# Independent transversals via entropy compression (18)

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## 1 Introduction

## 2 The setup

Let $G$ be a graph and $\pi\colon V(G) \to [k]$ be a (proper) coloring of $G$. For $i \in [k]$, put $P_i := \pi^{-1}(i)$. For each $xy \in E(G)$, recursively define $T_G(xy)$ to be the tree with root $xy$ having as children $T_G(e)$ for all $e \in E(G)$ such that $e$ intersects $P_{\pi(x)} \cup P_{\pi(y)}$. We call $T_G(xy)$ the *dependency tree* rooted at $xy$. Note that, by definition, $T_G(xy)$ is a child of $xy$ and in particular $T_G(xy)$ is an infinite tree.

## 3 Balanced partitions

**Lemma 3.1.** *If $|P_i| \geq 2\Delta(G)$ for all $i \in [k]$, then $\pi$ has an independent transversal.*

*Proof.* Suppose the lemma is false. Then $\pi$ has no independent transversal and we may as well have $|P_i| = 2\Delta(G)$ for all $i \in [k]$. Put $\Delta := \Delta(G)$. Number the vertices of each $P_i$ with $1, \ldots, 2\Delta$. Then a transversal of $\pi$ is represented by an element of $[2\Delta]^k$. Let $A$ be an arbitrary transversal of $\pi$ and $xy$ an edge contained in $A$. For $h \in \mathbb{N}$, let $R_h$ be the collection of subtrees of $T_G(xy)$ rooted at $xy$ having $h$ edges.

For each $h \in \mathbb{N}$, we construct an injection $f_h\colon [2\Delta]^{2h} \hookrightarrow R_h \times [2\Delta]^k$. Fix $(s_1, t_1, s_2, t_2, \ldots, s_h, t_h) \in [2\Delta]^{2h}$. Put $A_0 := A$, $x_0 := x$ and $y_0 := y$. For $j \in [h]$, to form $A_j$ replace the $\pi(x_{j-1})$th slot of $A_{j-1}$ with $s_j$ and the $\pi(y_{j-1})$th slot of $A_{j-1}$ with $t_j$. $\square$

## 4 Lopsided partitions