Please thank the referees for reading our paper carefully, and suggesting a number of improvements. We have implemented nearly all of their suggestions. Below are our detailed responses. (We first address the more detailed reported.)

0 Abstract

(1) Changed the sentence to read "The problem of determining the minimum number of edges in a k-critical graph with n vertices has been widely studied..."

(2) Changed the sentence to read "In this paper, we improve the best known lower bound on the number of edges in a k-list-critical graph."

(3) Added the following sentence: " In fact, our result on k-list-critical graphs is derived from a lower bound on the number of edges in a graph that is critical with respect to its Alon–Tarsi number."

1 Introduction

(1) Changed capitalization as requested.

(2) Added citations to Brooks, Dirac, and Gallai.

(3) Added missing parentheses.

(4) Changed "Alon-Tarsi" to "Alon–Tarsi".

(5) Changed the sentence to "In contrast, Krivelevich's proof [14] does not easily translate to list coloring, since it uses a lemma of Stiebitz [19], which says that in a color-critical graph, the subgraph induced by vertices of degree at least k has no more components than the subgraph induced by vertices of degree $k-1$ (but no analogous lemma is known for list coloring)."

(6) Changed the sentence to read "The discharging argument is more intuitive and we believe that it may be easier to modify in the future for use with new reducibility lemmas."

(7) Changed "... f-assignment such that $f(v) = k$..." to "... f-assignment where $f(v) = k$..."

(8) We kept this notation as d_0-assignment to remain with consistent in previous papers. (We avoid d-assignment to avoid confusion with k-assignment, where $f(v) = k$ for all $v \in V(G)$.)

(9) We added the definition of $d_G(v)$ and $d(v)$ at the start of Section 2.

(10) Changed "called Gallai Trees" to "known as *Gallai Trees*".

(11) Now, just after Lemma 1.2, we write: " From the definitions and Lemma 1.2, it is easy to check that always $\chi(G) \le \chi_\ell(G) \le \chi_{\mathrm{OL}}(G) \le AT(G) \le \Delta(G) + 1$."

(12)

(13) The first time we mention the discharging method, we added a citation of the paper by Cranston and West.

2 Gallai's bound via discharging

(1) We have kept the (repeated) definition of Gallai Tree. Our motivation is to gather all of the definitions in one place for easy reference. (If readers read papers from start to end at one sitting this would be less necessary. But that is not the case.)

(2) Changed "...precisely the d_0-choosable..." to "...precisely the non-d_0-choosable..."

(3) Changed "...set of all Gallai Trees of degree..." to "...set of all Gallai Trees of maximum degree..."

(4) Added the following to the proof of Theorem 2.1: (Suppose there exists $v \in V(G)$ with $d(v) \le k-2$. Since G is k-AT-critical, $G - v$ has an orientation D' with $d^+_{D'}(w) < k$ for all $w \in V(G) - v$ and $EE(D') \ne EO(D')$. Orienting all edges incident to v away from v gives an orientation D of G also with $d^+_D(w) < k$ for all $w \in V(G)$ and $EE(D) \ne EO(D)$. So, we assume that $\delta(G) \ge k-1$.)

(5) Added to the proof of Theorem 2.1 the sentence "Note that the sum of hte charge assigned to the vertices is preserved by these operations and equals the sum of the degrees. So..."

(6) Changed $ch^*(v)$ to $\mathrm{ch}^*(v)$.

(7) Removed "instead" from the start of the sentence.

(8) Changed previous wording to "The total charge received by vertices of T from the k^+-vertices is precisely"

(9) Defined $d_T(v)$ at start of Section 2.

(10) Added the following to the proof of Theorem 2.1: Recall from the introduction that a connected graph is not d_0-choosable precisely when it is a Gallai Tree. So T must be a Gallai Tree; otherwise, we can $(k-1)$-color $G - T$ (since G is k-critical), and extend the coloring to T, since T is d_0-choosable.

(11) Added citation to Gallai paper for Lemma 2.2.

3 A refined bound on $\|T\|$

(1) Changed (2) to (ii) in last sentence of first paragraph on page 6.

(2) Changed 1., 2., 3., and 4. in statement of Lemma 3.1 to (1), (2), (3), and (4).

(3) The first line of Lemma 3.1 already states: "Let $p : \mathbb{N} \to \mathbb{R}$, $f : \mathbb{N} \to \mathbb{R}$. For all $k \geq 5 \ldots$"

(4) Changed $t \in \{2, k-2\}$ to $t \in \{2, \ldots, k-2\}$.

(5) Changed start of second paragraph in proof of Lemma 3.1 to read as follows. "Suppose the lemma is false and choose a counterexample T minimizing $|T|$. If T is K_t for some $t \in \{2, \ldots, k-2\}$, then $t(t-1) > (k-3+p(k))t + f(k)$. After substituting $p(k) \geq \frac{-f(k)}{k-2}$ from (1), this implies that $-t(k-2)(k-2-t) > (k-2-t)f(k)$. When $t = k-2$, both sides are 0, which is a contradiction. Otherwise, dividing by $k-2-t$ gives that $-t(k-2) > f(k)$, which contradicts (3)."

(6) Revised paragraph in proof of Lemma 3.1 beginning "To handle…" to the following. "Suppose next that B is K_{k-2} and notice that, since T has maximum degree at most $k-1$ and $T \neq K_{k-2}$, this implies that $d_T(x_B)$ is either $k-2$ or $k-1$. In the former case, we set $D = B$ and obtain.

$$(k-2)(k-3) + 2 > (k-3+p(k))(k-2),$$

contradicting (4)."

(7) Changed 1., 2.,… in statement of Lemma 3.1 to (1), (2), …

4 Discharging

(1) Changed the sentence in the first paragraph of the discharging section to the following. "It is helpful to view our proof here as a refinement and strengthening of the proof of Gallai's bound in Section **??**, Theorem **??**." (The point of using the term "Gallai's bound" is that it is likely easier for the reader to know what is meant, without looking back to the earlier section of the paper.)

(2) The term $(k-1)$-neighbor is defined at the start of Section 2.

(3) On page 11 (in the paragraph starting "First, note…"), changed Q to be italicized.

5 Reducible Configurations

6 References

(1) Added the missing accent to Král'.

Second Referee

- p1: Dirac proved that every k-critical graph other than K_k [...]

  Changed as requested.

- p2.14: I do not think the term Ore degree is established. Maybe use "degree sums"?

  We added the following footnote: The *Ore degree* of an edge xy is the sum $d(x) + d(y)$. The Ore degree of a graph is the maximum Ore degree of its edges.

- p2.15: use "the" Alon-Tarsi number through out.

  Changed as requested.

- p3: Why do you mention the bounds for k-critcal graphs, $k \leq 10$ from [8]? They are worse than previous bounds, and somewhat confusing in the table.

  The other obvious option would be to use a '—'. However, our interpretation of a '—' (if we were unaware of the history of this problem) would be that [8] had not proved any bounds for these values of $d(G)$, which is incorrect. Thus, we include them.

- p5: Include a short argument that T' is in T_k in both cases, it tripped me up for a minute.

- p6: Just write "The proof mirrors that of [...]". No need to say that this sentence is the proof outline.

  Changed as requested.

- p7.13: "require" is maybe a bit too strong here. You don't give an argument (and you shouldn't) that (3), (4) and (5) are absolutely necessary.

  Changed "require" to "rely on"

- p7.18: delete "need to"

  Changed as requested.

- p7.23: each $\rightarrow$ every

  Changed as requested.

- p9.1: see $\rightarrow$ study ?

  Changed as requested.

- p9.7: How is this "instead"?

  Here we are first choosing the value of $p(k)$ (which then constrains the value of $h(k)$, instead of first choosing the value of $h(k)$ (which then constrains the value of $p(k)$).

  Alternately, the first paragraph assigns one pair of values to $p(k)$ and $h(k)$. Instead, the second paragraph assigns them a different pair of values.

- p9: can you make the formulas in Cor3.3 and Lem3.4 look even more similar by multiplying out some factors?

- p10.22: Should it be k-list-critical here?

  Changed to k-AT-irreducible.

- p10.27: the proof $\rightarrow$ our proof ?

  Changed as requested.

- p11.5: This sounds like your believe is the unfortunate part...

  Changed to the following. "However, we believe this is false. Fortunately, something similar is true."

- p15: I am sure you have discussed this, but in my opinion you should switch sections 4 and 5. You frequently refer to section 5 in section 4, and it would be comforting to know at that point that all results in section 5 were established before elsewhere.