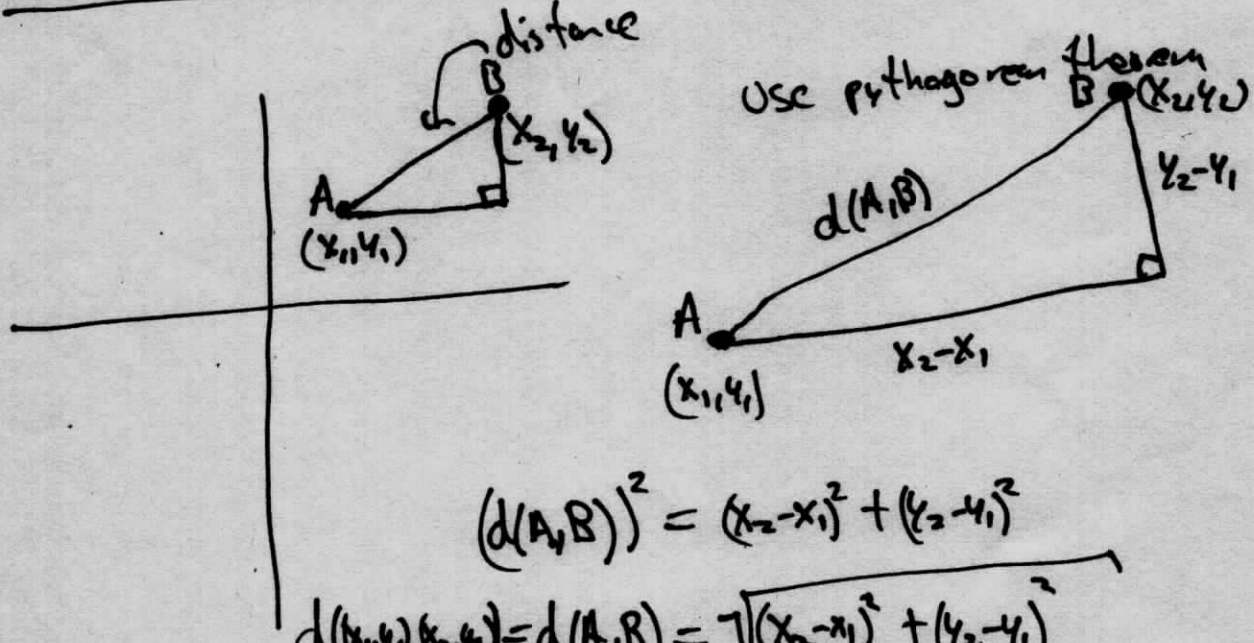
1.8 Distance between two points in the plane.

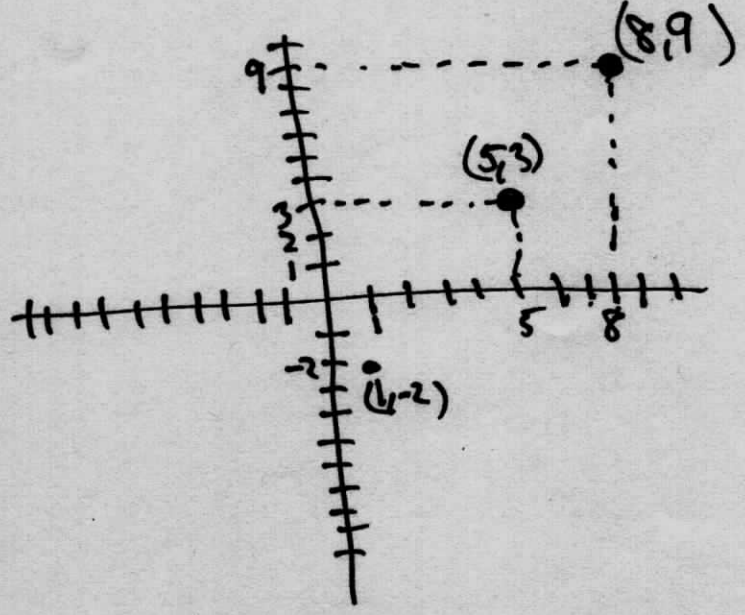


$$(d(A,B))^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

$$d((x_1,y_1),(x_2,y_2)) = d(A,B) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$\boxed{d((x_1,y_1),(x_2,y_2)) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$$

ex Which point is closer to $(5,3)$ $(1,-2)$ or $(8,9)$?



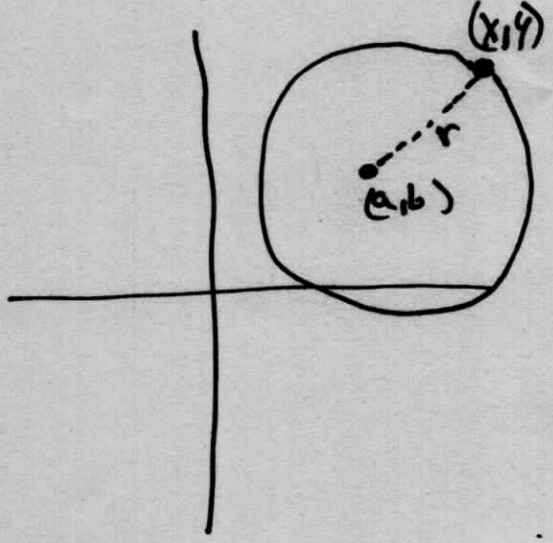
$d((5,3),(8,9)) = \ldots$

$d((5,3),(1,-2)) = \ldots$

"graph" of equation
"picture"

$y = 3x+2$
all (x,y) satisfying equation

Circles



« circle with radius r centered at (a,b)

how can we describe this with an equation?
All points at distance r from (a,b).

All (x,y) such that

$$d((a,b),(x,y)) = r.$$

$$\sqrt{(x-a)^2+(y-b)^2} = r$$

$$\boxed{(x-a)^2+(y-b)^2 = r^2}$$

ex1: draw graph of

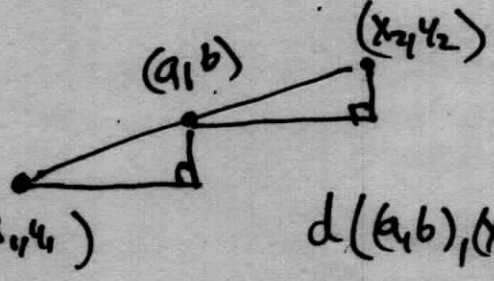
(a) $x^2+y^2 = 25$

(b) $(x-2)^2+(y+1)^2 = 25$

ex2: (a) Find the equation of the circle with radius 3 and center $(2,-5)$

(b) Find the equation of the circle that has the points $(1,8)$ and $(5,-6)$ as the endpoints of a diameter.

need to be able to find the midpoint of a line segment:



$$d((a,b),(x_1,y_1)) = d((a,b),(x_2,y_2))$$

$$\sqrt{(x_1-a)^2+(y_1-b)^2} = \sqrt{(x_2-a)^2+(y_2-b)^2}$$

do some algebra

Could have known this with no work →

$$a = \frac{x_1+x_2}{2}, \quad b = \frac{y_1+y_2}{2}$$

halfway x-direction; halfway y-direction.

Midpoint of line segment between (x_1,y_1) and (x_2,y_2) is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

to Solve ex 2 part b,
- find midpoint P
- find distance from P to one end to get radius
- plug into circle equation

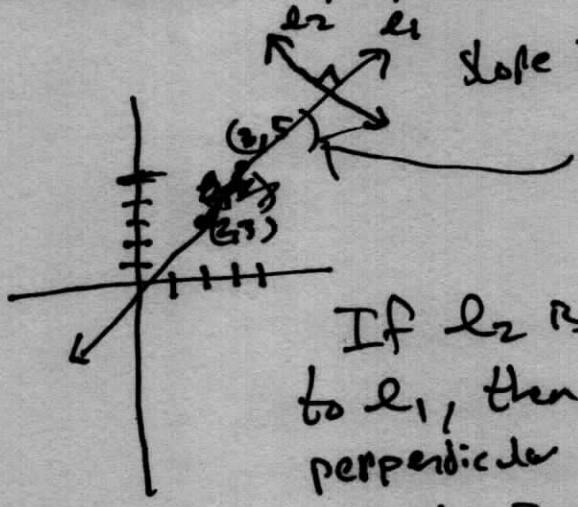
ex 3: Show that the graph of the equation $x^2+y^2+2x-6y+7=0$ is a circle and find its center and radius.

- complete the square twice.

1.10 Perpendicular lines

The graph of the equation $y-3=2(x-2)$ is a line ℓ_1

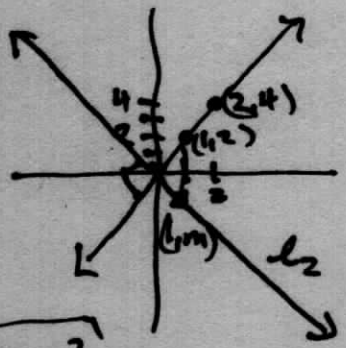
- Find the equation of some line perpendicular to ℓ_1.



slope 2 / so $\frac{\text{rise}}{\text{run}}=\frac{2}{1}$

over 1 up 2 from (2,3) to (3,5)

If ℓ_2 is perpendicular to ℓ_1, then ℓ_2 is also perpendicular to graph of $y=2x$: only the slope matter

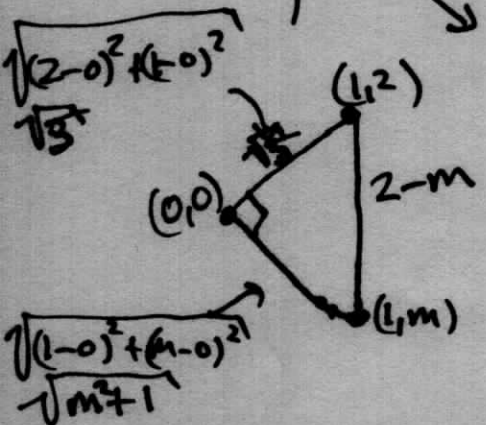


what is slope of ℓ_2?

Say slope of ℓ_2 is m, what is m?

ℓ_2: $y=mx$

point $(1,m)$ is on ℓ_2



$(\sqrt{5})^2+(\sqrt{m^2+1})^2=(2-m)^2$

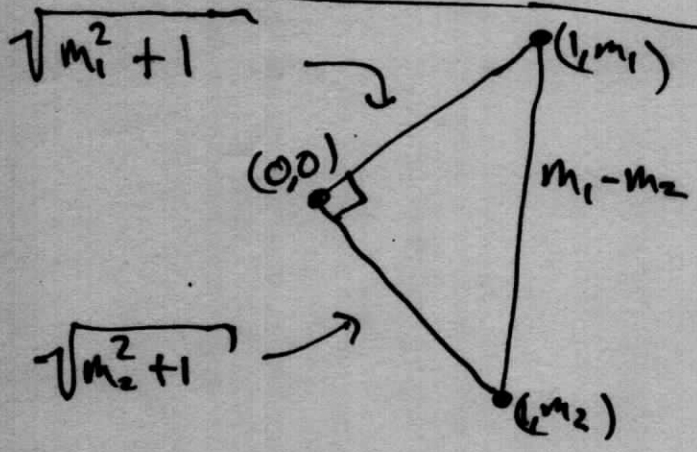
$5+m^2+1=2^2-2\cdot2\cdot m+m^2$

$4m=-2$

$m=-\frac{1}{2}$

Two lines with slopes m_1 and m_2 are perpendicular if and only if

$$m_2 = \frac{-1}{m_1}$$



$$\left(\sqrt{m_1^2+1}\right)^2 + \left(\sqrt{m_2^2+1}\right)^2 = (m_1 - m_2)^2$$

$$m_1^2 + 1 + m_2^2 + 1 = m_1^2 - 2m_1m_2 + m_2^2$$

$$2m_1m_2 = -2$$

$$m_2 = \frac{-2}{2m_1} = \frac{-1}{m_1}$$

Ex. Find an equation of the line that is perpendicular to the line $4x+6y+5=0$ and passes through the point $(1,2)$

Ex. Show that the points $(3,3)$, $(8,17)$ and $(11,5)$ are the vertices of a right triangle.