π-WALK IDEAS

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ABSTRACT.

1. π-WALKS

Unless otherwise noted we mean *proper coloring* when we say *coloring*. The codomain of all our k-colorings is $[k]$.

Definition 1. Let G be a graph and π a k-coloring of G. A π-*walk* is a walk $x_0 x_1 \cdots x_r$ in G with the following two properties:

(1) for $i \in [r]$, $\pi(x_i) \notin \pi\left(N(x_i) - \{x_j \mid j \in [i]\}\right)$;
(2) for $i, j \in [r]$, if $\pi(x_i) = \pi(x_j)$ then $x_{i-1}x_{j-1} \notin E(G)$.

Let G be a graph, π a k-coloring of G and $W := x_0 x_1 \cdots x_r$ a π-walk in G. For $v \in V(G)$ put $i_W(v) := \min\{i \in [r] \cup \{0\} \mid v = x_i\}$ if $v \in V(W)$ and $i_W(v) := -1$ otherwise.

Lemma 1. *The function $\pi_W \colon V(G) \to [k+1]$ given by the following is a $(k+1)$-coloring of G:*

$$\pi_W(v) := \begin{cases} \pi(v) & \text{if } i_W(v) = -1; \\ \pi(x_{i_W(v)+1}) & \text{if } 0 \leq i_W(v) \leq r-1; \\ k+1 & \text{if } i_W(v) = r. \end{cases}$$

Lemma 2. *Any π-walk beginning at a vertex $v \in V(G)$ with $|\pi^{-1}(\pi(v))| = 1$ is a path or a cycle.*