

# A common generalization of Hall's theorem and Vizing's edge-coloring theorem

landon rabern

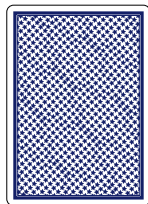
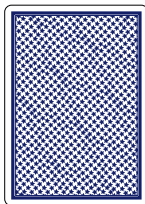
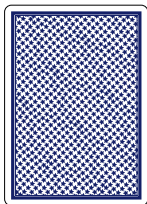
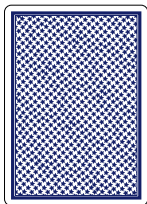
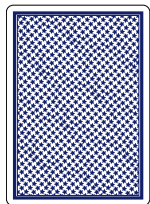
LBD Data

Miami University Colloquium  
November 6, 2014

# some card games

the simplest variation

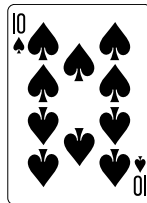
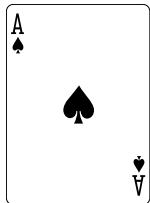
- two players, Dealer and Player



# some card games

## the simplest variation

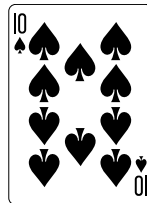
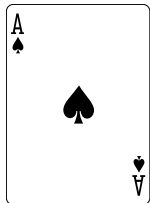
- two players, Dealer and Player
- the deck has just many copies of the high spade cards



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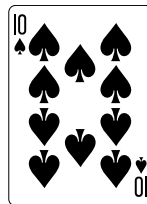
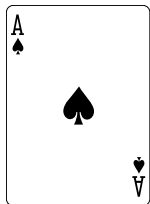
- two players, Dealer and Player
- the deck has just many copies of the high spade cards
- Dealer makes 5 stacks of cards with no duplicates, all cards face-up



# some card games

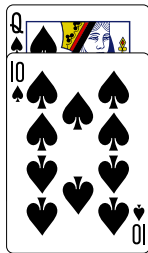
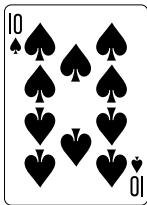
## the simplest variation

- two players, Dealer and Player
- the deck has just many copies of the high spade cards
- Dealer makes 5 stacks of cards with no duplicates, all cards face-up
- Player wins if he can pick a Royal Flush, one card from each stack



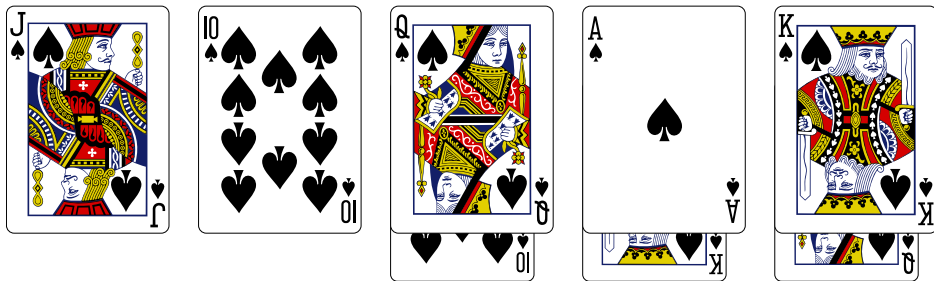
# some card games

example, a Player win



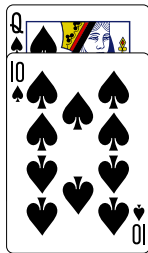
# some card games

example, a Player win



# some card games

example, a Dealer win





# some card games

## winning condition

- Player cannot win if there is a set of  $k$  stacks that together have fewer than  $k$  different cards

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# some card games

winning condition

- Player cannot win if there is a set of  $k$  stacks that together have fewer than  $k$  different cards
- Hall's theorem says: **Player wins otherwise**



# some card games

making things harder for Dealer

- this isn't a fun game, far too easy for Dealer to win

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## Player's Move

*Player can pick any card  $A$  from the deck and swap it for another card  $B$  in one stack (not containing  $A$ ).*

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## Dealer's Move

*Dealer can either do nothing or swap  $A$  and  $B$  in at most one other stack.*

# some card games

making things harder for Dealer

- this isn't a fun game, far too easy for Dealer to win
- to make a better game, we allow Player to modify some of the stacks

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*Player can pick any card A from the deck and swap it for another card B in one stack (not containing A).*

## Dealer's Move

*Dealer can either do nothing or swap A and B in at most one other stack.*

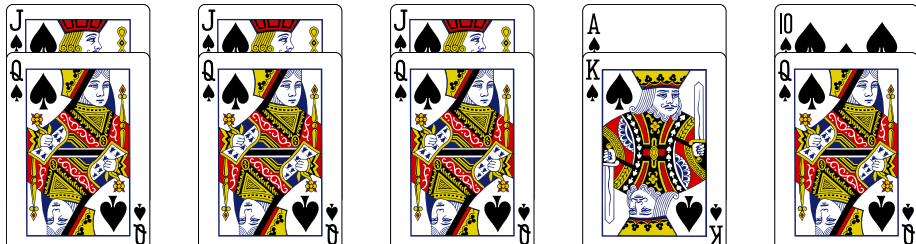
## Winning

*Player wins if he can pick a Royal Flush at the start of one of his turns, otherwise Dealer wins.*



# some card games

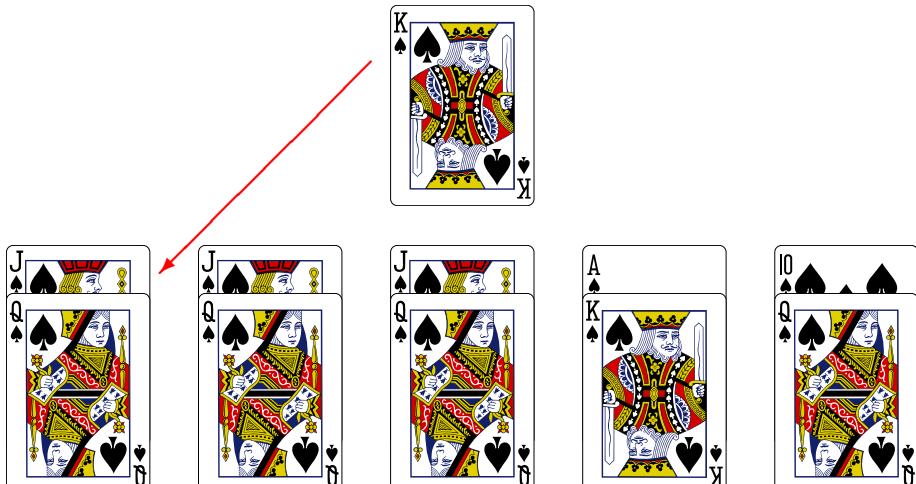
example, a Player win



# some card games

example, a Player win

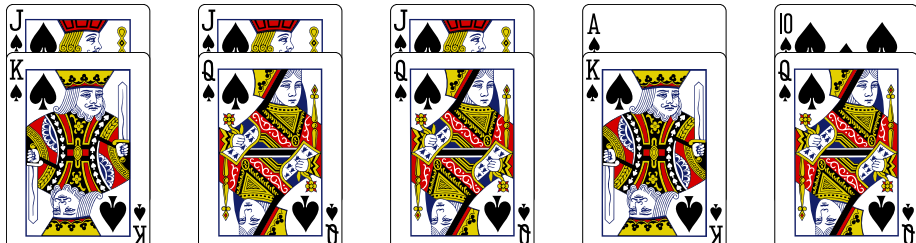
- Player picks a King from the deck and swaps it for a Queen in the first stack



# some card games

example, a Player win

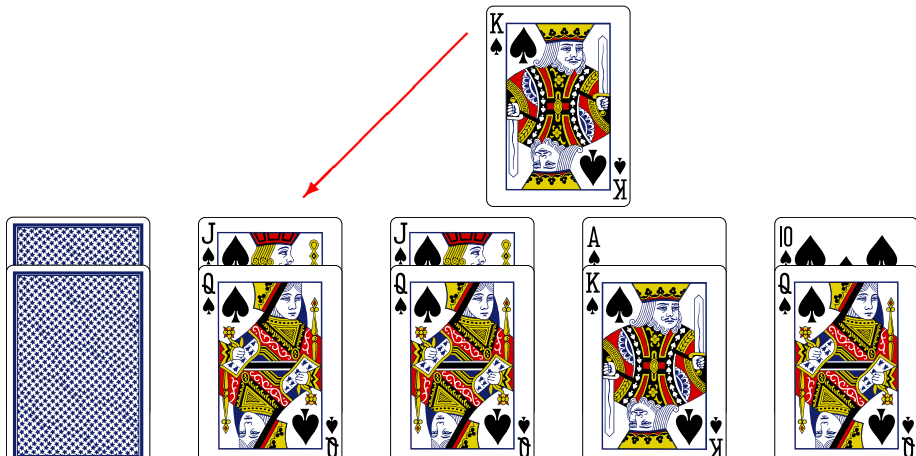
- Player picks a King from the deck and swaps it for a Queen in the first stack



# some card games

example, a Player win

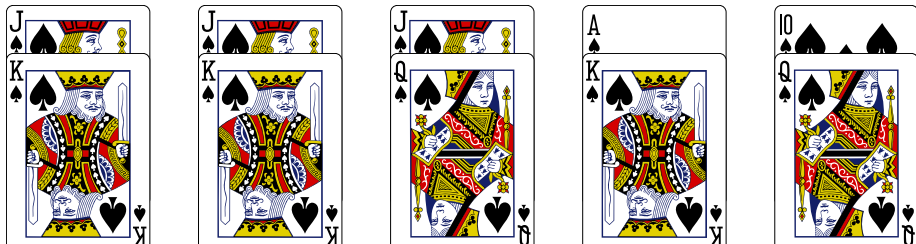
- Player picks a King from the deck and swaps it for a Queen in the first stack
- Dealer can swap a King and Queen in one of the other stacks



# some card games

example, a Player win

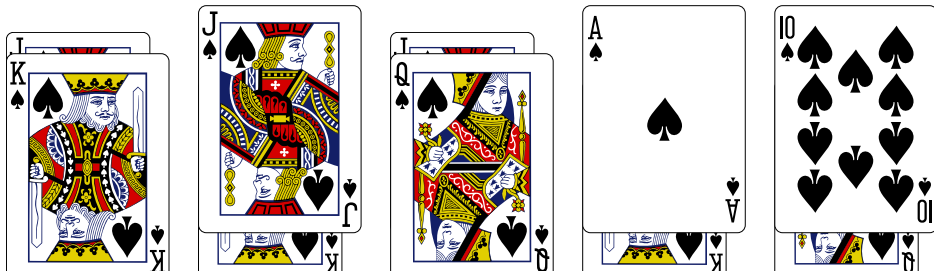
- Player picks a King from the deck and swaps it for a Queen in the first stack
- Dealer can swap a King and Queen in one of the other stacks



# some card games

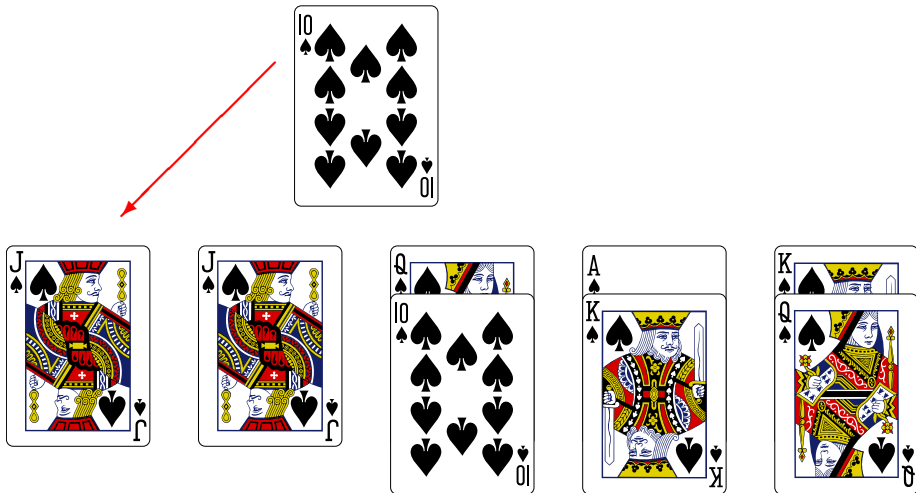
example, a Player win

- Player picks a King from the deck and swaps it for a Queen in the first stack
- Dealer can swap a King and Queen in one of the other stacks
- Player wins no matter what Dealer does



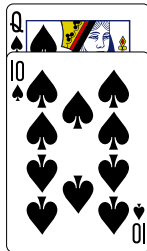
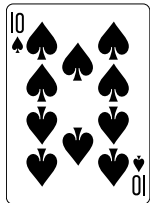
# some card games

example, a Dealer win



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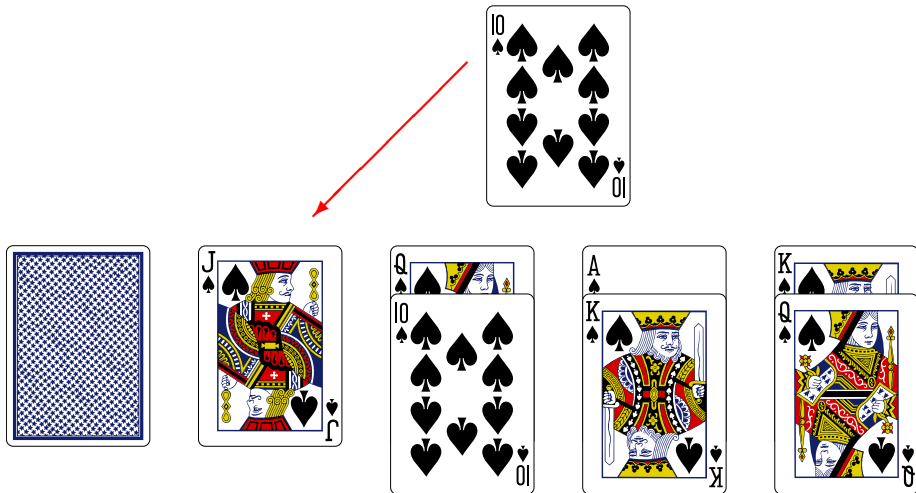
example, a Dealer win





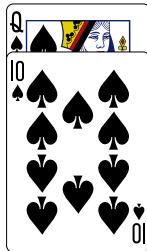
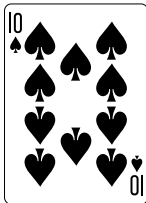
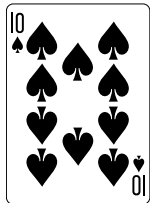
# some card games

example, a Dealer win



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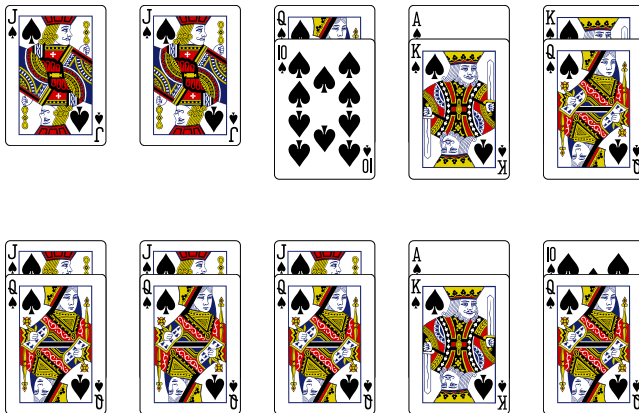
what was the difference?



# some card games

what was the difference?

- in the top game, Dealer can prevent player from increasing the number of different cards in the first two stacks



# some card games

what was the difference?

- in the top game, Dealer can prevent player from increasing the number of different cards in the first two stacks
- in the bottom game, Dealer cannot prevent this

