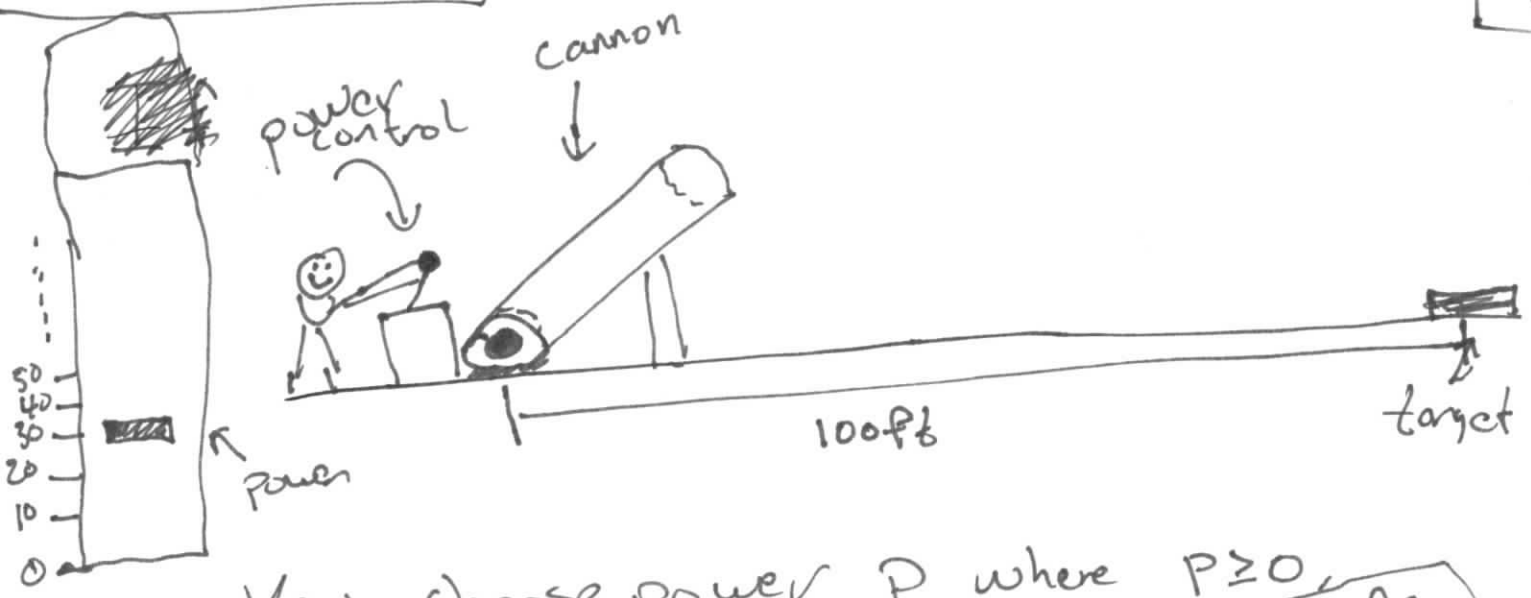

You choose power p where $p \geq 0$,
cannon ball moves horizontally at: $4p$ ft/s
cannon ball height at time t given by

$$h(t) = 3pt - 10t^2 \quad \text{(in feet)}$$

what should you set p at to hit the target?

ball hits ground when

$$h(t) = 0.$$

$$0 = h(t) = 3pt - 10t^2$$

$$0 = 3pt - 10t^2$$

$$0 = t(3p - 10t)$$

either $t = 0$ or $3p - 10t = 0$

$t = 0$ ↑ start

$3p = 10t$

$t = \frac{3p}{10}$

Want ball to be 100 ft away

$100 = 4pt$

$t = \frac{100}{4p} = \frac{25}{p}$

$\frac{25}{p} = \frac{3p}{10}$

$p^2 = \frac{250}{3}$

$p = \sqrt{\frac{250}{3}} \approx 9.1$

what if the target is moving away at 2 ft/s?

$h(t)=0$ still gives $t=\frac{3p}{10}$.

but now want

$$4pt = 100+2t \iff 4pt-2t=0$$

$$(4p-2)t = 100$$

$$10\cdot(4p-2)\left(\frac{3p}{10}\right)=100\cdot 10$$

$$(4p-2)(3p)=1000$$

$$\boxed{12p^2-6p-1000=0}$$ what is p?

---

in general, can we solve

$$ax^2+bx+c=0 \quad \text{for } x?$$

let's solve the $a=1$ case first.

$$x^2+bx+c=0$$

$$x^2+bx=\left(x+\frac{b}{2}\right)^2-\frac{b^2}{4}$$

$$\left(x+\frac{b}{2}\right)^2-\frac{b^2}{4}+c=0$$

$$\left(x+\frac{b}{2}\right)^2=\frac{b^2}{4}-c$$

$$x+\frac{b}{2}=\pm\sqrt{\frac{b^2}{4}-c}$$

$$x=\frac{-b}{2}\pm\sqrt{\frac{b^2}{4}-c}=\frac{-b}{2}\pm\sqrt{\frac{1}{4}(b^2-4c)}$$

$$=\frac{-b}{2}\pm\left(\sqrt{\tfrac{1}{4}}\cdot\sqrt{b^2-4c}\right)$$

$$=\frac{-b}{2}\pm\frac{1}{2}\sqrt{b^2-4c}$$

$$X = \frac{-b \pm \sqrt{b^2-4c}}{2}$$

general a?

$$ax^2+bx+c=0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$X=\frac{-\frac{b}{a}\pm\sqrt{\left(\frac{b}{a}\right)^2-4\frac{c}{a}}}{2} = \frac{-\frac{b}{a}\pm\sqrt{\frac{1}{a^2}(b-4ac)}}{}$$

$$= \frac{-\frac{b}{a}\pm\left(\sqrt{\frac{1}{a^2}}\right)^2\sqrt{b-4ac}}{2}$$

$$= \frac{-\frac{b}{a}\pm\frac{1}{a}\sqrt{b^2-4ac}}{2}$$

$$= \boxed{\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

Our case: $12p^2-6p-1000=0$

$$p = \frac{-(-6)\pm\sqrt{(-6)^2-4(12)(-1000)}}{2(12)} = \frac{6\pm\sqrt{36+48000}}{24}$$

want + since $p \geq 0$

$$p = \frac{6+\sqrt{48036}}{24} \approx 9,38.$$

① what if target moves away at 50 ft/s?

② what if target moves away at 50 ft/s and up at 10 ft/s? starting from 5 ft high.

$4pt = 100 + 50t$

$\boxed{(4p-50)t = 100} \Rightarrow t = \frac{100}{4p-50}$

also $h(t) = 10t + 5$

$5 + 10t = h(t) = 3pt - 10t^2$

$-10t^2 + 3pt - 10t - 5 = 0$

$-10t^2 + (3p-10)t - 5 = 0$

$-10\left(\frac{100}{4p-50}\right)^2 + (3p-10)\frac{100}{(4p-50)} - 5 = 0$

$-100000 + (3p-10)(100)(4p-50) - 5(4p-50)^2 = 0$

: algebra

$1120p^2 - 17000p - 62500 = 0$

$p = \frac{-(-17000) \pm \sqrt{(-17000)^2 - 4(1120)(-62500)}}{2(1120)}$

$p \approx -3,05$ or $p \approx 18,2$