fixable proofs

May 30, 2015

1 Proofs

Lemma 1.1. *The graph in Figure 1 is reducible.*

Proof. Let $X = \{0, 1\}$, $Y = \{0, 2\}$ and $Z = \{1, 2\}$. Then with the vertex ordering in Figure 1, a string such as YXYZZY, represents a possible list assignment on $V(H)$ arising from a 3-edge-coloring of $G - E(H)$. By an X-Kempe change, we mean flipping colors 0 and 1 on a two-colored path in $G - E(H)$. We call such a path an X-path. Any endpoint of an X-path in H must end at a Y or Z vertex. The meanings of Y-Kempe change, Z-Kempe change, Y-path and Z-path are analogous. Note that if there are an odd number of Y's and Z's, then at least one X-path has only one endpoint in H. We use shorthand notation like $\mathcal{K}_{X,2}(YXYZZY, 5, 6) \Rightarrow YYYXZY, ZZZXYZ$ (Case 1). This means the X-Kempe change on YXYZZY starting at the second vertex and ending at the fifth and sixth result in boards YYYXZY and ZZZXYZ respectively and these are handled by Case 1. The ∞ symbol means starting (or ending) outside H.

We need to handle all boards up to permutations of $\{X, Y, Z\}$, so it will suffice to handle all boards of the form $\bigstar\bigstar\bigstar\bigstar YZ$, $\bigstar\bigstar\bigstar YZZ$, $\bigstar\bigstar YZZZ$, $\bigstar YZZZZ$, $YZZZZZ$ or $ZZZZZZ$.

Case 1. *B is one of $\bigstar Z\bigstar\bigstar YZ$, $\bigstar Y\bigstar YZZ$, $\bigstar X\bigstar YZZ$, $\bigstar\bigstar ZYYZ$, $\bigstar\bigstar XYYZ$, $Y\bigstar Y\bigstar YZ$, $\bigstar ZYZZZ$, $X\bigstar ZZYZ$, $X\bigstar XZYZ$, $Y\bigstar YZZZ$, $Y\bigstar ZZYZ$, $Y\bigstar XXYZ$, $Z\bigstar ZXYZ$, $Z\bigstar XXYZ$, $XYZZZZ$, $XZYYZZ$, $XZXYZZ$, $YYZZZZ$, $ZZZZZZ$, $ZZZYZZ$ or $ZZYYZZ$.*

In all these cases, H is immediately colorable from the lists.

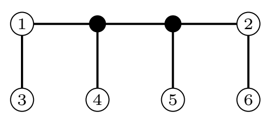

Figure 1: Solid vertices have lists of size 3 and the labeled vertices have lists of size 2.

Case 2. *B is one of $XYY\bigstar YZ$, $ZY\bigstar ZYZ$, $XXYZYZ$, $XXYXYZ$, $XYYZZZ$, $XZZYZZ$, $YYXZYZ$, $YZYYZZ$, $YZZYZZ$, $YZZZZZ$, $ZXYZYZ$, $ZXYYYZ$, $ZXZZYZ$, $ZXYZZZ$, $ZYYYYZ$, $ZYYZZZ$ or $ZYZZZZ$.*

$\mathcal{K}_{Y,3}(YZZZZZ,\infty,4,5,6) \Rightarrow XZYZZZ$, $XZYYZZ$, $XZYZYZ$, $XYZYYZ$(Case 1).
$\mathcal{K}_{Y,5}(YZZZZZ,\infty,4,6) \Rightarrow XZZZYZ$, $XZZYYZ$, $XYYYZZ$(Case 1).
$\mathcal{K}_{Y,2}(YZZZZZ,\infty) \Rightarrow XYZZZZ$(Case 1).
$(214365) \Rightarrow ZYZZZZ$
$\mathcal{K}_{X,3}(ZYYZZZ,2,4,6) \Rightarrow ZZZZZZ$, $ZYZYZZ$, $YZYYYZ$(Case 1).
$\mathcal{K}_{X,4}(ZYYZZZ,2,6) \Rightarrow ZZYYZZ$, $YZZZYZ$(Case 1).
$\mathcal{K}_{X,2}(ZYYZZZ,6) \Rightarrow YYZYYZ$(Case 1).
$(125436) \Rightarrow ZYZZYZ$
$(214365) \Rightarrow YZZYZZ$
$(216345) \Rightarrow ZYYYYZ$
$\mathcal{K}_{X,6}(ZXYZZZ,\infty,1,3,4,5) \Rightarrow YXZYYZ$, $ZXZYYZ$, $YXYYYZ$, $YXZZYZ$, $YXZYZZ$(Case 1).
$\mathcal{K}_{X,4}(ZXYZZZ,\infty,3) \Rightarrow ZXYYZZ$, $ZXZYZZ$(Case 1).
$\mathcal{K}_{X,1}(ZXYZZZ,\infty) \Rightarrow YXYZZZ$(Case 1).
$(125436) \Rightarrow ZXZZYZ$
$(214563) \Rightarrow XZZYZZ$
$(216543) \Rightarrow XYYYYZ$
$\mathcal{K}_{X,3}(XYYZZZ,\infty,4,6) \Rightarrow XYZZZZ$, $XYZYZZ$, $XZYYYZ$(Case 1).
$\mathcal{K}_{X,4}(XYYZZZ,\infty,6) \Rightarrow XYYYZZ$, $XZZZYZ$(Case 1).
$\mathcal{K}_{X,2}(XYYZZZ,\infty) \Rightarrow XZYZZZ$(Case 1).
$\mathcal{K}_{X,6}(XYYZZZ,\infty) \Rightarrow XZZYYZ$(Case 1).
$(216345) \Rightarrow ZXYYYZ$
$\mathcal{K}_{X,4}(YZYYZZ,2,5,6) \Rightarrow YYYZZZ$, $YZYZYZ$, $ZYZYYZ$(Case 1).
$\mathcal{K}_{X,2}(YZYYZZ,1,3,5,6) \Rightarrow ZYYYZZ$, $YYZYZZ$, $YYYYYZ$, $ZZZZYZ$(Case 1).
$\mathcal{K}_{X,3}(YZYYZZ,5) \Rightarrow YZZYYZ$(Case 1).
$(126453) \Rightarrow ZYYZYZ$
$\mathcal{K}_{X,6}(ZXYZYZ,\infty,1,3,4,5) \Rightarrow YXZYZZ$, $ZXZYZZ$, $YXYYZZ$, $XYZZZZ$, $YXZYYZ$(Case 1).
$\mathcal{K}_{X,1}(ZXYZYZ,\infty) \Rightarrow YXYZYZ$(Case 1).
$\mathcal{K}_{X,4}(ZXYZYZ,3,5) \Rightarrow ZXZYYZ$, $ZXYYZZ$(Case 1).
$(214563) \Rightarrow XYYZYZ$
$\mathcal{K}_{X,4}(ZYXZYZ,\infty,2,5,6) \Rightarrow ZYXYYZ$, $ZZXYYZ$, $ZYXYZZ$, $XZYZZZ$(Case 1).
$\mathcal{K}_{X,2}(ZYXZYZ,\infty,5,6) \Rightarrow ZZXZYZ$, $ZZYZZZ$, $YYXYZZ$(Case 1).
$\mathcal{K}_{X,5}(ZYXZYZ,6) \Rightarrow YZXYYZ$(Case 1).
$(143265) \Rightarrow YYXZYZ$
$(216543) \Rightarrow XYYXYZ$
$(256134) \Rightarrow XXYXYZ$
$\mathcal{K}_{X,4}(XXYZYZ,5,6) \Rightarrow XXYYZZ$, $YYZZZZ$(Case 1).
$\mathcal{K}_{X,5}(XXYZYZ,6) \Rightarrow XXZYYZ$(Case 1).

Case 3. *B is one of $XYZXYZ$, $XXXXYZ$, $XXZXYZ$, $XYXXYZ$, $XXYYYZ$, $XXYZZZ$, $YXXZYZ$, $YYZXYZ$, $YZXYZZ$, $ZXYXYZ$, $ZZXYZZ$, $ZXXZYZ$ or $ZYYXYZ$.*

$\mathcal{K}_{Z,\infty}(XXYZZZ, 1, 2, 3) \Rightarrow YXYZZZ$, $XYYZZZ$, $YYYZZZ$(Case 1 and 2).
$(126345) \Rightarrow XXYYYZ$
$(143256) \Rightarrow YZXYZZ$
$(164235) \Rightarrow ZYYXYZ$
$(235146) \Rightarrow ZXXZYZ$
$(256134) \Rightarrow XYXXYZ$
$(364125) \Rightarrow YYZXYZ$
$(365124) \Rightarrow XXZXYZ$
$\mathcal{K}_{X,4}(ZZXYZZ, \infty, 2, 5, 6) \Rightarrow ZZYZZZ$, $ZXYZZZ$, $ZZXZYZ$, $YYXYYZ$(Case 1 and 2).
$\mathcal{K}_{X,2}(ZZXYZZ, \infty) \Rightarrow ZYXYZZ$(Case 1).
$\mathcal{K}_{X,5}(ZZXYZZ, \infty) \Rightarrow ZZXYYZ$(Case 1).
$\mathcal{K}_{X,6}(ZZXYZZ, \infty) \Rightarrow YYXZYZ$(Case 2).
$(216534) \Rightarrow XXXXYZ$
$\mathcal{K}_{X,4}(YXXZYZ, 5, 6) \Rightarrow YXXYZZ$, $ZYYZZZ$(Case 1 and 2).
$\mathcal{K}_{X,5}(YXXZYZ, 6) \Rightarrow ZXXYYZ$(Case 1).
$(125346) \Rightarrow XYZXYZ$
$(135264) \Rightarrow ZXYXYZ$

Case 4. *B is one of $YXZXYZ$.*

$\mathcal{K}_{X,3}(YXZXYZ, 5, 6) \Rightarrow XYXYZZ$, $ZYZYZZ$(Case 1).
$\mathcal{K}_{X,5}(YXZXYZ, 6) \Rightarrow ZXYXYZ$(Case 3).

$\square$