

## Benchmark 1: Analytical. Solitary Wave on a Simple Beach

The goal of this benchmark is to verify a model by comparing numerical and analytical solutions that describe propagation and runup of a 1-D solitary wave. The analytical solution to the solitary wave runup on a sloping beach was derived by Synolakis (1986, 1987). In this problem, the wave of height  $H$  is initially centered at distance  $L$  from the beach toe and is schematically shown in Figure 1. The beach bathymetry consists of an area of constant depth  $d$ , connected to a plane sloping beach of angle  $\beta = \text{arccot}(19.85)$ . Note that the  $x$  coordinate increases monotonically seaward,  $x = 0$  is the initial shore location, and the toe of the beach is located at  $x = X_0 = d \cot \beta$ .

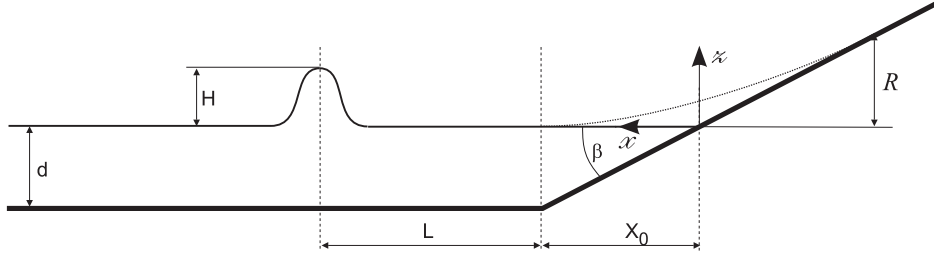


Figure 1: Non-scaled sketch of a canonical beach with a wave climbing up.

The value of  $L = \text{arccosh}(\sqrt{20})/\gamma$  is the half-length of the solitary wave, and the initial depth profile is given by

$$\eta(x, 0) = H \text{sech}^2(\gamma(x - X_1)/d), \quad (1)$$

where  $X_1 = X_0 + L$ , and  $\gamma = \sqrt{3H/4d}$ .

More information regarding this benchmark can be found in (Synolakis et al., 2007), or at the web-site <http://nctr.pmel.noaa.gov/benchmark/Analytical/> of the NOAA Center for Tsunami Research.

This benchmark problem is focused on modeling runup of an incident non-breaking solitary wave such that its height  $H$  satisfies the condition:  $H/d = 0.019$ . In the computer experiment, this wave can be simulated by specifying the initial wave profile according to formula (1) and by setting the initial wave-particle velocity, following Titov and Synolakis (1995) as:

$$u(x, 0) = -\sqrt{g/d} \eta(x, 0).$$

It is recommended to set the non-reflective boundary condition at the left side of the computational domain.

Figure 2 shows profiles and time series of the water level at the left and right plots, respectively. Extreme positions of the shoreline - the maximum runup and rundown occur  $t \approx 55(d/g)^{1/2}$  and  $t \approx 70(d/g)^{1/2}$ , respectively. The water level dynamics at the locations  $x/d = 0.25$ , near the initial shoreline, predicts that the water retreats from  $t \approx 67(d/g)^{1/2}$  to  $t \approx 82(d/g)^{1/2}$ . This point location temporally becomes dry, while the point  $x/d = 9.95$  remains wet throughout the entire length of the computer experiment.

### To accomplish this benchmark, it is suggested to:

1. Compare the numerically and analytically computed water level profiles at  $t = 25(d/g)^{1/2}$ ,  $t = 35(d/g)^{1/2}$ ,  $t = 45(d/g)^{1/2}$ ,  $t = 55(d/g)^{1/2}$ , and  $t = 65(d/g)^{1/2}$ .  
Note that the numerical model must be run in appropriate (i.e. non-linear, non-dispersive, no friction) mode for the comparison and verification purposes.
2. Compare the numerically and analytically computed water level dynamics at locations  $x/d = 0.25$  and  $x/d = 9.95$  during propagation and reflection of the wave,
3. Compute the maximum runup,
4. (Optional) Show convergence of the numerical solution to the analytical one at  $t = 55\sqrt{d/g}$  and  $t = 70\sqrt{d/g}$ .

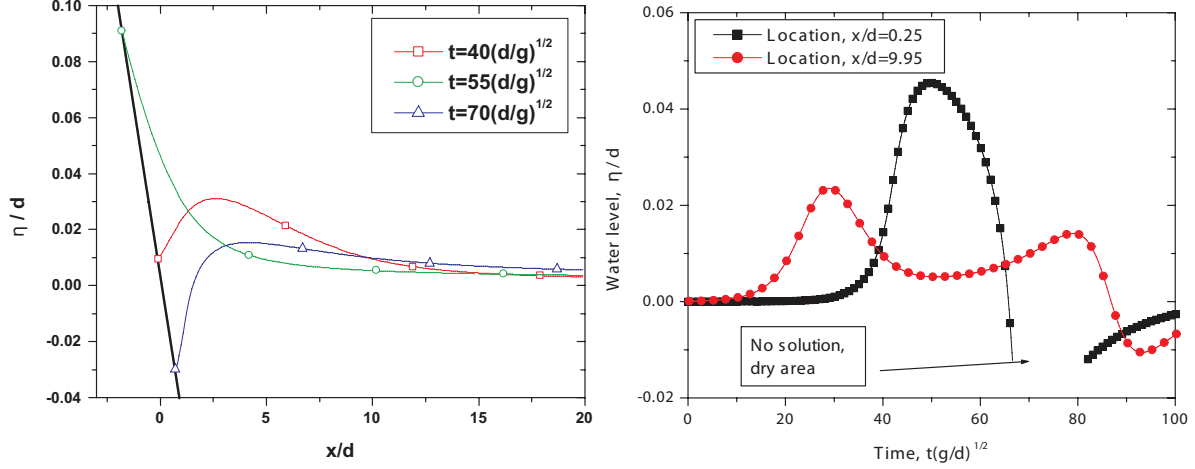


Figure 2: Left plot: the water level profiles during runup of the non-breaking wave in the case of  $H/d = 0.019$  on the 1:19.85 beach.. Right plot: the water level dynamics at two locations  $x/d = 0.25$  and  $x/d = 9.95$ . The analytical solution is according to Synolakis (1986).

### Description of the data files

The analytically computed water level profiles at  $t = 25(d/g)^{1/2}, \dots, t = 70(d/g)^{1/2}$  are provided in file `canonical_profile.txt`. The analytically computed water level dynamics at two locations  $x/d=0.25$  and  $x/d=9.95$  are provided in file `canonical_ts.txt`. In both files the units are non-dimensional.

### References

- Synolakis, C., 1986. The Runup of Long Waves. Ph.D. thesis, California Institute of Technology, Pasadena, California, 228 pp..
- Synolakis, C., 1987. The runup of solitary waves. *Journal of Fluid Mechanics* 185, 523–545.
- Synolakis, C., Bernard, E., Titov, V., K  noğlu, U., Gonz  lez, F., 2007. Standards, criteria, and procedures for NOAA evaluation of tsunami numerical models. OAR PMEL-135 Special Report, NOAA/OAR/PMEL, Seattle, Washington, 55 pp..
- Titov, V., Synolakis, C., 1995. Evolution and runup of breaking and nonbreaking waves using VTSC2. *Journal of Waterway, Port, Coastal and Ocean Engineering* 121 (6), 308–316.