A common generalization of Hall's theorem and Vizing's edge-coloring theorem

landon rabern

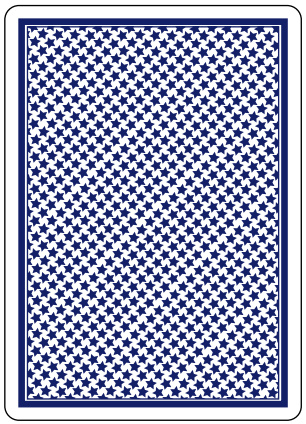
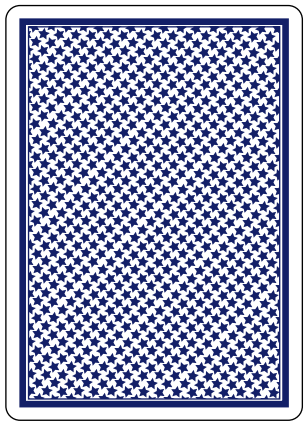
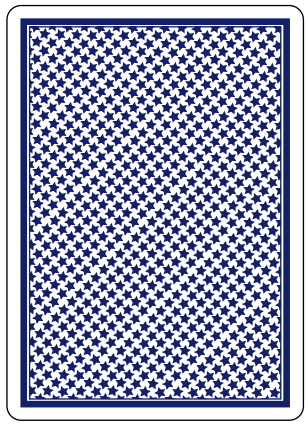
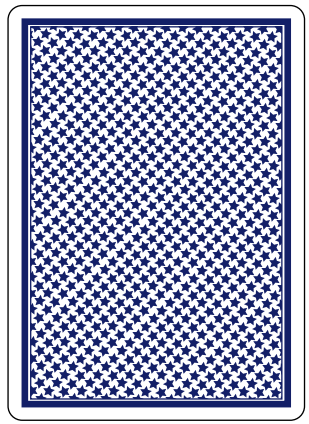
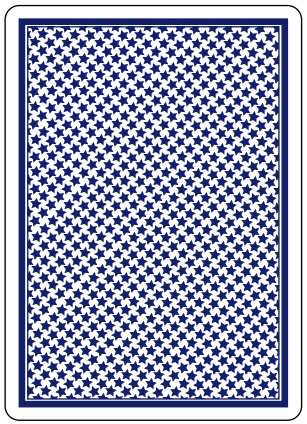
LBD Data

Miami University Colloquium
November 6, 2014

some card games

the simplest variation

- two players, Dealer and Player

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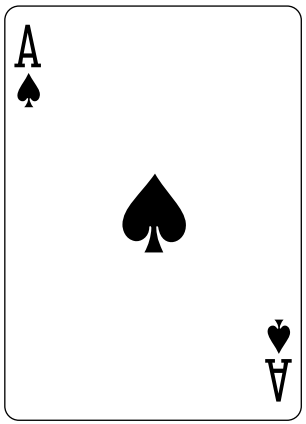
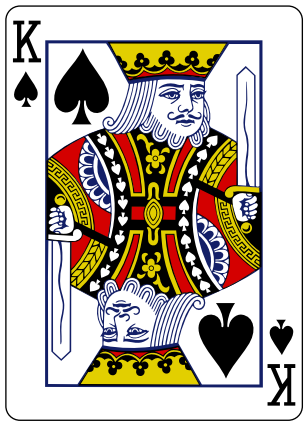
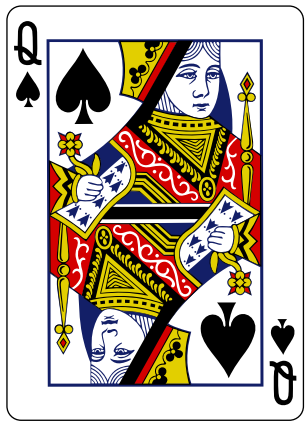
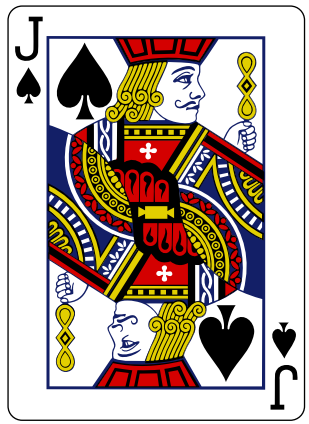
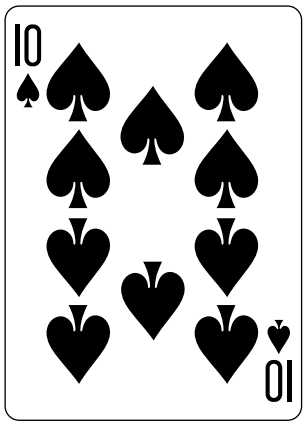

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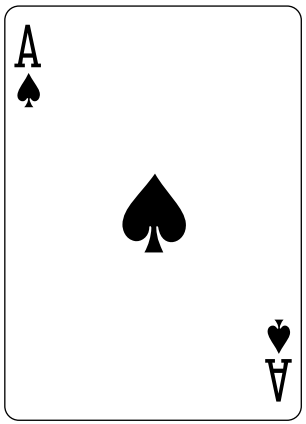

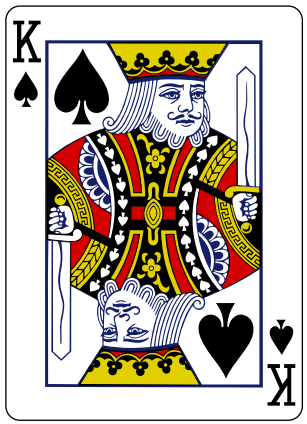

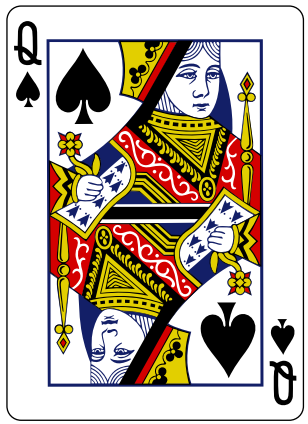

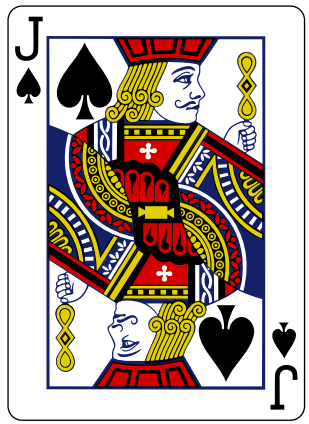

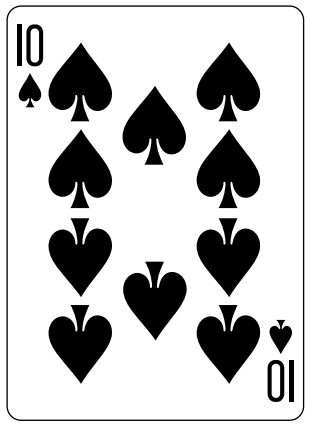

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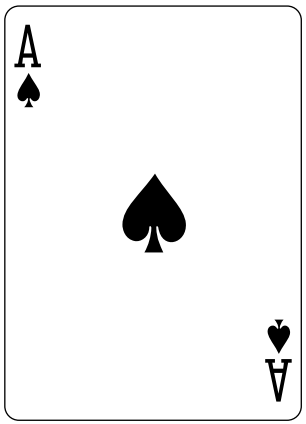

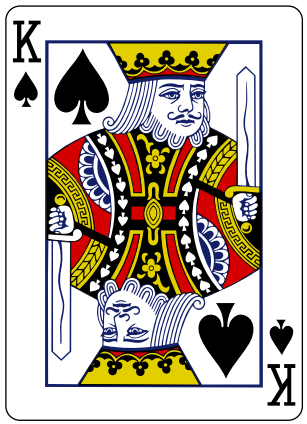

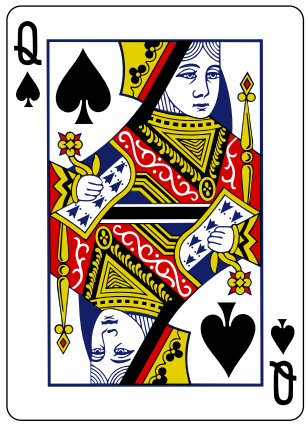

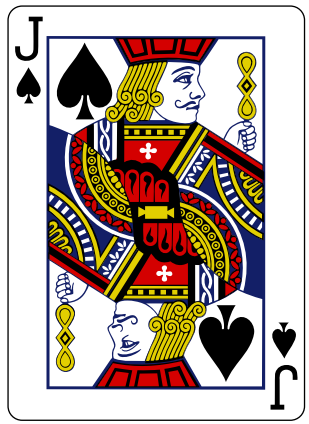

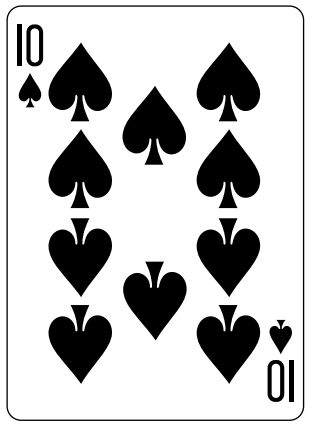

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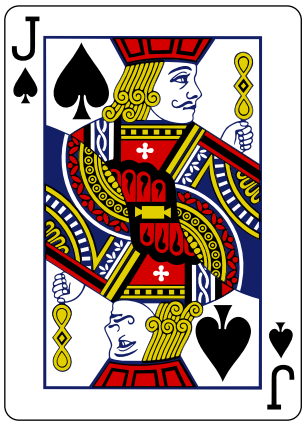
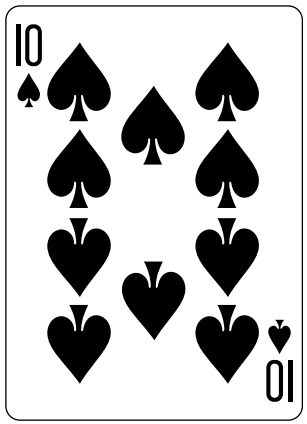
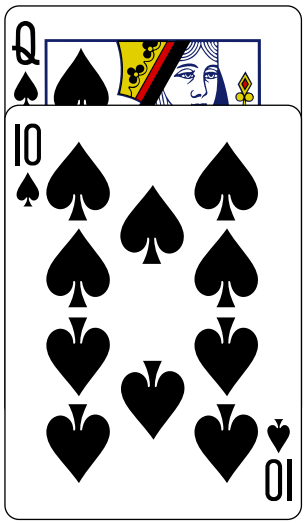
example, a Player win

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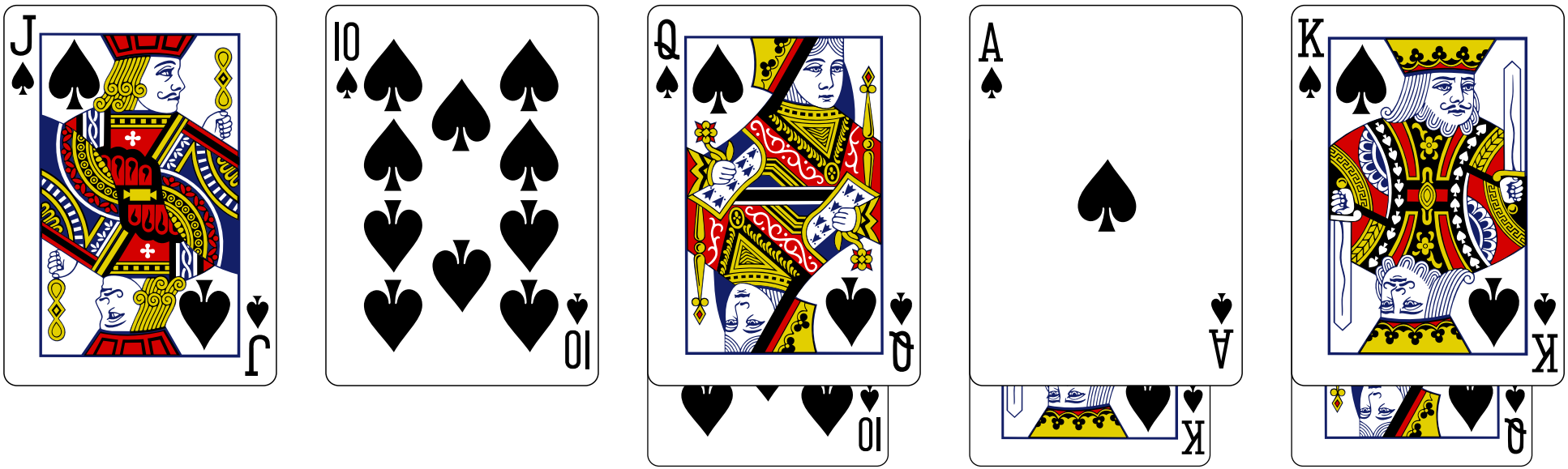
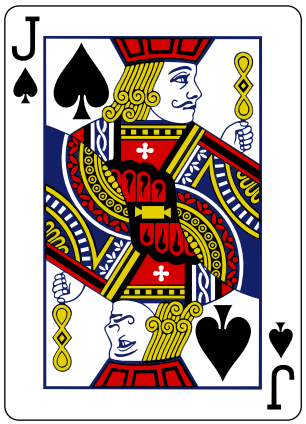
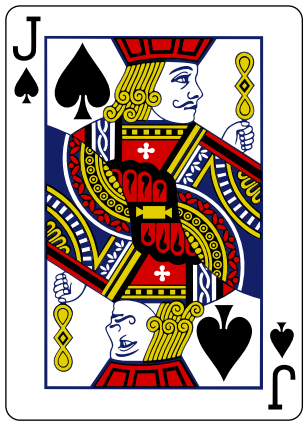
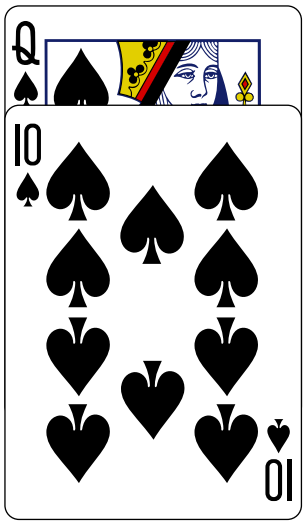


some card games

example, a Dealer win







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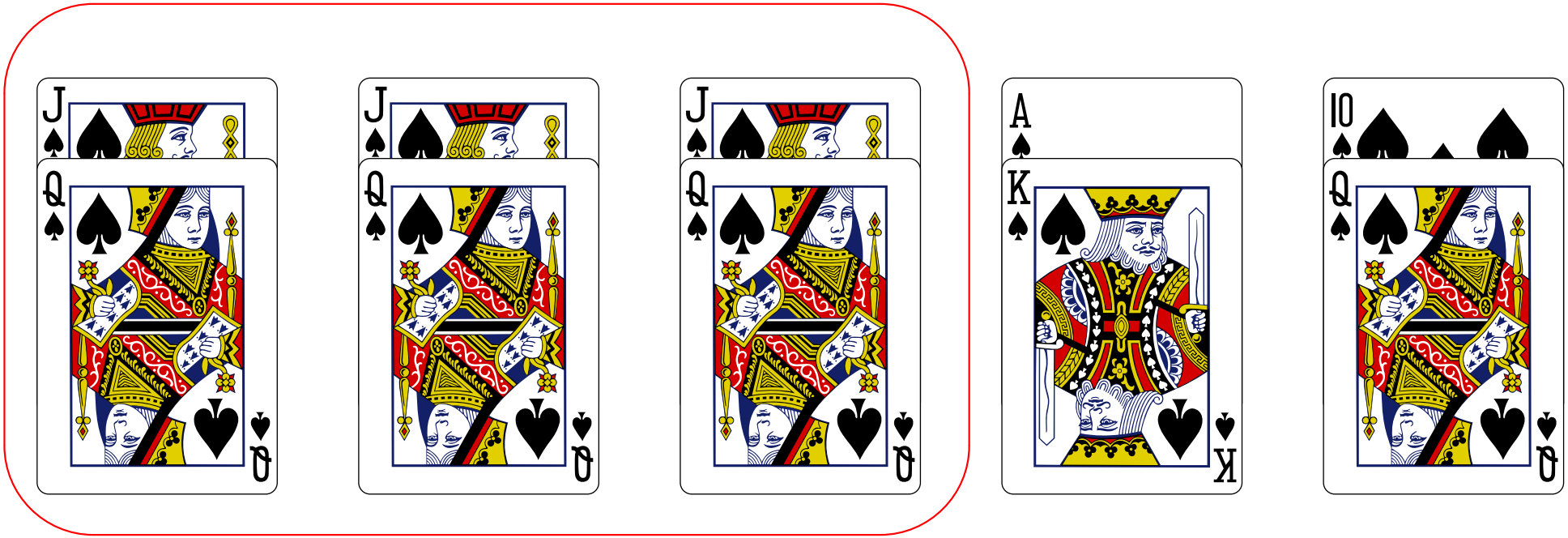

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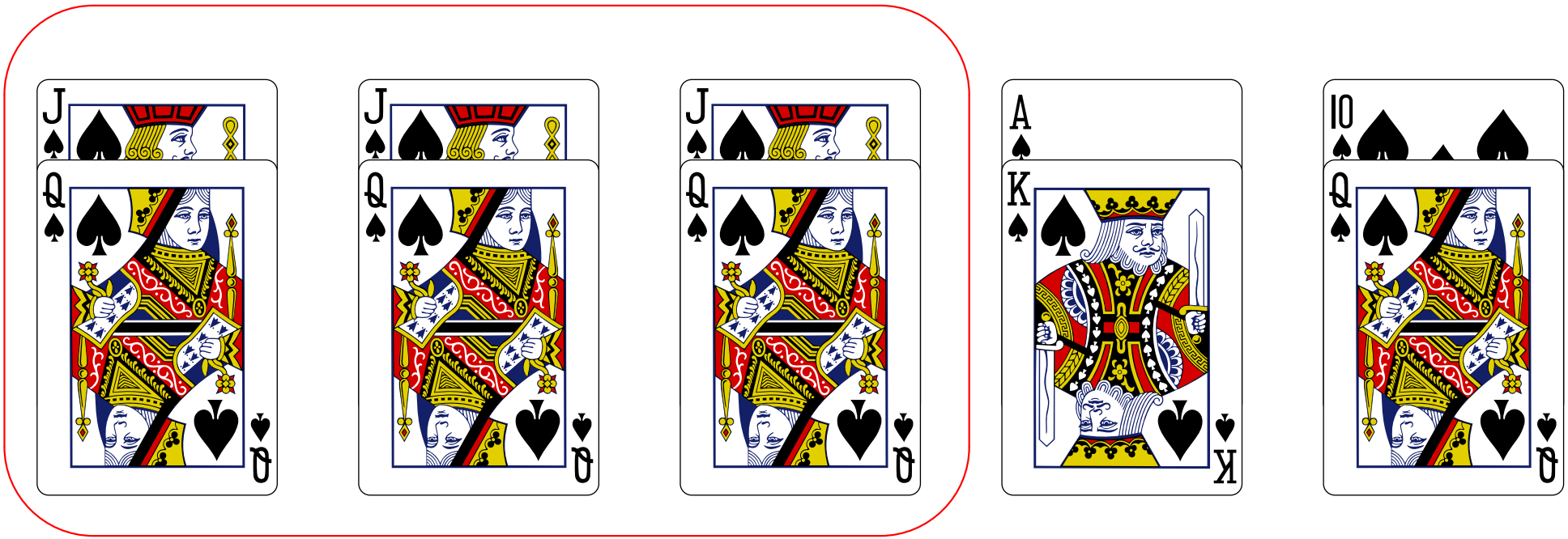
- Hall's theorem says: **Player wins otherwise**



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Winning

Player wins if he can pick a Royal Flush at the start of one of his turns, otherwise Dealer wins.

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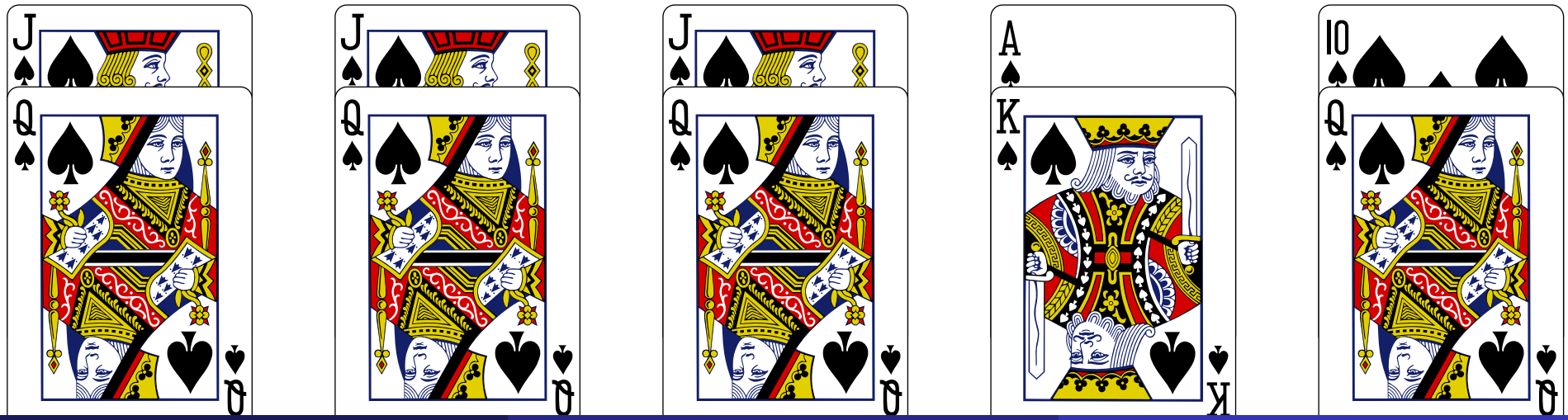

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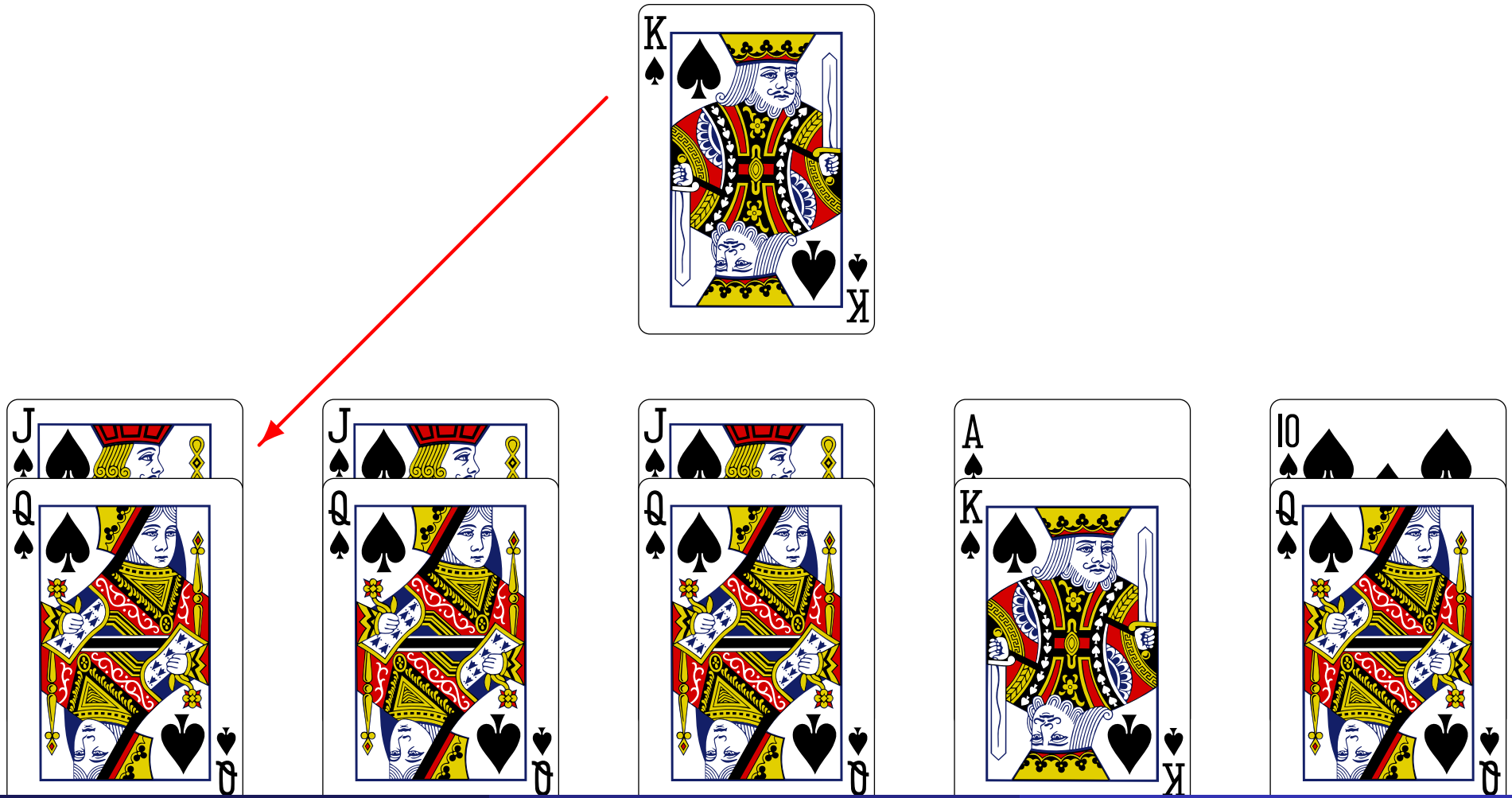

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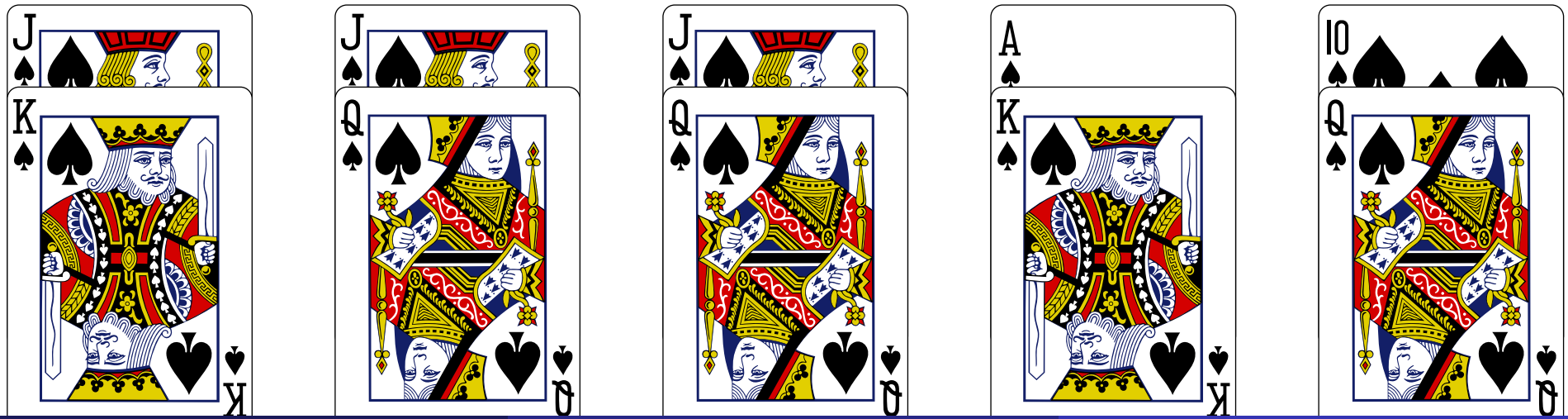

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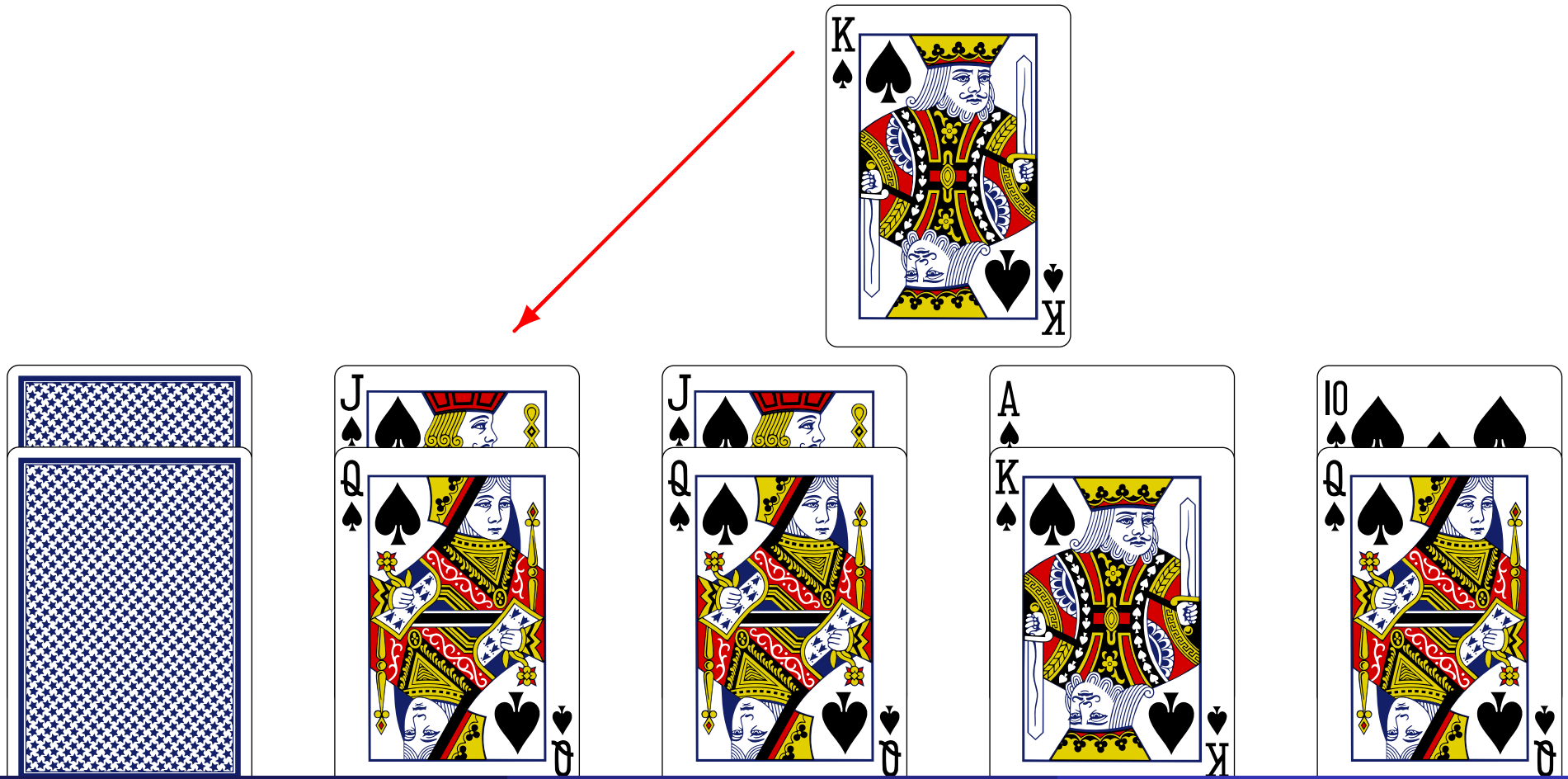

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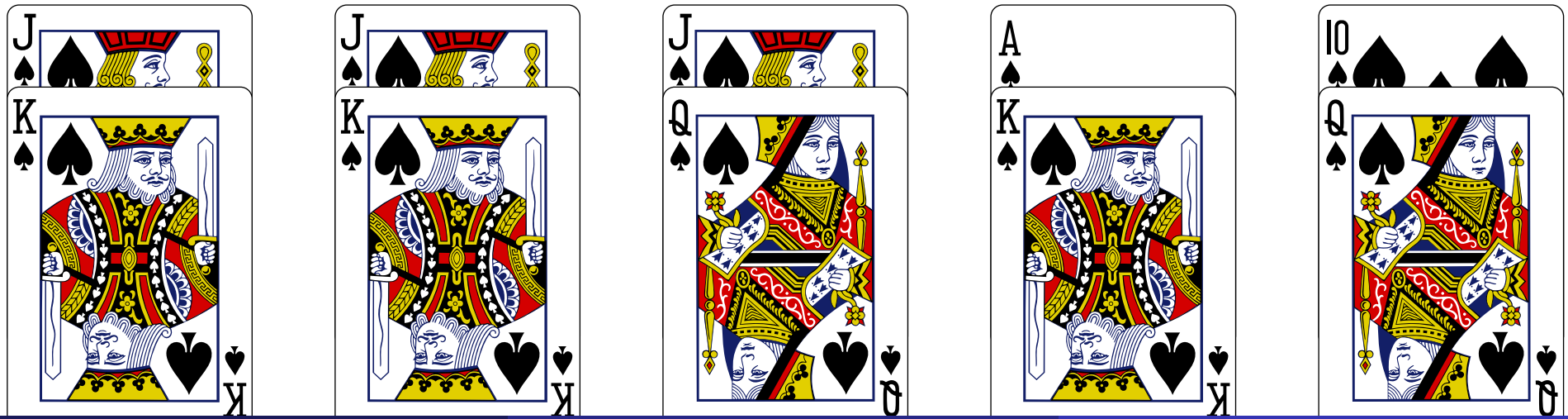

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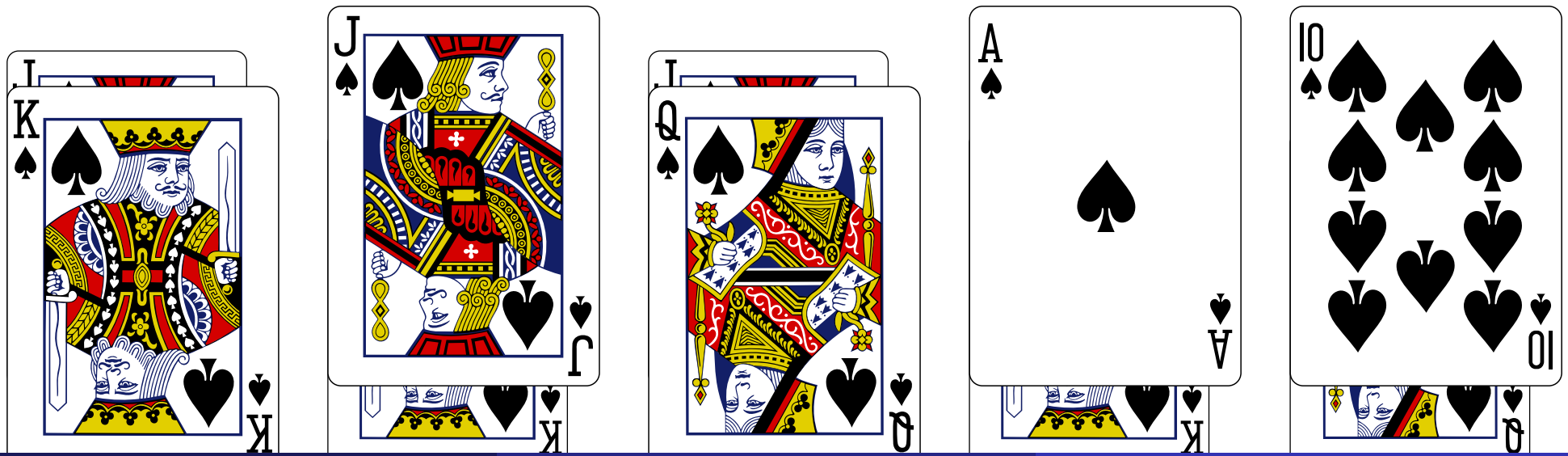
- Player wins no matter what Dealer does



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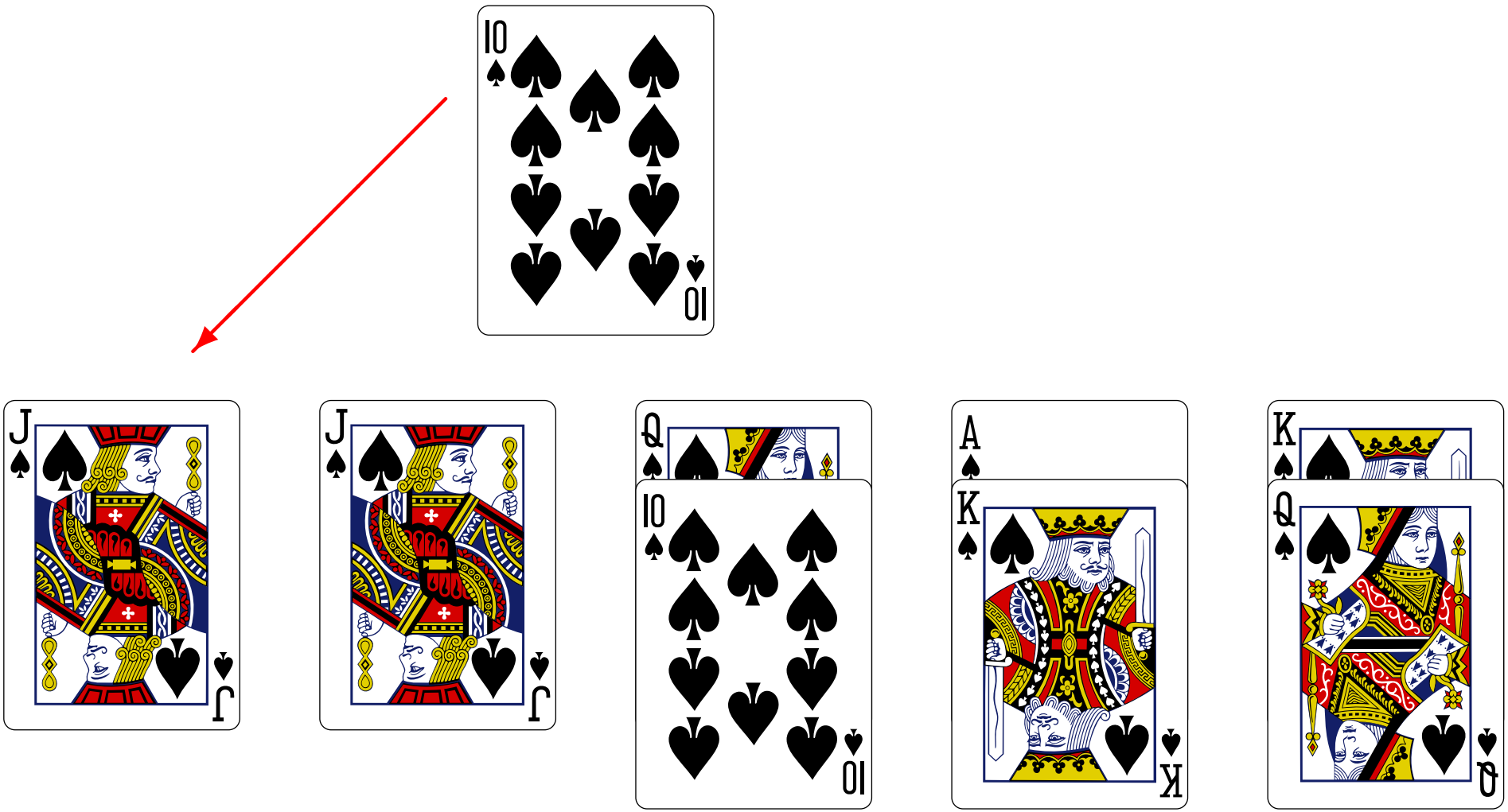

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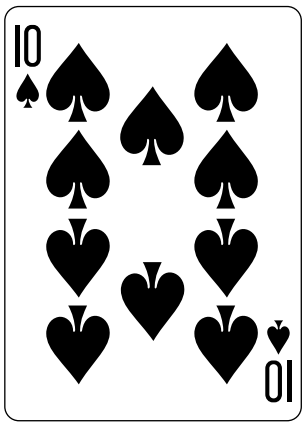

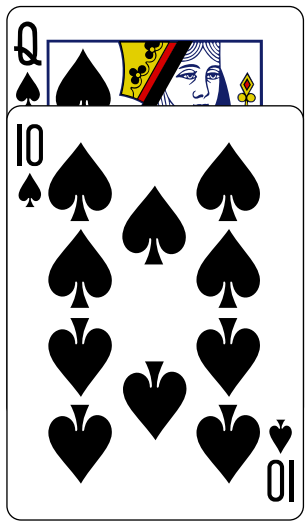

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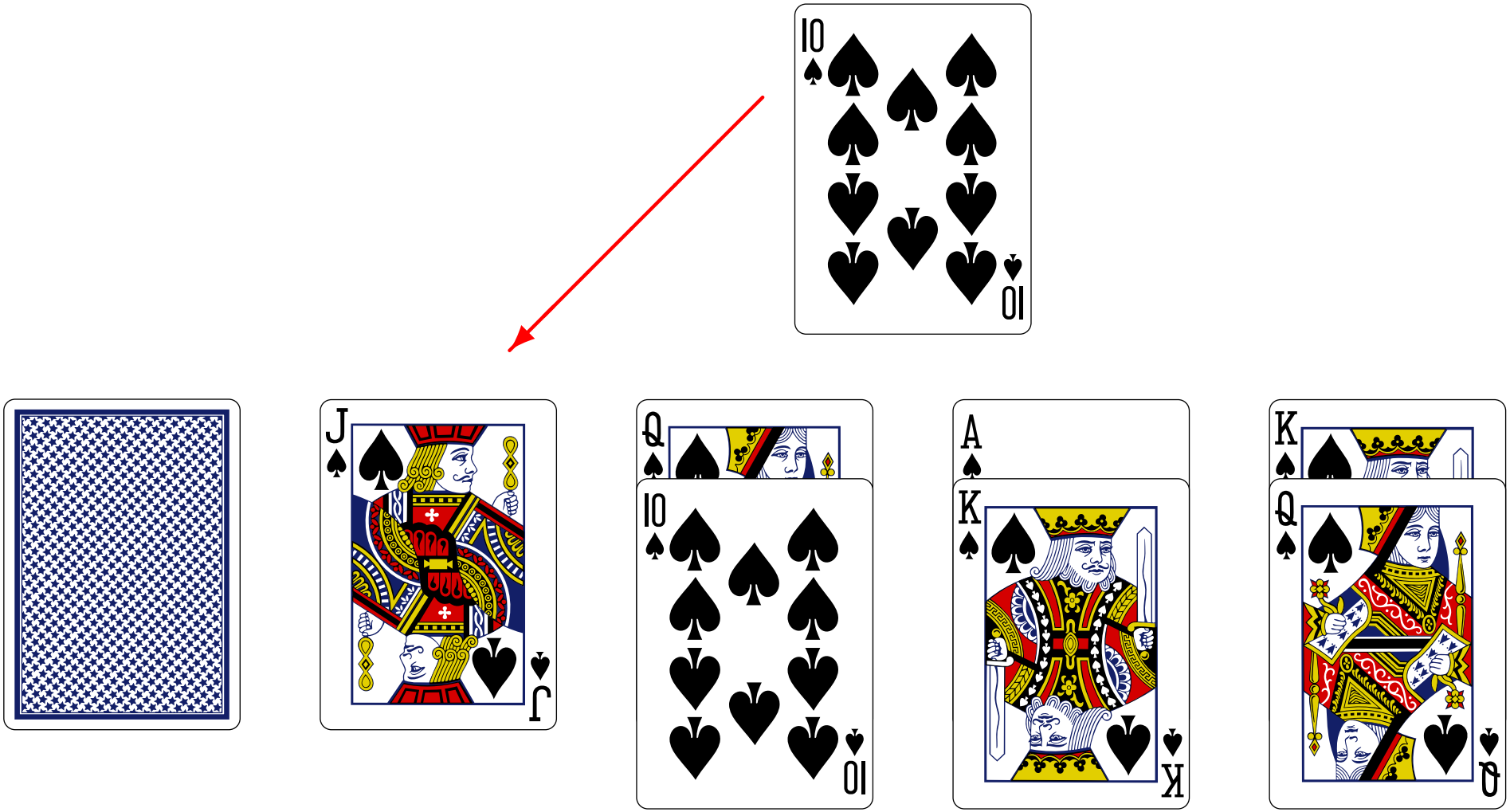

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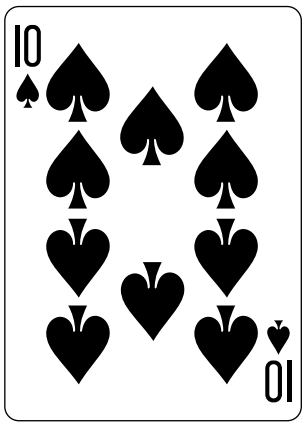
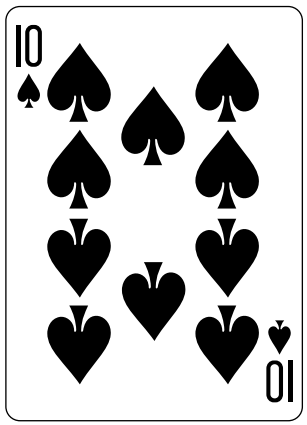
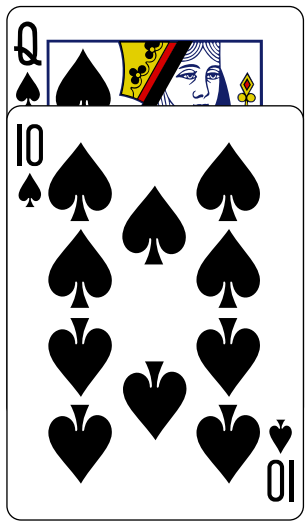

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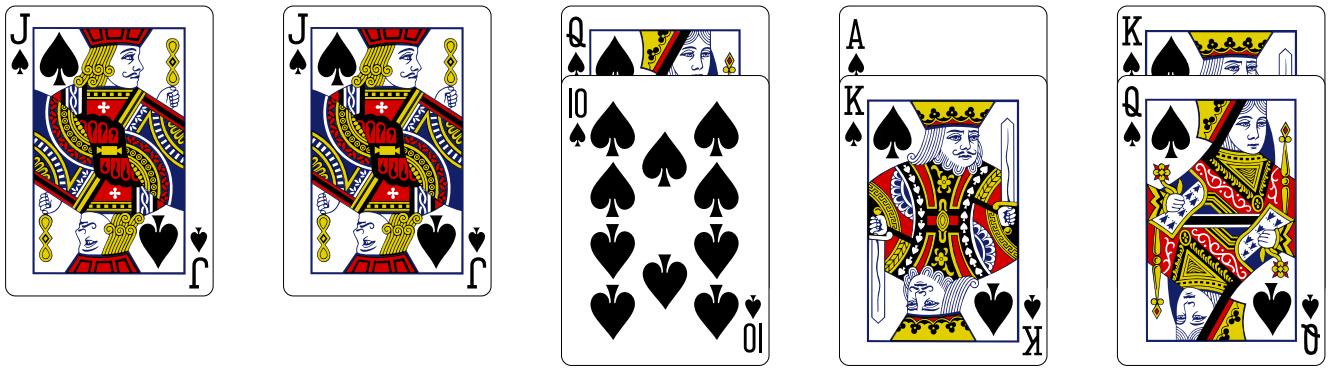

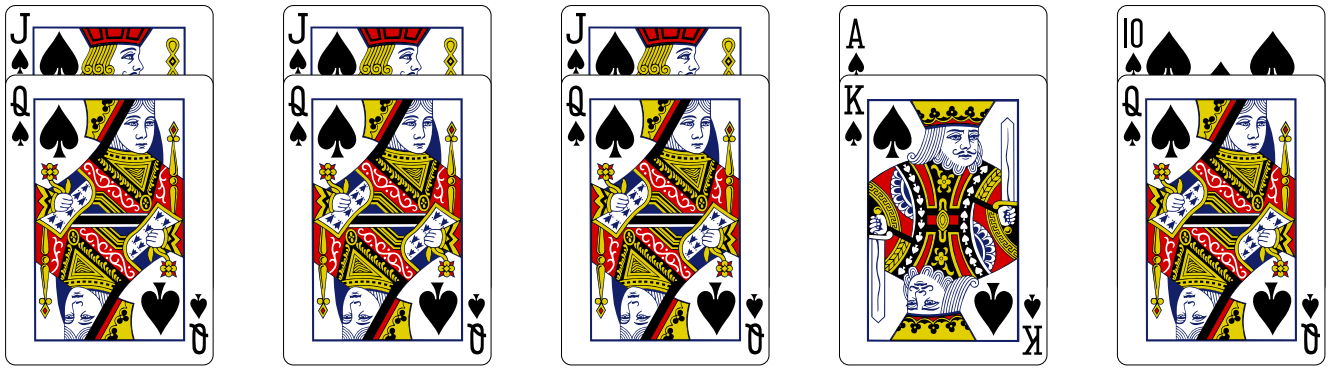

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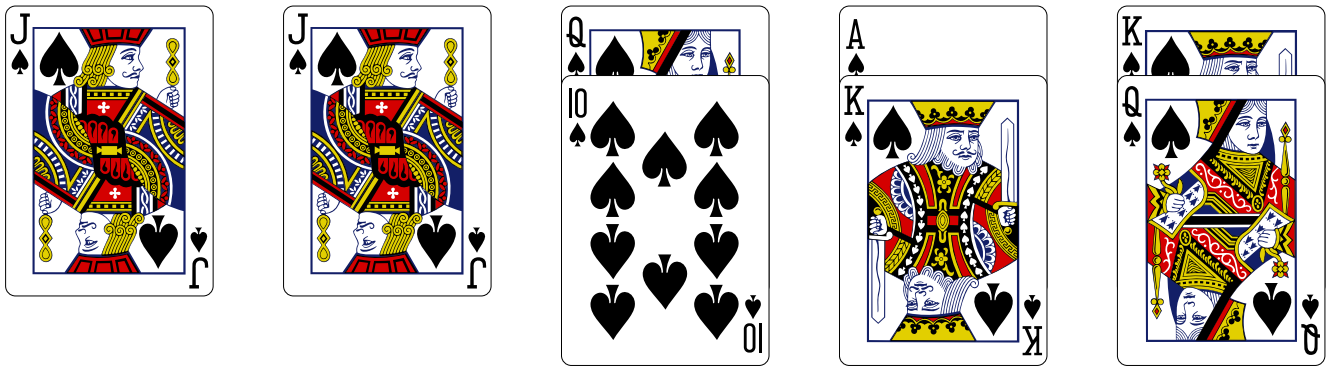

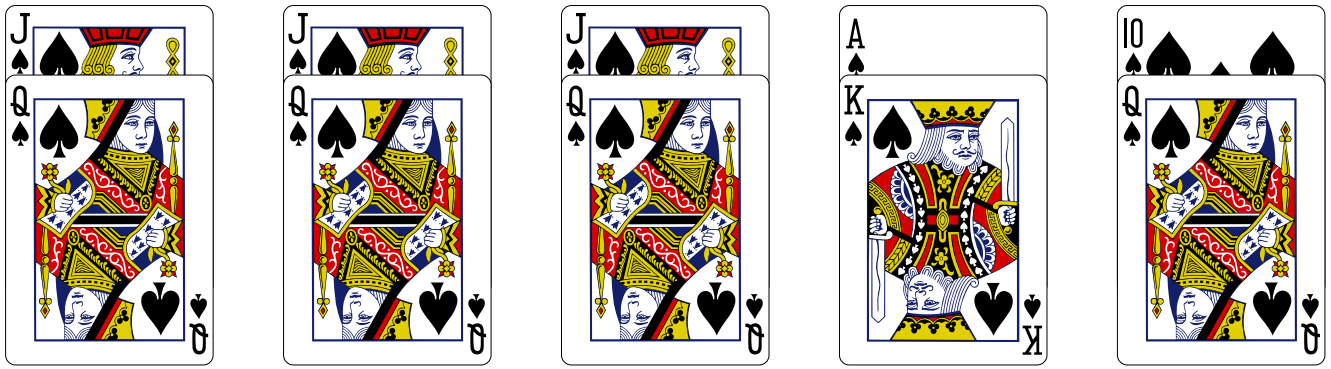

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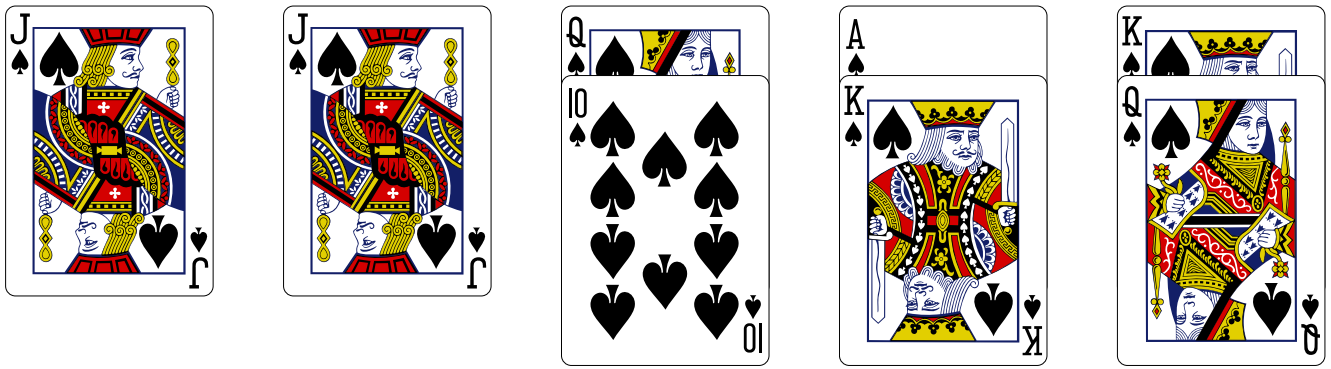
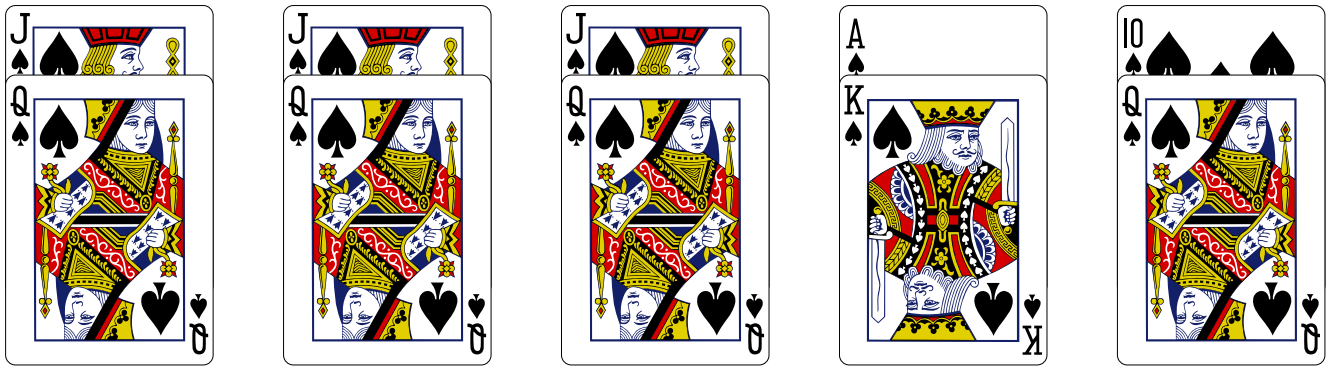
what was the difference?

- in the top game, Dealer can prevent Player from increasing the number of different cards in the first two stacks
- in the bottom game, Dealer cannot prevent this





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Necessary Condition

If Player has a winning strategy, then for every set of stacks S we must have

$$\sum_{C \in \bigcup S} \left\lceil \frac{d_S(C)}{2} \right\rceil \geq |S|.$$