graph theory notes*

Schauz's combinatorial interpretation of Alon and Tarsi's algebraic technique

In [1], Alon and Tarsi introduced a beautiful algebraic technique for proving the existence of list colorings. In [2], Uwe Schauz gave a new combinatorial proof that extended the technique to online list coloring. Suppose we have graph G and $f\colon V(G) \to \mathbb{N}$ for which we would like to show that G is f-choosable. We could show this easily by just being greedy if we could find an *acyclic* orientation of G such that $d^+(v) < f(v)$ for every $v \in V(G)$. Being acyclic is a strong requirement, so it is natural to seek weaker conditions that allow us to conclude f-choosability. That is, we want to find properties $\mathcal{P}$ such that if G has a $\mathcal{P}$-orientation such that $d^+(v) < f(v)$ for every $v \in V(G)$, then G is f-choosable. A simple inductive argument shows that "kernel-perfect" is such a $\mathcal{P}$.

References

[1] N. Alon and M. Tarsi, *Colorings and orientations of graphs*, Combinatorica **12** (1992), no. 2, 125–134.

[2] Uwe Schauz, *Flexible color lists in Alon and Tarsis theorem, and time scheduling with unreliable participants*, the electronic journal of combinatorics **17** (2010), no. 1, R13.

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