Possibly true things i'd like to prove

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First the main motivating conjectures for the study of graphs with $\chi = \Delta$.

Conjecture 1 (Borodin & Kostochka). *Every graph satisfying $\chi \geq \Delta \geq 9$ contains a K_Δ.*

It is easy to show that the following conjecture of Grünbaum holds for $\chi \geq \Delta \geq 7$, so the 5 and 6 cases remain. The 6 case should be easier – the 5 case implies the 6.

Conjecture 2 (Grünbaum). *There are no triangle free graphs with $\chi \geq \Delta \geq 5$.*

The following with 8 replaced by 9 is equivalent to Borodin-Kostochka – i think the stronger statement is true. One idea for proving Borodin-Kostochka is to try to find a more general statement that holds for $\Delta = 7$, $\Delta = 6$, etc and get down to a manageable max degree base case.

Conjecture 3. *Every graph satisfying $\chi \geq \Delta \geq 8$ contains a $K_3 \vee H$ where H is some graph on $\Delta - 3$ vertices.*

Conjecture 4. *In the complement of any k-mule with $k \geq 8$ the only minimum cardinality vertex cuts are neighborhoods. i think the bound on k might be reduced if we allow a finite list of exceptions.*

Conjecture 5. *No k-mule with $k \geq 8$ contains a K_{k-1}.*

Conjecture 6. *The Borodin-Kostochka conjecture holds for circular interval graphs, quasi-line graphs, and claw-free graphs – in order of class containment.*

Conjecture 7. *The only connected counterexample to Borodin-Kostochka with $\Delta = 8$ that is the line graph of a multigraph is $L(3C_5)$. Is this also the only circular interval graph counterexample with $\Delta = 8$? quasi-line? claw-free?*

The following can be viewed as an improvement of Vizing's theorem.

Conjecture 8. *If G is the line graph of a multigraph H, then*

$$\chi(G) \leq \max\left\{\omega(G), \frac{\Delta(G)+2}{2} + \mu(H)\right\}.$$

This is motivated in that if true so would be the following tight bound.

Conjecture 9. *If G is the line graph of a multigraph, then*

$$\chi(G) \leq \max\left\{\omega(G), \frac{5\Delta(G)+8}{6}\right\}.$$

i think this next one might be false, but i'd like to know. It is false for claw-free graphs; in fact, no bound like this at all can hold for claw-free since $C_5 \vee K_t$ has $\omega = t+2$, $\chi = t+3$, and $\Delta = t+4$.

Conjecture 10. *If G is a quasi-line graph then*

$$\chi(G) \leq \max\left\{\omega(G), \frac{5\Delta(G)+8}{6}\right\}.$$