3.1

Maximum and minimum of quadratic functions.

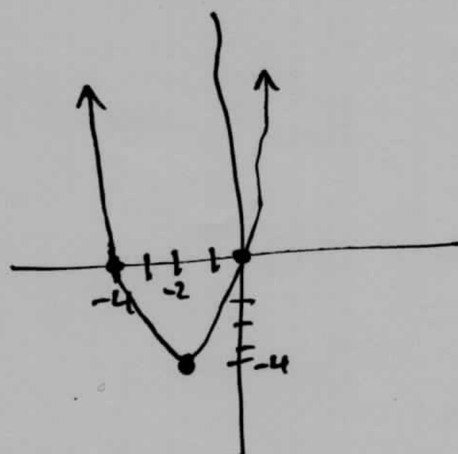
ex! how small does $f(x)=x^2+4x$ get?

$f(x) = x(x+4)$

$f(0)=0$

$f(-4)=0$

$f(-2)=(-2)(-2+4)=(-2)(2)=-4$



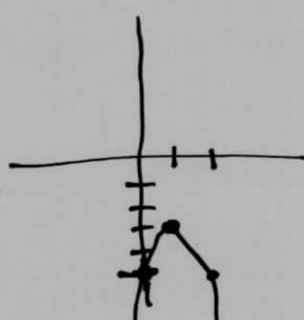
$\boxed{-4}$

ex2 how big does $f(x)=-2x^2+4x-5$ get?

$f(0)=-5$

$f(1)=(-2)(1)^2+4(1)-5=-3$

$f(2)=(-2)(2)^2+4\cdot2-5=(-2)(4)+8-5=-5$



$f(1)=\boxed{-3}$

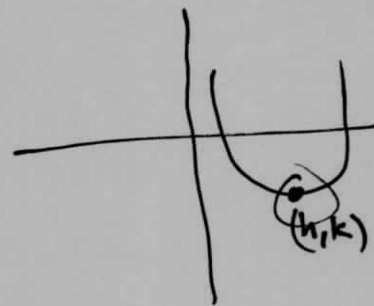
guessing from the picture works ok, but we can do it much simpler with a little algebra.

$f(x) = ax^2+bx+c$ ← every quadratic function looks like that.

is a parabola

Find peak/valley

how?

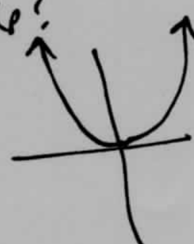


rewrite ax^2+bx+c

as $\underline{a(x-h)^2+k}$

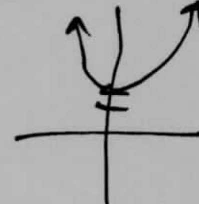
how does this help?

$f(x) = x^2$



shift up 2

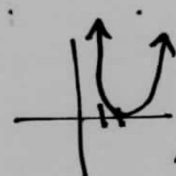
$f(x) = x^2+2$



Shift right 2

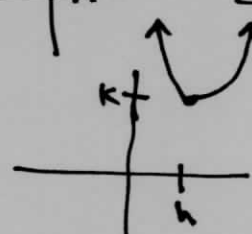
$f(x) = (x-2)^2$

$f(x) = 3(x-2)^2$



shift right 2 and squash

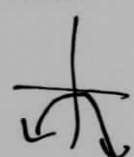
$f(x) = (x-h)^2+k$



right h, up k.

flip over

$f(x) = -x^2$



$f(x) = a(x-h)+k$ ← shift right h, up k, flip over if $a<0$ (and squash)

If we can write $f(x)=ax^2+bx+c$ as

$f(x) = a(x-h)+k$,

we can find (h,k).

$$f(x) = ax^2+bx+c = a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$$

$$\boxed{h = \frac{-b}{2a}}$$

$$k = \left(\frac{-b}{2a}\right)^2 + \frac{c}{a} = f(h)$$

To find the min/max of $f(x) = ax^2+bx+c$, look at the point $(h, f(h))$ where $h = \frac{-b}{2a}$.

if $a>0$ it is a min

if $a<0$ it got flipped over, so it is a max.

ex. Sally has 500 ft of fencing and wants to build a rectangular corral for her horses

what dimensions should she make it to have the largest area?