

# A common generalization of Hall's theorem and Vizing's edge-coloring theorem

landon rabern

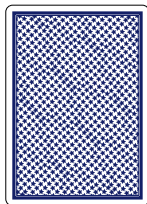
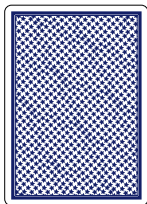
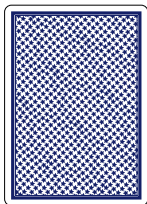
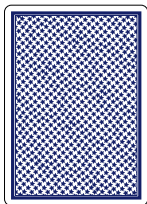
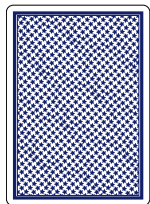
LBD Data

Miami University Colloquium  
November 6, 2014

# some card games

the simplest variation

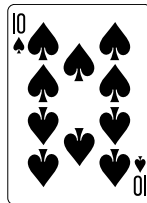
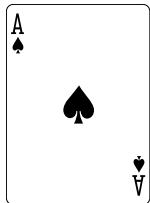
- two players, Dealer and Player



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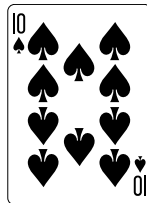
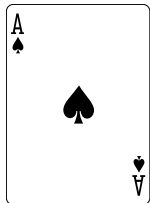
- two players, Dealer and Player
- the deck has just many copies of the high spade cards



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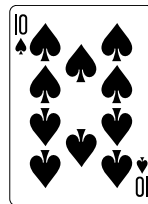
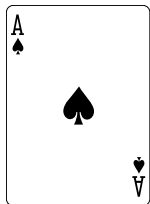
- two players, Dealer and Player
- the deck has just many copies of the high spade cards
- Dealer makes 5 stacks of cards with no duplicates, all cards face-up



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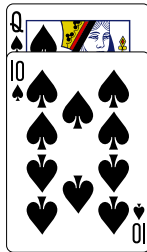
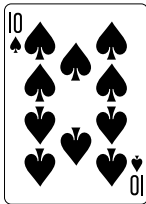
## the simplest variation

- two players, Dealer and Player
- the deck has just many copies of the high spade cards
- Dealer makes 5 stacks of cards with no duplicates, all cards face-up
- Player wins if he can pick a Royal Flush, one card from each stack



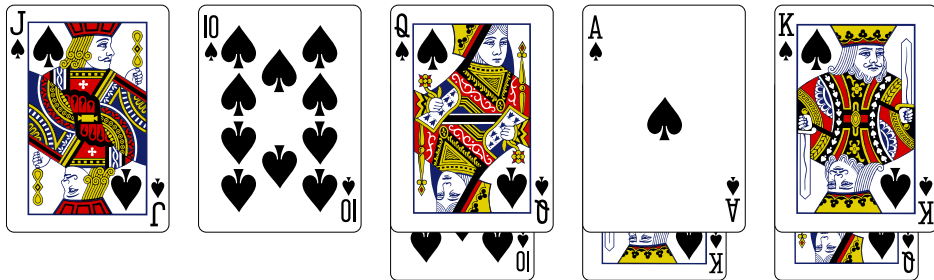
# some card games

example, a Player win



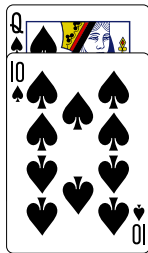
# some card games

example, a Player win



# some card games

example, a Dealer win





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winning condition

- Player cannot win if there is a set of  $k$  stacks that together have fewer than  $k$  different cards

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## winning condition

- Player cannot win if there is a set of  $k$  stacks that together have fewer than  $k$  different cards
- Hall's theorem says: **Player wins otherwise**



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making things harder for Dealer

- this isn't a fun game, far too easy for Dealer to win

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## Player's Move

*Player can pick any card  $A$  from the deck and swap it for another card  $B$  in one stack (not containing  $A$ ).*

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*Dealer can either do nothing or swap  $A$  and  $B$  in at most one other stack.*

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*Dealer can either do nothing or swap A and B in at most one other stack.*

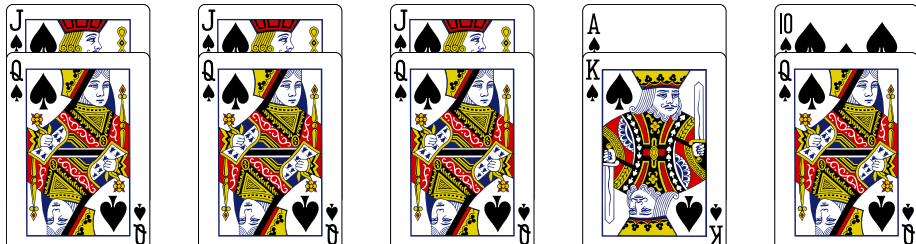
## Winning

*Player wins if he can pick a Royal Flush at the start of one of his turns, otherwise Dealer wins.*



# some card games

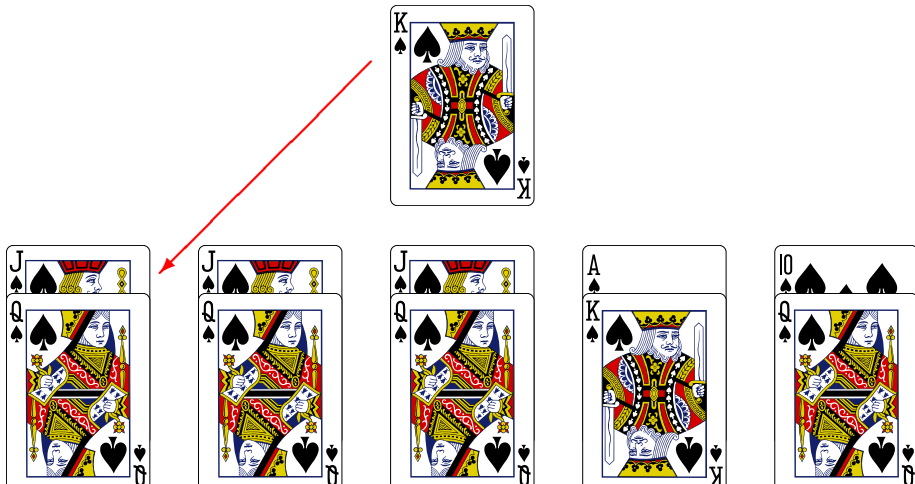
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example, a Player win

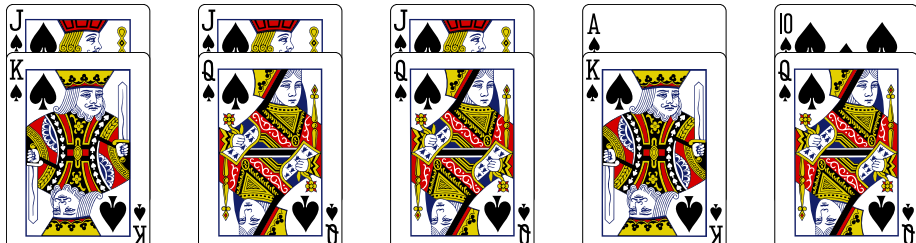
- Player picks a King from the deck and swaps it for a Queen in the first stack



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example, a Player win

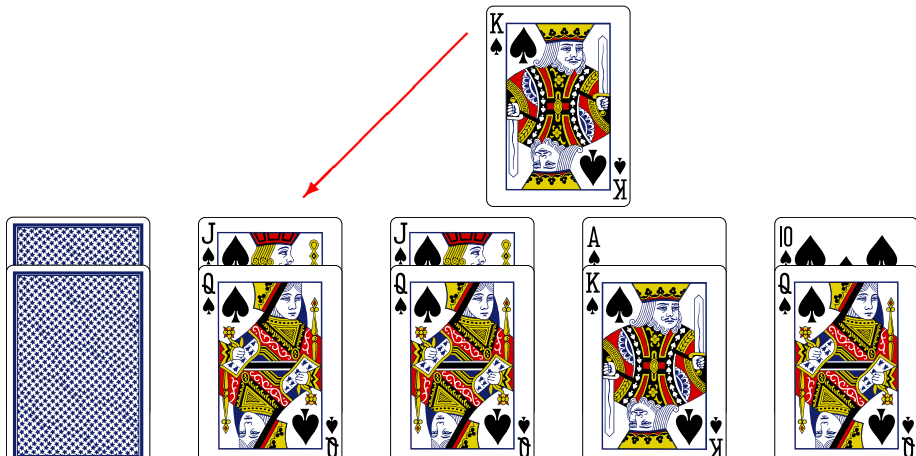
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example, a Player win

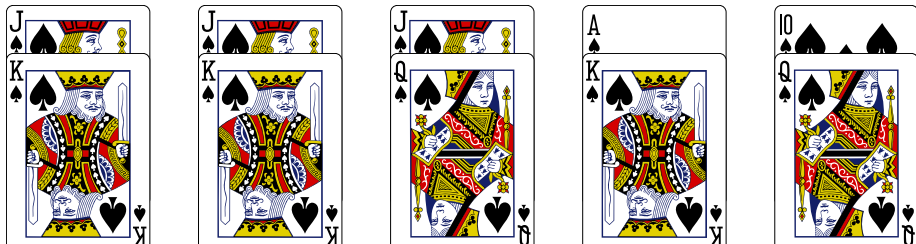
- Player picks a King from the deck and swaps it for a Queen in the first stack
- Dealer can swap a King and Queen in one of the other stacks



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example, a Player win

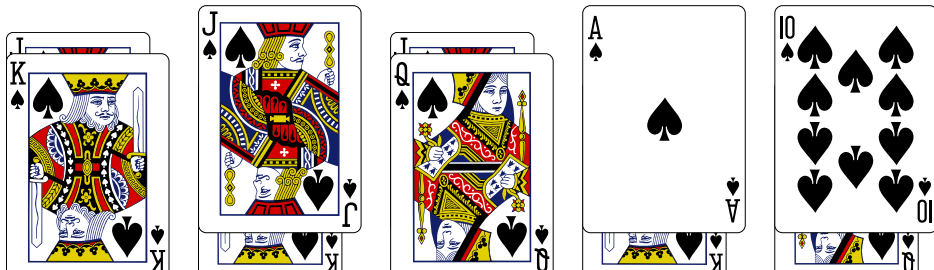
- Player picks a King from the deck and swaps it for a Queen in the first stack
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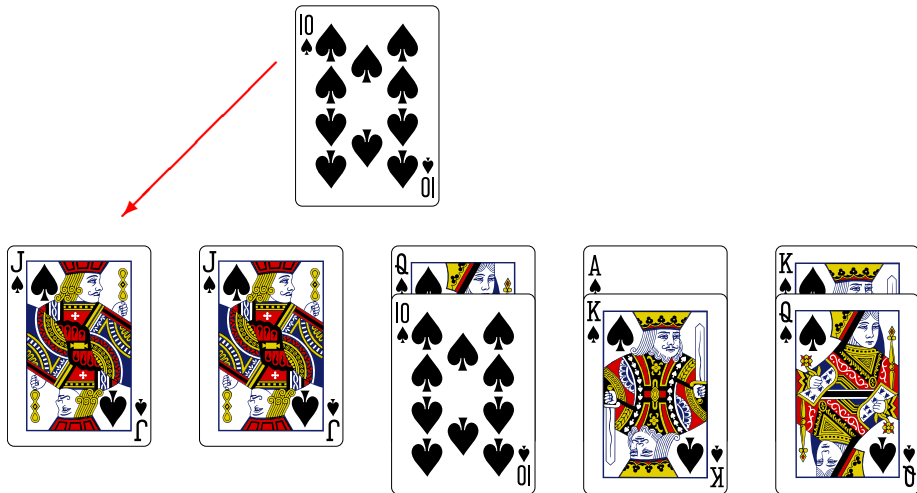
example, a Player win

- Player picks a King from the deck and swaps it for a Queen in the first stack
- Dealer can swap a King and Queen in one of the other stacks
- Player wins no matter what Dealer does



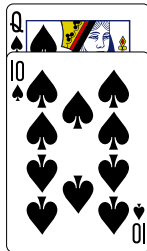
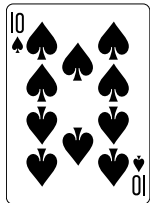
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example, a Dealer win



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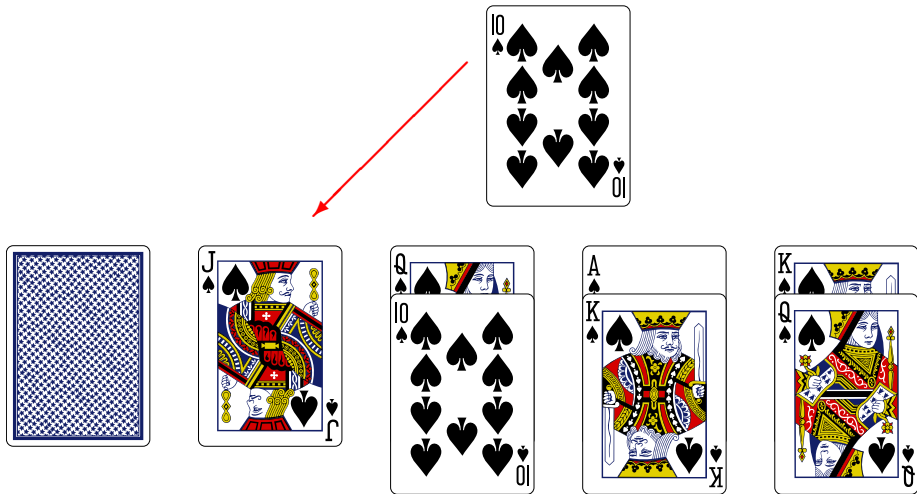
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