① what is the 487th derivative of $e^x + e^{-x}$?

② what is the 487th derivative of $e^{ix} + e^{-ix}$ where $i = \sqrt{-1}$?

③ Let $f(x) = \frac{1}{2}(e^{ix} + e^{-ix})$ and $g(x) = \frac{1}{2i}(e^{ix} - e^{-ix})$.

Compute $f^{(n)}(0)$ for all $n \geq 0$ and $g^{(n)}(0)$ for all $n \geq 0$.

($f^{(n)}(0)$: nth derivative at 0)

④ Let $f(x) = \cos(x)$ and $g(x) = \sin(x)$

Compute $f^{(n)}(0)$ and $g^{(n)}(0)$ for all $n \geq 0$.

Conclude $\cos(x) = \frac{1}{2}(e^{ix} + e^{-ix})$

$\sin(x) = \frac{1}{2i}(e^{ix} - e^{-ix})$.

⑤ Simplify $\cos(x) + i\sin(x)$ using ④.

⑥ what is $e^{\pi i}$?

⑦ Derive the product rule from the chain rule by considering $\left((a(x)+b(x))^2\right)'$

⑧ Using just the product rule and the fact that $(x)' = 1$, show that $\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$. Hint: consider $x \cdot \frac{1}{x} = 1$.

⑨ Use the product rule, chain rule and ⑧ to derive the quotient rule.