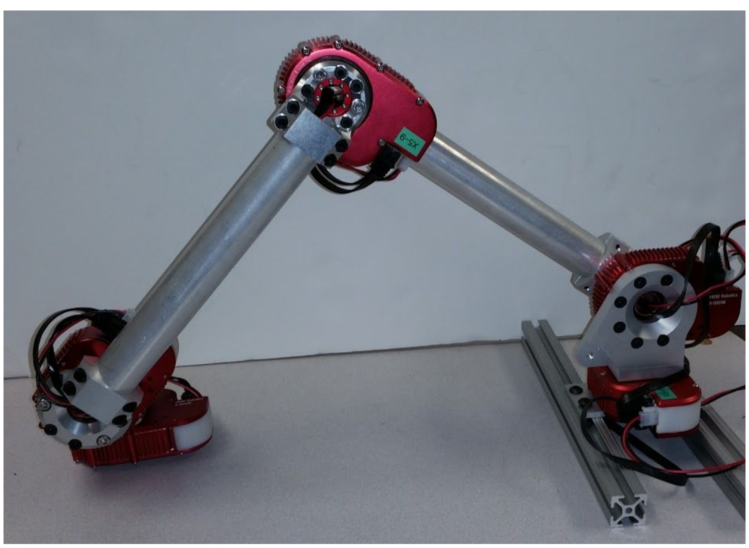
Engineering Optimization Project Proposal: Robot Limb Design Optimization
Julian Whitman, Ryan Edmonson, Ky Woodard, Alex Kowalski

Abstract

Modular robotic systems have the potential to be adapted to a wide variety of tasks and to increase reusability of individual components; however, no practical modular robots currently exist that allow manufacturers this flexibility. Instead, companies are forced to buy highly specialized and expensive assemblies that are not suited for reuse and can be cost inhibitive for smaller organizations resulting in a time to production of weeks if not months. A solution to this problem is to implement modular robots that are easily reconfigurable and introduce software to solve the complex setup process given a relatively simple set of conditions about the workspace. The optimization process involved in this software would seek reduction of cost, torque, and distance from specified configurations by analyzing necessary joints, length of intermediary links, and the base configuration. Given this objective, the expected result of the modularity optimization project is a system that reacts similarly to if not better than a roboticist who is familiar with the modular robots and assembling them for general tasks.

Introduction

Modular robotic systems have the potential to be adapted to varying tasks using a single platform and enable customizable robots to be developed faster and more economically than conventional robots. Currently, no practical modular robots exist, and even the best engineering experience, calculations, and intuition do not reveal how to arrange modules for a given task. An optimally designed modular manipulator would be more effective and require fewer redundant components than would a generic manipulator, and one could easily reconfigure its joints, links, or end effector to respond to a change in task.



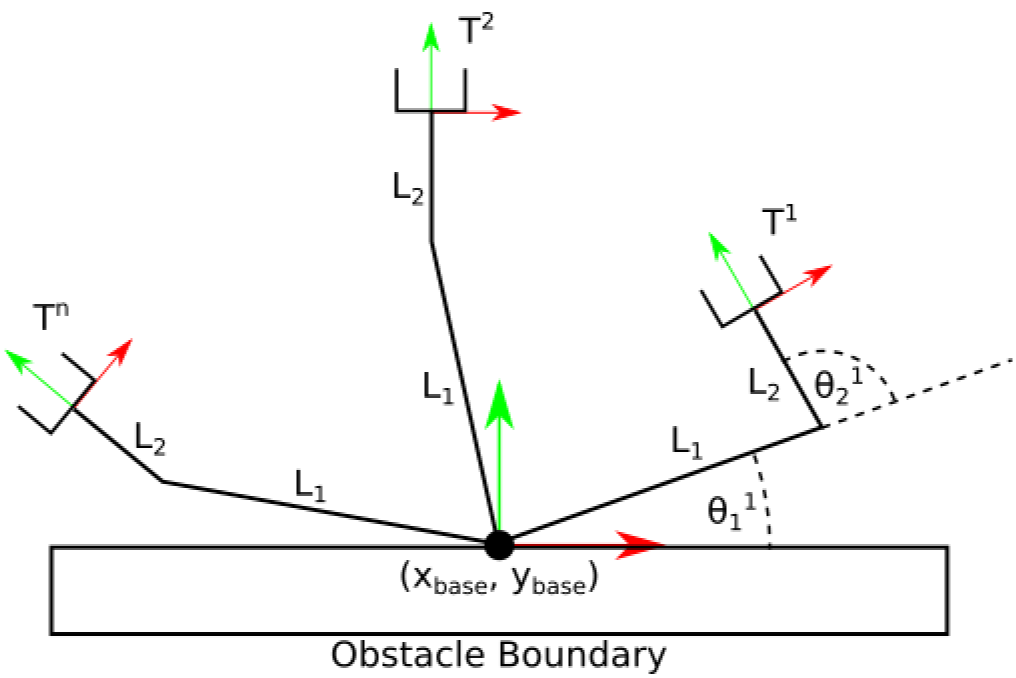
A five degree-of-freedom manipulator arm constructed from modular actuators and easily resized static links.

A modular robot arm can be reconfigured frequently and easily. In this work we assume that the user's desired task has been specified in terms of a series of poses in a workspace that should be reached by an end effector of a fixed base manipulator. Some traditional manipulator design principles would not work well for an arm constructed from modules. For example, an often-used kinematic chain of a 2-joint shoulder, 1-joint elbow, and 3-joint wrist configuration which maximizes reachable workspace [1] requires three distal joints. If each joint is a module with relatively high mass, then the shoulder joints will have to operate at high torque while the wrist joints are underutilized.

In other cases, only a few distinct end effector poses must be reached. Rather than design a manipulator with full workspace coverage, it could be less costly to use a minimal degree-of-freedom arm. However, even experienced roboticists may have difficulty identifying the the necessary design for an arbitrary sequence of poses. In this work, we will find minimal arm designs for such scenarios.

We will make use of gradient-based constrained optimization techniques. Some other past work makes use of genetic algorithms [1] which are suited for instances in which the design arrangements variables are all integer, i.e. the link lengths and mounting angles must be chosen from a discrete set. Others have used simulated annealing to find lengths of segments which optimize for manipulability criterion, [3] but which do not seek to minimize manipulator cost, are not usually looking to minimize number of joints, or do not work with the differences inherent in designs using modular components

Problem Statement



An illustration of the problem with two joints, J=2

Given a set of *N* poses in the workspace, we seek the lengths and joint angles for an arm with *J* joints. The proposed optimization problem is to minimize the position and orientation error of the end effector to the waypoints specified with respect to the number of joints and length of links and subject to joint constraints and link length limits. The decision variables **x** joint angles (θ) for each waypoint, and link lengths (L) which are the same for each pose. The objective function f(**x**) is to minimize the position and orientation error of the end effector, which consists of the squared distance to the final pose at each waypoint and the Frobenius norm of the orientation offset from the desired pose effector angle, where the rotations are parameterized by rotation matrix R. The inequality constraints g(**x**) ensure that the joints do not exceed their limits and a lower bound on the link lengths so the values are physically meaningful. There are constraints and costs on the joint torques ($\boldsymbol{\tau}$). We may also add costs on changing the joint angles significantly between poses, and on using long links, which we anticipate to reduce the number of local minima. If time permits, we will also add a constraint that there be no physical collisions between the arm and some obstacles with positions specified in the workspace.

Negative null form:

$$\min_x f(x) = \sum_{i=1}^{N} \left((x_i - x_{fk}(x))^2 + (y_i - y_{fk}(x))^2 + ||I - R_{fk}(x) R_i^T||_F \right)$$

W.r.t

$$x = (\theta_1^1, \dots \theta_J^1, \dots \theta_1^N, \dots \theta_J^N, L_1, L_2, \dots L_J)^T \in \mathbb{R}^{nJ+J}$$

S.t.

$$-\theta_j^i - \theta_{j,min} \leq 0, \quad \forall i = \{1 \dots N\}, \quad j = \{1 \dots J\}$$
$$\theta_j^i - \theta_{j,max} \leq 0, \quad \forall i = \{1 \dots N\}, \quad j = \{1 \dots J\}$$
$$-L_j \leq 0, \quad j = \{1 \dots J\}$$
$$L_j - L_{max} \leq 0, \quad j = \{1 \dots J\}$$

Minimize the end effector distance from the desired poses;
With respect to joint angles and link lengths;
Subject to joint angle limits and positive lengths

Additional costs:

$$f_\tau = \sum_{j=1}^{J} \sum_{i=1}^{N} (\tau_j^i)^2 \qquad f_L = \sum_{j=1}^{J} (L_j) \qquad f_\theta = \sum_{j=1}^{J} \sum_{i=1}^{N} \sum_{k=i}^{N} (\theta_j^i - \theta_j^k)^2$$

Cost on high joint torques, Cost on link lengths, Cost on changing angles between poses.

Additional constraints:

$$\tau_j^i - \tau_{max} \leq 0, \quad \forall i = \{1 \dots N\}, \quad j = \{1 \dots J\}$$

Prevent motor torque saturation.

Model Parameters

Symbol	Description	Value	Unit
θ_{min}	Joint angle lower bound	**-π**	Radians
θ_{max}	Joint angle upper bound	**π**	Radians
x_i	End effector x-position desired for pose i	Depends on task	m
y_i	End effector y-position desired for pose i	Depends on task	m
R_i	End effector rotation matrix desired for pose i	Depends on task	SO(2)
L_{max}	Upper bound on link length	1	m
$\boldsymbol{\tau}_{max}$	Maximum joint torque	7	N-m

Analysis of Problem Statement

The complexity of this optimization problem is highly dependent on the number of configuration waypoints that are included. Due to this, the number of decision variables scales at J*(N+1) with N being the number of initial goal configurations and J the number of joints in the current optimization. Additionally, the inequality constraints scale at the rate of J*(2*N*+1) since every joint has both an upper and lower bound and the joint lengths have natural limitations due to the attachment hardware between joints. Although the modules for this project will have the ability to continuously rotate, the power and communication lines between modules limit the amount of rotation possible. This however is still a practical constraint since the alternative is to design and build a cost inhibitive slip ring joint to solve this problem. An example feasible point would be a series of links with zeros joint angles and maximum link lengths, x = [0, 0, . . ., L_{max}, L_{max} …]. Maximum link lengths and torques are practical constraints, while positive link lengths is a natural constraint.

This project does not take into consideration some physical limits, such as gear train backlash, communication/power limitations, or noise. Since the search range of the number of joints in the optimization will be limited, it is safe to assume that these effects will not contribute considerably to the result, and a solution that is good enough for all practical purposes can still be found. In order to find attainable values that make up the pareto set, a parametric study across the joint values will be performed to capture and compare all of the minima from each configuration. As posed, the optimization problem would be of the mixed integer nonlinear programming form since the number of joints is fixed to an integer value. In order to circumvent this potential issue, the optimization will be run separately over a series of increasing integer number of joints and then compared retrospectively. Since the expected solutions will require a relatively low number of joints, this is a practical approach to reducing the mixed integer problem to a standard nonlinear programming (NLP) one. The system will overall be nonconvex, and it is expected that there will be many local minima since robotic arms of more than a couple joints can have infinite solutions to the inverse kinematics problem. Based on the initial findings, more constraint functions might need to be imposed to restrict the number of solutions to a more workable range.

The objective function is expected to be smooth and continuous, but the optimizer may seek singular configurations. Although these may seem ideal from an analytical sense, the small region in which they are ideal quickly disappears in a real world scenario where small errors and noisy sensors will prevent operation in this regime. One of the supplemental costs described above may need to be imposed on the singularities themselves to reduce the likelihood of convergence to them.

References

[1] Paden, Brad, and Shankar Sastry. "Optimal kinematic design of 6R manipulators." *The International Journal of Robotics Research* 7, no. 2 (1988): 43-61.

[2] Yang, Guilin, and I-Ming Chen. "Task-based optimization of modular robot configurations: minimized degree-of-freedom approach." *Mechanism and machine theory* 35, no. 4 (2000): 517-540.

[3] Hammond, Frank L. "Configuring kinematically redundant robotic manipulators to increase effective task-specific motion resolution." In *2011 IEEE International Conference on Mechatronics and Automation*, pp. 34-39. IEEE, 2011.