stolidity

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1 Introduction

For a graph G, let $\mathcal{C}_G$ be the $\chi(G)$-colorings of G. For a graph G and $S \subseteq V(G)$, the *stolidity* of S is

$$\text{stol}_G(S) := \frac{1}{|\mathcal{C}_G|} \sum_{\pi \in \mathcal{C}_G} \frac{|\pi(S)|}{|S|}.$$

Of particular interest are graphs G where $\text{stol}_{G-v}(N_G(v))$ is near 1 for all $v \in V(G)$. For $k \in \mathbb{N}$, a graph G is *k-stolid* if $\text{stol}_{G-v}(N_G(v)) \geq 1 - \frac{k}{\Delta(G)}$ for all $v \in V(G)$.

Theorem 1. *A connected graph G is 0-stolid just in case G is complete or an odd cycle.*

Proof. Let G be a 0-stolid graph. If $v \in V(G)$ has maximum degree, then $\text{stol}_{G-v}(N_G(v)) \geq 1$ implies that every $\chi(G-v)$-coloring of $G-v$ uses $\Delta(G)$ colors on $N_G(v)$. In particular, $\chi(G) \geq \Delta(G) + 1$. Now Brooks' theorem shows that G is complete or an odd cycle. $\square$

Problem. Classify the 1-stolid graphs.