Taming the Borodin-Kostochka monster

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We seek to dispel a myth. That myth is that there exists a graph M with $\Delta(M) = \chi(M) = 9$ and $\omega(M) < 9$. So that we have an object at which to point our ire, let us assume that such a beast does in fact exist. Let's look for the bestest such beast we can play with, call him $\mathcal{M}$ and let $\mathcal{M}$ have the least number of vertices among such beasts. We may as well specify our beast $\mathcal{M}$ further by letting him be the one, amongst those with the least vertices, with the most edges.

What do we know about $\mathcal{M}$?

$$|\mathcal{M}| \geq 71$$

$$3 \leq \alpha(\mathcal{M}) \leq \frac{2}{9}\,|\mathcal{M}|$$

$$\kappa(\overline{\mathcal{M}}) = \delta(\overline{\mathcal{M}})$$

$$\mathcal{L}(\mathcal{M}) \text{ is complete and } |\mathcal{L}(\mathcal{M})| \leq 7$$

References