notes on the Borodin-Kostochka conjecture

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1 Introduction

The goal here is to prove Borodin and Kostochka's conjecture from 1977. If proving the full conjecture is unfeasable, we aim to prove the conjecture for large classes of graphs.

Conjecture 1 (Borodin and Kostochka [2])**.** *Every graph G with $\Delta(G) \geq 9$ satisfies $\chi(G) \leq \max\{\omega(G), \Delta(G) - 1\}$.*

2 Excluded induced subgraphs by d_1-choosability

A graph G is *d_r-choosable* if G can be L-colored from every list assingment L with $|L(v)| \geq d_G(v) - r$ for all $v \in V(G)$. Every graph is d_{-1}-choosable. The d_0-choosable graphs were classified by Borodin [1] and independently by Erdős, Rubin, and Taylor [5] as those graphs whose every block is either complete or an odd cycle (a connected such graph is a *Gallai tree*). Classifying the d_r-choosable graphs for any $r \geq 1$ appears to be a hard problem. However, we can get useful sufficient conditions for a graph to be d_1-choosable. For example, all of the graphs here are d_1-choosable (the vertex color indicates components of the complement): `https://landon.github.io/graphdata/borodinkostochka/offline/index.html`

3 Claw-free graphs

In [4], Cranston and R. proved the Borodin-Kostochka conjecture for claw-free graphs using some of the d_1-choosable graphs in Section 2 combined with the structure theorem for quasi-line graphs of Chudnovsky and Seymour [3].

References

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