6, 6, 6, 6, 1}	94 097	{5, 5, 5, 5, 5, 4, 3}	285 354
{6, 6, 6, 6, 3, 3}	78 437	{5, 5, 5, 4, 4, 4, 4}	283 516	{5, 5, 5, 5, 5, 5, 2}	286 302
{6, 6, 6, 6, 4, 2}	78 507	{5, 5, 5, 5, 4, 4, 3}	284 394	{6, 5, 5, 4, 4, 4, 4}	298 150
{6, 6, 6, 6, 5, 1}	79 463	{5, 5, 5, 5, 5, 3, 3}	285 272	{6, 5, 5, 5, 4, 4, 3}	299 028

Figure 30: 10 Lowest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements When Inter-Group Diagnostic Complexity = 0

Arrangement	System Complex
{27, 3}	160 059 109 085 386 090 080 713 531
{27, 1, 1, 1}	160 059 109 085 386 090 080 713 531
{2, 2, 1}	11 972 515 182 562 019 788 602 740 02
{3, 1}	11 972 515 182 562 019 788 602 740 02
{28, 2}	11 972 515 182 562 019 788 602 740 02
{28, 1, 1}	11 972 515 182 562 019 788 602 740 02
{2, 1}	928 074 647 171 094 496 152 036 391 0
{29, 1}	928 074 647 171 094 496 152 036 391 0
{1, 1}	74 462 898 441 675 122 902 293 018 227
{30}	74 462 898 441 675 122 902 293 018 227

Arrangement	System Complex
{28, 3}	11 972 515 182 562 019 788 602 740 02
{28, 1, 1, 1}	11 972 515 182 562 019 788 602 740 02
{2, 2, 1}	928 074 647 171 094 496 152 036 391 0
{3, 1}	928 074 647 171 094 496 152 036 391 0
{29, 2}	928 074 647 171 094 496 152 036 391 0
{29, 1, 1}	928 074 647 171 094 496 152 036 391 0
{2, 1}	74 462 898 441 675 122 902 293 018 227
{30, 1}	74 462 898 441 675 122 902 293 018 227
{1, 1}	6176 733 962 839 470 000 000 000 00
{31}	6176 733 962 839 470 000 000 000 00

Arrangement	System Complex
{29, 3}	928 074 647 171 094 496 152 036 391 0
{29, 1, 1, 1}	928 074 647 171 094 496 152 036 391 0
{2, 2, 1}	74 462 898 441 675 122 902 293 018 227
{3, 1}	74 462 898 441 675 122 902 293 018 227
{30, 2}	74 462 898 441 675 122 902 293 018 227
{30, 1, 1}	74 462 898 441 675 122 902 293 018 227
{2, 1}	6176 733 962 839 470 000 000 000 00
{31, 1}	6176 733 962 839 470 000 000 000 00
{1, 1}	529 144 398 052 420 314 716 929 9
{32}	529 144 398 052 420 314 716 929 9

Figure 31: 10 Highest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements When Inter-Group Diagnostic Complexity = 0

Arrangement	System Complex	Arrangement	System Complex	Arrangement	System Complex
$\{2, 1\}$	5	$\{2, 2\}$	7	$\{3, 2\}$	15
$\{3\}$	11	$\{3, 1\}$	13	$\{2, 2, 1\}$	15
$\{1, 1, 1\}$	11	$\{2, 1, 1\}$	13	$\{3, 1, 1\}$	21
		$\{4\}$	85	$\{4, 1\}$	87
		$\{1, 1, 1, 1\}$	85	$\{2, 1, 1, 1\}$	87
				$\{5\}$	1029
				$\{1, 1, 1, 1, 1\}$	1029

Figure 32: System Complexity Values for Modular Arrangements of 3, 4, and 5 Functional Elements When Inter- and Intra-Group Diagnostic Complexity = 0

Arrangement	System Complx	Arrangement	System Complx	Arrangement	System Complx
{2, 2, 2}	17	{3, 2, 2}	25	{3, 3, 2}	33
{3, 3}	23	{3, 3, 1}	31	{2, 2, 2, 2}	93
{3, 2, 1}	23	{2, 2, 2, 1}	91	{4, 2, 2}	99
{4, 2}	89	{4, 3}	97	{3, 2, 2, 1}	99
{2, 2, 1, 1}	89	{4, 2, 1}	97	{4, 3, 1}	105
{4, 1, 1}	95	{3, 2, 1, 1}	97	{3, 3, 1, 1}	105
{3, 1, 1, 1}	95	{4, 1, 1, 1}	169	{4, 4}	171
{5, 1}	1031	{5, 2}	1033	{4, 2, 1, 1}	171
{2, 1, 1, 1, 1}	1031	{2, 2, 1, 1, 1}	1033	{2, 2, 2, 1, 1}	1035
{6}	15 631	{5, 1, 1}	1039	{5, 3}	1041
{1, 1, 1, 1, 1, 1}	15 631	{3, 1, 1, 1, 1}	1039	{5, 2, 1}	1041
		{6, 1}	15 633	{3, 2, 1, 1, 1}	1041
		{2, 1, 1, 1, 1, 1}	15 633	{5, 1, 1, 1}	1113
		{7}	279 943	{4, 1, 1, 1, 1}	1113
		{1, 1, 1, 1, 1, 1, 1}	279 943	{6, 2}	15 635
				{2, 2, 1, 1, 1, 1}	15 635
				{6, 1, 1}	15 641
				{3, 1, 1, 1, 1, 1}	15 641
				{7, 1}	279 945
				{2, 1, 1, 1, 1, 1, 1}	279 945
				{8}	5764 809
				{1, 1, 1, 1, 1, 1, 1, 1}	5764 809

Figure 33: System Complexity Values for Modular Arrangements of 6, 7, and 8 Functional Elements When Inter- and Intra-Group Diagnostic Complexity = 0

Arrangement	System Complx	Arrangement	System Complx	Arrangement	System Complx
{3, 3, 2, 2}	109	{3, 3, 3, 2}	117	{3, 3, 3, 3}	125
{4, 3, 3}	115	{4, 3, 2, 2}	183	{4, 3, 3, 2}	191
{3, 3, 3, 1}	115	{4, 4, 3}	189	{4, 4, 2, 2}	257
{4, 2, 2, 2}	175	{4, 3, 3, 1}	189	{4, 4, 4}	263
{4, 4, 2}	181	{4, 4, 2, 1}	255	{4, 4, 3, 1}	263
{4, 3, 2, 1}	181	{3, 2, 2, 2, 2}	1047	{3, 3, 2, 2, 2}	1055
{4, 4, 1, 1}	253	{3, 3, 2, 2, 1}	1053	{3, 3, 3, 2, 1}	1061
{2, 2, 2, 2, 2}	1039	{5, 3, 3}	1059	{4, 2, 2, 2, 2}	1121
{3, 2, 2, 2, 1}	1045	{3, 3, 3, 1, 1}	1059	{5, 3, 2, 2}	1127
{5, 3, 2}	1051	{5, 2, 2, 2}	1119	{4, 3, 2, 2, 1}	1127

Figure 34: 10 Lowest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements When Inter-and Intra-Group Diagnostic Complexity = 0

Arrangement	System Complx	Arrangement	System Complx	Arrangement
{7, 1, 1, 1}	280 027	{8, 1, 1, 1}	5 764 893	{9, 1, 1, 1}
{4, 1, 1, 1, 1, 1, 1}	280 027	{4, 1, 1, 1, 1, 1, 1, 1}	5 764 893	{4, 1, 1, 1, 1, 1, 1, 1, 1}
{8, 2}	5 764 813	{9, 2}	134 217 741	{10, 2}
{2, 2, 1, 1, 1, 1, 1, 1, 1}	5 764 813	{2, 2, 1, 1, 1, 1, 1, 1, 1, 1}	134 217 741	{2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1}
{8, 1, 1}	5 764 819	{9, 1, 1}	134 217 747	{10, 1, 1}
{3, 1, 1, 1, 1, 1, 1, 1, 1}	5 764 819	{3, 1, 1, 1, 1, 1, 1, 1, 1, 1}	134 217 747	{3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
{9, 1}	134 217 739	{10, 1}	3 486 784 413	{11, 1}
{2, 1, 1, 1, 1, 1, 1, 1, 1, 1}	134 217 739	{2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	3 486 784 413	{2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
{10}	3 486 784 411	{11}	100 000 000 011	{12}
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	3 486 784 411	{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}	100 000 000 011	{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}

Figure 35: 10 Highest System Complexity Values for Modular Arrangements of 10, 11, and 12 Functional Elements When Inter-and Intra-Group Diagnostic Complexity = 0

Arrangement	System Complx	Arrangement	System Complx	Arrangement	System Complx
{5, 5, 5, 5, 5}	21 799	{6, 5, 5, 5, 5}	36 401	{6, 6, 5, 5, 5}	51 003
{6, 5, 5, 5, 5, 4}	35 457	{6, 6, 5, 5, 5, 4}	50 059	{6, 6, 6, 5, 5, 4}	64 661
{6, 6, 5, 5, 4, 4}	49 115	{6, 6, 6, 5, 4, 4}	63 717	{6, 6, 6, 6, 4, 4}	78 319
{6, 6, 5, 5, 5, 3}	49 985	{6, 6, 6, 5, 5, 3}	64 587	{6, 6, 6, 6, 5, 3}	79 189
{6, 6, 6, 4, 4, 4}	62 773	{6, 6, 6, 6, 4, 3}	78 245	{6, 6, 6, 6, 6, 2}	93 783
{6, 6, 6, 5, 4, 3}	63 643	{6, 6, 6, 6, 5, 2}	79 181	{5, 5, 5, 5, 4, 4, 4}	284 307
{6, 6, 6, 5, 5, 2}	64 579	{6, 6, 6, 6, 6, 1}	93 781	{5, 5, 5, 5, 5, 4, 3}	285 177
{6, 6, 6, 6, 3, 3}	78 171	{5, 5, 5, 4, 4, 4, 4}	283 363	{5, 5, 5, 5, 5, 5, 2}	286 113
{6, 6, 6, 6, 4, 2}	78 237	{5, 5, 5, 5, 4, 4, 3}	284 233	{6, 5, 5, 4, 4, 4, 4}	297 965
{6, 6, 6, 6, 6}	79 179	{5, 5, 5, 5, 5, 3, 3}	285 103	{6, 5, 5, 5, 4, 4, 3}	298 835

Figure 36: 10 Lowest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements When Inter-and Intra-Group Diagnostic Complexity = 0

Figure 37: 10 Highest System Complexity Values for Modular Arrangements of 30, 31, and 32 Functional Elements When Inter- and Intra-Group Diagnostic Complexity = 0

8 Conclusion and Next Steps

Simulation is not exhaustive. Got to figure out the architectural techniques that reduce inter- and intra-group coordination and diagnostic complexity. A lot can be learned without using any semantic information like business synergy. The functional complexity of a single element is not an issue from the standpoint of architectural complexity. It's the wiring diagram that drives complexity. A balanced distribution is best even if that means leaving certain groups empty. When you reduce intra- and inter-group coordination complexity, the difference between the best and worst case complexity is not that great. This gives you more freedom to design according to business and technology constraints (not just functionality) while still keeping complexity in check.

Therefore, it is important to understand the architectural techniques that will result in minimizing diagnostic and coordination complexity – both intra- and inter-group.

While SIP is a reliable way to get to the lowest possible complexity (if that is true), there are other ways to get there as well – following the rule of thumb of balanced distribution gives you a guide to how to carve up the business functionality into groups/categories and how to distribute the functionality elements within these categories. If you find the design is straying toward an imbalanced distribution, re-carve, re-distribute, or find ways to reduce diagnostic and/or coordination complexity.