Lemma 1. *Fixer has a winning strategy against Breaker in the chronicled game on G, with superabundant list assignment L where $|L(v_1)| = 2$, $|L(v_2)| = 2$, $|L(v_3)| = 3$ and $|L(v_4)| = 2$.*

Proof. We show that for each possible such list assignment L on G, Fixer has a winning strategy. Up to symmetry, the following cases cover all the possible list assignments that are not an immediate win for Fixer.

Case 1. *$L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 1, 2\}$ and $L(v_4) = \{2, 3\}$.*

Fixer gets a winning strategy by coloring v_3v_4 with 2 and applying Lemma 2.

Case 2. *$L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 1, 3\}$ and $L(v_4) = \{1, 3\}$.*

Fixer gets a winning strategy by coloring v_3v_4 with 3 and applying Lemma 2.

Case 3. *$L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 1, 3\}$ and $L(v_4) = \{1, 2\}$.*

Let S and A_S be as in Lemma 3 using colors 2 and 3. If the components of A_S have vertex sets $\{v_1\}$ and $\{v_2, v_3\}$, then Fixer should swap 2 and 3 at v_1. This results in a position with lists $L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 3\}$, $L(v_3) = \{0, 1, 3\}$ and $L(v_4) = \{1, 2\}$, but then Fixer can edge-color the graph. If the components of A_S have vertex sets $\{v_2\}$ and $\{v_1, v_3\}$, then Fixer should swap 2 and 3 at v_2. This results in a position with lists $L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 1, 2\}$ and $L(v_4) = \{1, 2\}$, but then Fixer can edge-color the graph. If the components of A_S have vertex sets $\{v_3\}$ and $\{v_1, v_2\}$, then Fixer should swap 2 and 3 at v_3. This results in a position with lists $L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 1, 3\}$ and $L(v_4) = \{1, 3\}$, but then Fixer wins by Case 2.

Case 4. *$L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 2, 3\}$ and $L(v_4) = \{1, 2\}$.*

Let S and A_S be as in Lemma 3 using colors 1 and 3. If the components of A_S have vertex sets $\{v_0\}$ and $\{v_2, v_3\}$, then Fixer should swap 1 and 3 at v_0. This results in a position with lists $L(v_1) = \{0, 3\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 2, 3\}$ and $L(v_4) = \{1, 2\}$, but then Fixer wins by Case 1. If the components of A_S have vertex sets $\{v_2\}$ and $\{v_0, v_3\}$, then Fixer should swap 1 and 3 at v_2. This results in a position with lists $L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 1, 2\}$ and $L(v_4) = \{1, 2\}$, but then Fixer can edge-color the graph. If the components of A_S have vertex sets $\{v_3\}$ and $\{v_0, v_2\}$, then Fixer should swap 1 and 3 at v_3. This results in a position with lists $L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 2, 3\}$ and $L(v_4) = \{2, 3\}$, but then Fixer can edge-color the graph.

Case 5. *$L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 1, 3\}$ and $L(v_4) = \{0, 1\}$.*

Let S and A_S be as in Lemma 3 using colors 0 and 2. If the components of A_S have vertex sets $\{v_0\}$ and $\{v_2, v_3\}$, then Fixer should swap 0 and 2 at v_0. This results in a position with lists $L(v_1) = \{1, 2\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 1, 3\}$ and $L(v_4) = \{0, 1\}$, but then Fixer can edge-color the graph. If the components of A_S have vertex sets $\{v_2\}$ and $\{v_0, v_3\}$, then Fixer should swap 0 and 2 at v_2. This results in a position with lists $L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{1, 2, 3\}$ and $L(v_4) = \{0, 1\}$, but then Fixer can edge-color the graph. If the components of A_S have vertex sets $\{v_3\}$ and $\{v_0, v_2\}$, then Fixer should swap 0 and 2 at v_3. This results in a position with lists $L(v_1) = \{0, 1\}$, $L(v_2) = \{0, 2\}$, $L(v_3) = \{0, 1, 3\}$ and $L(v_4) = \{1, 2\}$, but then Fixer wins by Case 3.

□

Lemma 2. *Let G be a multigraph and L a list assignment on G. Suppose we have an edge-coloring π of $H \subseteq G$ where $\pi(xy) \in L(x) \cap L(y)$ for all $xy \in E(H)$. Put $G' := G - E(H)$ and $L'(v) := L(v) - \pi(E_H(v))$ for all $v \in V(G')$. If Fixer has a winning strategy against*

Breaker in the chronicled game on G' with lists L', then Fixer has a winning strategy against Breaker in the chronicled game on G with lists L.

Lemma 3. *Let G be a multigraph, L a list assignment on G and $\alpha, \beta \in \mathrm{Pot}(L)$. Let $S \subseteq V(G)$ be those vertices v with $|\{\alpha, \beta\} \cap L(v)| = 1$. Then there is a graph A_S with vertex set S and $\Delta(A_S) \leq 1$ such that Fixer has a sequence of moves against Breaker in the chronicled game resulting in a list assignment where Fixer has chosen to swap α and β all or none of the vertices in each component of A_S.*

Proof. For each $v \in S$, Fixer should swap α and β at v twice in a row. Now every $v \in S$ is incident to an edge in $\mathcal{C}$; that is, as long as Fixer only does swaps with α and β, Breaker's moves are already foretold in the chronicle. Now add an edge in A_S for each $xy \in \mathcal{C} - \infty$ labeled $\{\alpha, \beta\}$. The lemma follows. $\square$