Chain rule: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$

$(f \circ g)' = (f' \circ g) \cdot g'$

ex if $f(x) = (x^2+3x+2) \cdot (x^3+4x^2+2x+1)$,

what is $f'(x)$?

• could multiply it out and use our $\boxed{(x^n)' = nx^{n-1}}$

better if we had a general method of handling products

product rule:

if $f(x) = a(x)b(x)$, then $\boxed{f'(x) = a(x) \cdot b'(x) + b(x)a'(x)}$

$\boxed{(a(x)b(x))' = a(x)b'(x) + b(x)a'(x)}$

why? we could do $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{a(x+h) \cdot b(x+h) - a(x)b(x)}{h}$

or, be more clever.

∴ work.

$$(a(x)+b(x))^2 = a(x)^2 + 2a(x)b(x) + b(x)^2$$

so by chain rule taking derivating of both sides,

$$2(a(x)+b(x)) \cdot (a'(x)+b'(x)) = 2a(x)a'(x) + 2(a(x)b(x))' + 2b(x)b'(x)$$

Kill 2.

Solve for $(a(x)b(x))' = (a(x)+b(x))(a'(x)+b'(x)) - a(x)a'(x) - b(x)b'(x)$

$= a(x)a'(x) + b(x)b'(x) + a(x)b'(x) + b(x)a'(x) - a(x)a'(x) - b(x)b'(x)$

$= a(x)b'(x) + b(x) \cdot a'(x).$

Quotient rule

$$\left(\frac{a(x)}{b(x)}\right)' = \frac{b(x)\,a'(x) - a(x)\,b'(x)}{(b(x))^2}$$

why? $\displaystyle\lim_{h\to 0} \frac{\frac{a(x+h)}{b(x+h)} - \frac{a(x)}{b(x)}}{h}$ manipulate a bunch.

or

$$\frac{a(x)}{b(x)} = a(x)\cdot(b(x))^{-1}$$ we can do this with product rule and chain rule.

but one thing, we don't know how to do

$$(x^{-1})'$$

$f(x) = x^{-1}$, what is $f'(x)$?

easy, use product rule: $(x^{-1}\cdot x) = 1$

$$(x^{-1}\cdot x)' = 0$$

$$x^{-1}\cdot 1 + x\cdot(x^{-1})' = 0$$

$$x\cdot(x^{-1})' = -x^{-1}$$

$$\boxed{(x^{-1})' = -x^{-2}} = \frac{-1}{x^2}$$

$$\begin{aligned}
(a(x)b(x)^{-1})' &= a(x)\,(b(x)^{-1})' + b(x)^{-1}\cdot a'(x) \\
&= a(x)\,(-1)\,b(x)^{-2}\cdot b'(x) + b(x)^{-1}a'(x) \\
&= \frac{-a(x)\,b'(x)}{b(x)^2} + \frac{a'(x)}{b(x)}\cdot\frac{b(x)}{b(x)} \\
&= \checkmark
\end{aligned}$$