A common generalization of Hall's theorem and Vizing's edge-coloring theorem

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LBD Data

Miami University Colloquium
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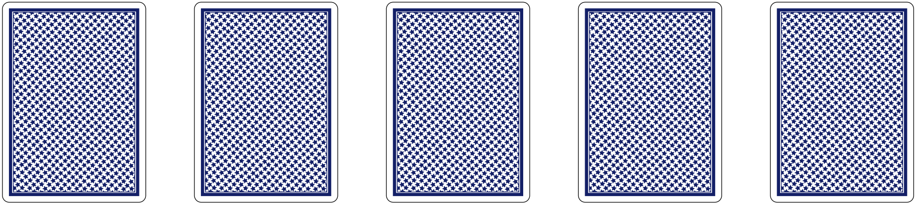
some card games

the simplest variation

- two players, Dealer and Player



some card games

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- the deck has just many copies of the high spade cards

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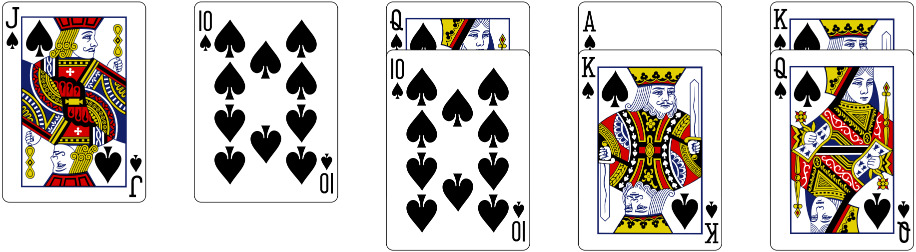
- two players, Dealer and Player
- the deck has just many copies of the high spade cards
- Dealer makes 5 stacks of cards with no duplicates, all cards face-up
- Player wins if he can pick a Royal Flush, one card from each stack

some card games

example, a Player win



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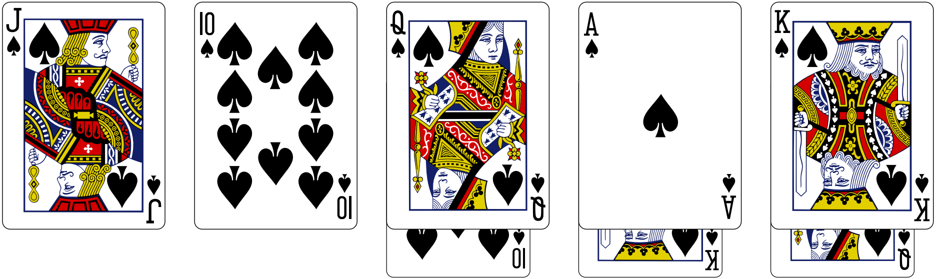
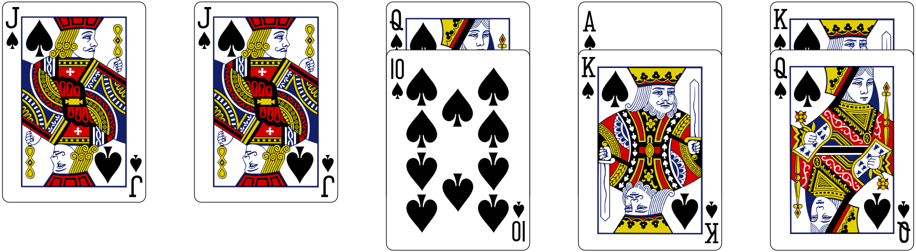
example, a Player win



example, a Dealer win



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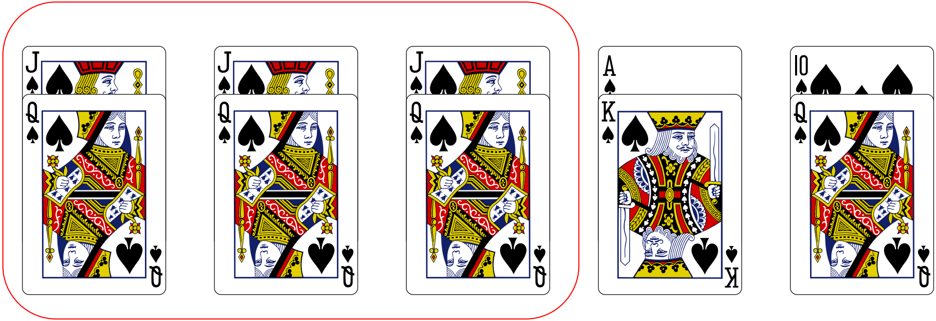

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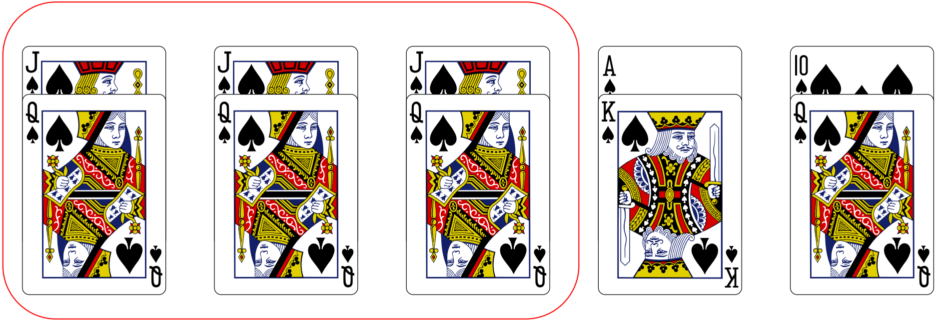
winning condition

- Player cannot win if there is a set of k stacks that together have fewer than k different cards
- Hall's theorem says: **Player wins otherwise**



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some card games

making things harder for Dealer

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Player's Move

Player can pick any card A from the deck and swap it for another card B in one stack (not containing A).

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Dealer can either do nothing or swap A and B in at most one other stack.

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Dealer's Move

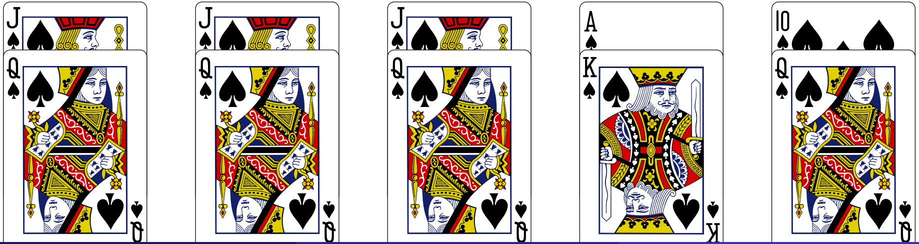
Dealer can either do nothing or swap A and B in at most one other stack.

Winning

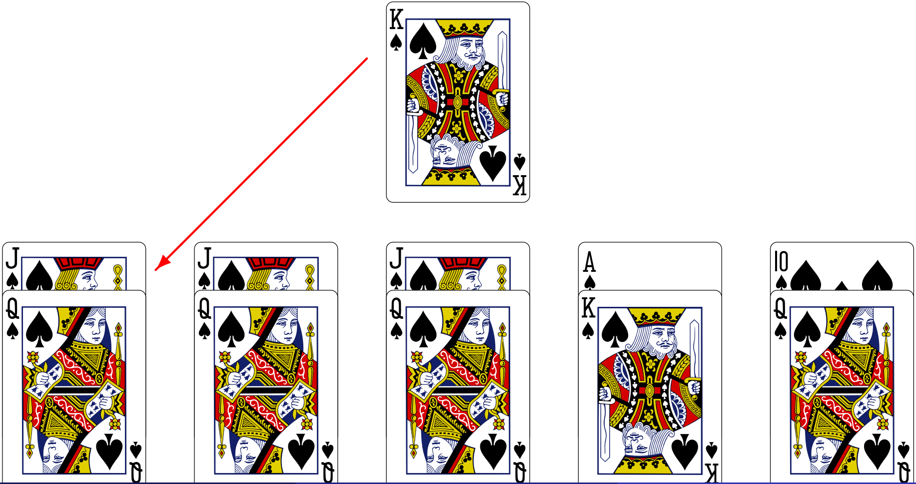
Player wins if he can pick a Royal Flush at the start of one of his turns, otherwise Dealer wins.

some card games

example, a Player win



- Player picks a King from the deck and swaps it for a Queen in the first stack



example, a Player win

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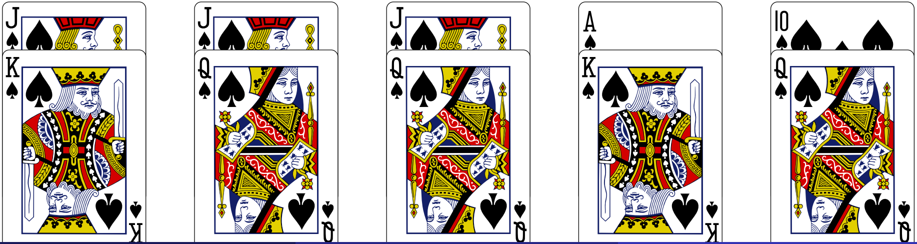

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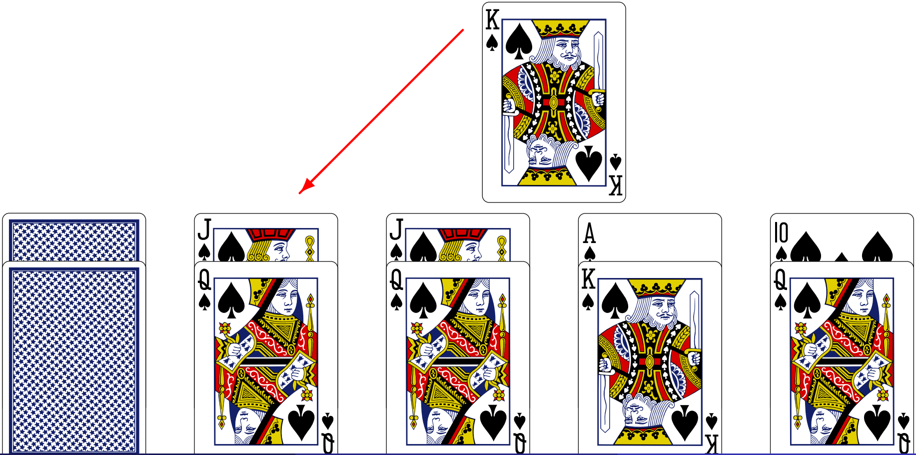
example, a Player win

- Player picks a King from the deck and swaps it for a Queen in the first stack
- Dealer can swap a King and Queen in one of the other stacks



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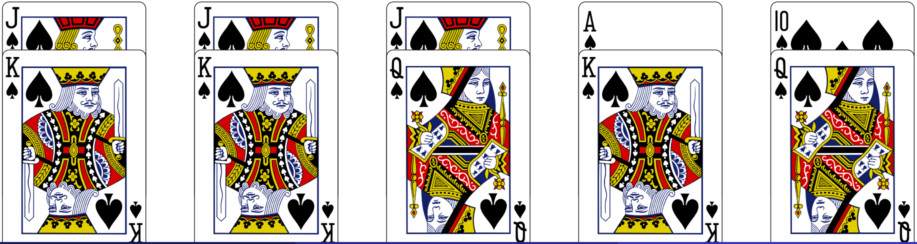

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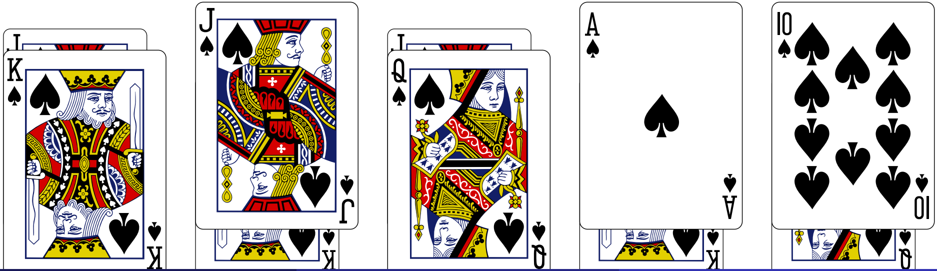
example, a Player win

- Player picks a King from the deck and swaps it for a Queen in the first stack
- Dealer can swap a King and Queen in one of the other stacks
- Player wins no matter what Dealer does



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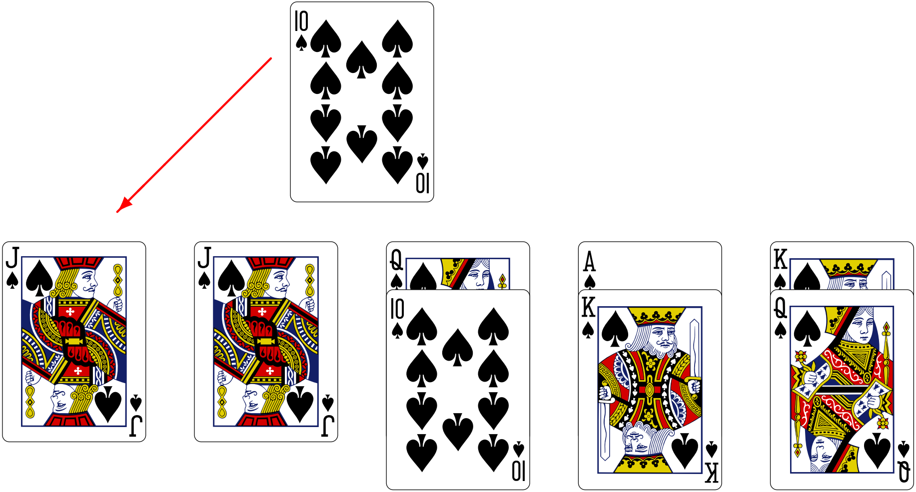
example, a Dealer win



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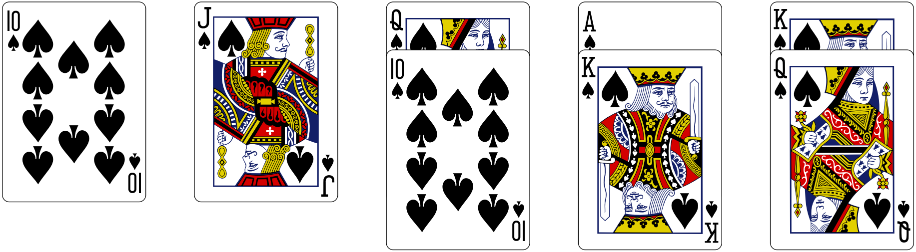

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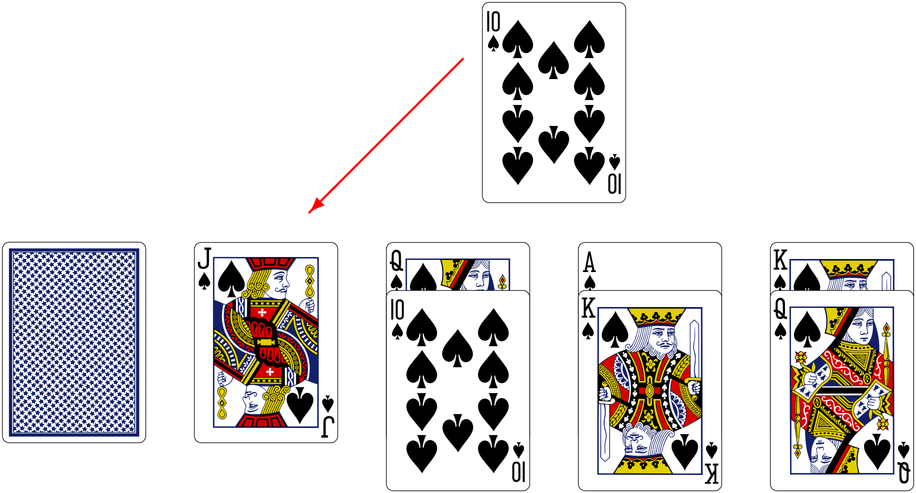

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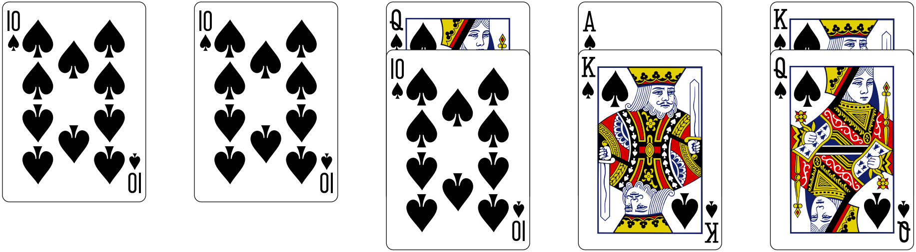
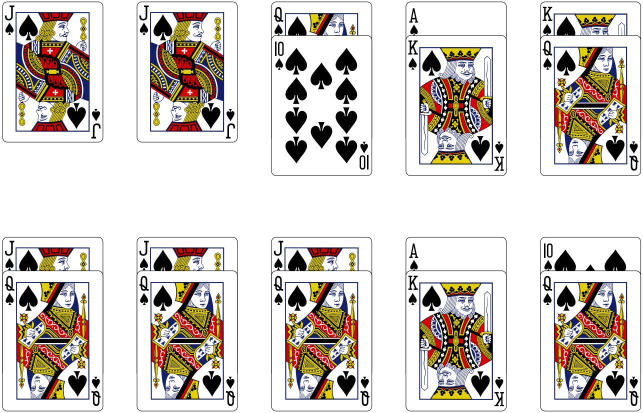


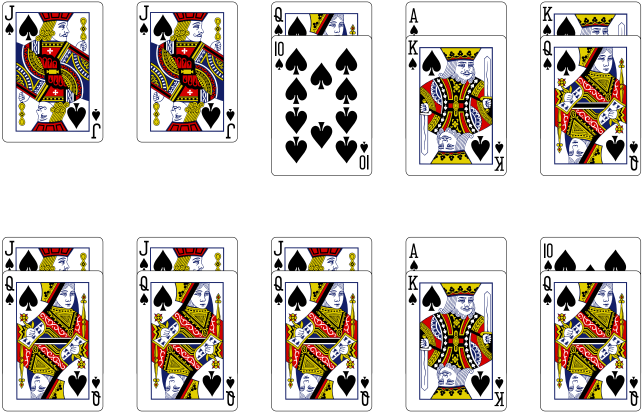
some card games

what was the difference?



some card games
what was the difference?

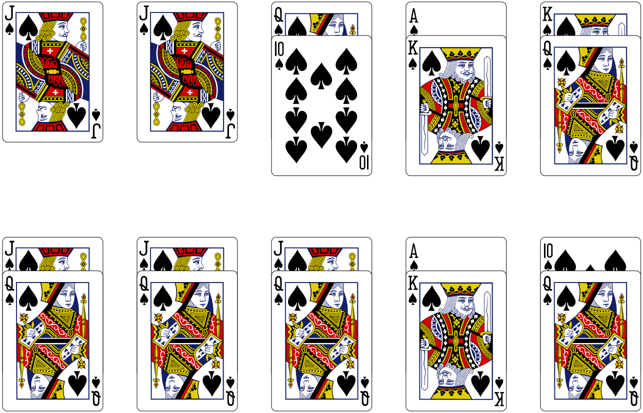
- in the top game, Dealer can prevent Player from increasing the number of different cards in the first two stacks



some card games

what was the difference?

- in the top game, Dealer can prevent Player from increasing the number of different cards in the first two stacks
- in the bottom game, Dealer cannot prevent this



some card games

necessary condition

- if the same card appears on three stacks, Player can force the addition of a new card to these stacks

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Degree

The *degree* of a card C in a set of stacks S is the number of times C appears in S. We write $d_S(C)$ for this quantity.

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Necessary Condition

If Player has a winning strategy, then for every set of stacks S we must have

$$\sum_{C \in \bigcup S} \left\lceil \frac{d_S(C)}{2} \right\rceil \geq |S|.$$

- **this necessary condition is also suffcient**

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Winning Condition

Player has a winning strategy if and only if for every set of stacks S we have

$$\sum_{C \in \bigcup S} \left\lceil \frac{d_S(C)}{2} \right\rceil \geq |S|.$$