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March 8, 2020

Dear Editor,

Thank you very much for your email and the review comments on our paper:

Ref. Number PHYCOM-2019-745
"Power Allocation and Subcarrier Pairing for SWIPT in Cognitive Relay System"

which was required major revisions before resubmission. Please find the substantially revised paper, which we would like to resubmit for possible publication in *Physical Communication*.

As kindly suggested by you and the reviewers, the paper has been seriously revised in accordance to the constructive and helpful comments from you and the reviewers for improving the quality further. All the modifications in the revision have been marked in red. Remarkably, on the basis of the reviewers' comments, we have improved the system model and added simulation contrast, but the ideas that our paper intend to express are not affected. For more information, please see the detailed Responses to Reviewers.

We would like to express our sincere appreciation to you for your prompt and professional handling of our manuscript.

Looking forward to hearing from you.

Yours sincerely,

Zhixin Liu

Response to Reviewers*

We would like to thank the reviewers for their careful assessments and constructive comments on our submission, particularly the time being spent. We take the reviewers' views very seriously, and have made every possible effort in order to address the concerns raised by the reviewers and modify the paper according to his/her suggestions and comments. We have corrected all the errors and typos. The details are explained below:

We hope this revised version is now suitable for publication in **Physical Communication**.

*For the paper, Ref. Number PHYCOM-2019-745, "Power Allocation and Subcarrier Pairing for SWIPT in Cognitive Relay System," submitted to *Physical Communication*.

Response to Reviewer 1

We would like to thank the reviewer for spending his/her time to assess the paper, and make some very constructive and detailed informative comments provided in the review. Our responses are given as follows:

1. **Question**: The scenario as well as the solution seem not new. What is the contribution of this paper?

   Answer: Thanks for the reviewer's comments! Wireless cognitive relay networks are common research scenarios. SWIPT is a new type of communication. SWIPT technology breaks through the limitations of traditional wired power supply methods, and is expected to achieve green and sustainable wireless communication. There are rich works on the resource optimization for SWIPT and relay network.

   Ref. [1] derived and optimized the achievable data rate of the PS-SWIPT-based point-to-point and two-hop DF relay channels subject to the non-decreasing decoding costs at the receiving nodes. The performance of the PtP system, relay channel with decode-and-forward (DF) and amplify-and-forward (AF) relay schemes were discussed. Ref. [2] jointly designed the energy interleaving and the constellation rotation in a multi-user NOMA-SWIPT system. In their design, the energy interleaving tensor as well as the constellation rotation angles were optimized for maximizing the actual energy carried by the superposition signals. Ref. [3] studied beamforming design problems for multi-cell multi-user downlink networks with SWIPT. Their work maximized the efficiency of energy harvesting, while guaranteeing the quality of service constraints of each information decoding user. For energy harvesting operations, they established ideal linear models and practical non-linear models for research. Cognitive Radio (CR) is a promising design technology to increase the utilization of spectrum and dedicated frequency bands [4]. The energy harvesting technology is effective to alleviate spectrum and energy stress in cognitive radio networks [5]. Therefore, we study the cognitive relay cooperative network based on SWIPT.

   Most of existing studies did not consider the uncertainty of the channel environment. This work takes into account the practical scenario, where the uncertainty of the channel gains exists. The robust resource allocation method is developted to approach practical applications. The joint resource optimization problem is formulated, which attempts to optimize jointly the relay transmission power, ratio and subcarrier allocation. The proposed allocation problem is distinguished from the previous allocation problems since the subcarrier allocation and energy harvesting are combined. Therefore, the energy efficiency of the secondary system can be effectively improved. To make the formulated nonconvex integer programming solvable, the joint methods including problem decomposition, Dinkelbach method and Hungarian algorithm are developed. And the practical power allocation and subcarrier allocation algorithm is given.

The main contributions of this paper are summarized as follows:

- The joint resource allocation problem is formulated, which attempts to optimize jointly the relay transmission power, ratio and subcarrier allocation. The proposed allocation problem is distinguished from previous allocation problems since the subcarrier allocation and energy harvesting are combined. Therefore, the energy efficiency of the secondary system can be effectively improved.
- The robust power allocation scheme based on imperfect CSI is proposed. The scheme attempts to tackle the practical situations that channel state information is dynamic and perfect CSI may not be available due to channel estimation error or quantization error.
- To tackle the original joint allocation problem, which is nonconvex integer programming, we use the Dinkelbach method to convert the objective function into a subtractive form and decompose the problem into sub-problems. The subcarrier allocation problem is solved by Hungarian algorithm.

[1] M. Abedi, H. Masoumi, and M. J. Emadi. Power Splitting-Based SWIPT Systems With Decoding Cost. *IEEE Wireless Communications Letters*, 8(2): 432-435, 2019.

[2] Y. Zhao, J. Hu, Z. Ding, and K. Yang. Joint Interleaver and Modulation Design For Multi-User SWIPT-NOMA. *IEEE Transactions on Communications*, 67(10): 7288-7301, 2019.

[3] S. Jang, H. Lee, S. Kang, T. Oh, and I. Lee. Energy Efficient SWIPT Systems in Multi-Cell MISO Networks. *IEEE Transactions on Wireless Communications*, 17(12): 8180-8194, 2018.

[4] J. Mitola, and G. Q. Maguire. Cognitive radio: Making Software Radios more Personal. *IEEE Personal Communications*, 6(4): 13–18, 1999.

[5] X. Lu, P. Wang, N. Dusit, and H. Ekram. Dynamic Spectrum Access in Cognitive Radio Networks with RF Energy Harvesting. *IEEE Wireless Communications*, 21(3): 102-110, 2014.

2. **Question**: In Introduction, related works on SWIPT CR/SWIPT relay CR are not introduced properly.

   Answer: Thanks for the reviewer's comments! We have revised the presentation on SWIPT CR/SWIPT relay CR and added some relevant literature. The revised presentation is described as follows:

   "Cognitive radio allows secondary users (SUs) to opportunistically share the spectrum of the primary user (PU) while satisfying the quality of service (QoS) requirements of the PU. Meanwhile, energy harvesting is used as a renewable energy source, in order to ensure self-sustainability and reduce the chances of short network life caused by the hardware failure[19], [20]. [21] studied the performance optimization of cognitive radio networks with RF energy harvesting capabilities for secondary users. When the channel was idle, the secondary user transmitted data packets on the channel licensed to the primary user. When the channel was busy, the secondary user obtained RF energy from the transmission of the primary user. [22] proposed a multi-channel selection strategy for RF energy harvesting cognitive radio networks, where multiple channels were available. Each secondary user harvested energy from a channel with an active neighbouring primary user and transmitted data on another channel which was not occupied by neighbouring primary users."

[21] D. T. Hoang, D. Niyato, P. Wang, and D. I. Kim. Performance Optimization for Cooperative Multiuser Cognitive Radio Networks with RF Energy Harvesting Capability. *IEEE Transactions on Wireless Communications*, 14(7): 3614-3629, 2015.

[22] M. Xu, M. Jin, Q. Guo, and Y. Li. Multichannel Selection for Cognitive Radio Networks with RF Energy Harvesting. *IEEE Wireless Communications Letters*, 7(2): 178-181, 2018.

3. **Question**: In System Model, why the S node does not interfere with the PU?; If I did not miss something, the equations (12),(13),(14) are not used. However, the total rates received by the SUs cannot exceed the rate received at the R node; The considered system model is more like a cognitive system not a cognitive relay system.

   Answer: Thanks for the reviewer's comments! We are sorry for our negligence. In the original system model diagram, the line type of the S-to-PU interference link was incorrectly drawn as the line type of the transmission link. The revised system model is shown in Fig. 5. Also, we have fixed the error in the revision.

   In the cognitive radio system, the primary network and the secondary network coexist, and there are two typical working modes for the cognitive users, overlay and underlay. In this paper, the underlay mode is assumed, thus the secondary users share the same frequency resource and the caused interference to the primary user must be below the allowed level. The signal transmission of primary user is not considered and the source node of primary user is omitted in the figure. The interference constraint is described as

$$\sum_{k=1}^{K}\sum_{i=1}^{N}\beta_k^i P_{r,k}^i g_{rpu,k}^i \leq I_{th}, \tag{0.1}$$

   and it is the constraint $C2$ in the optimization problem (19).

   In this paper, the decode-and-forward(DF) relay system is considered. The first hop is from the source node relay node, the SNR and the rate are given in Eq.(12) and Eq.(14), respectively. The second hop is from the relay node to destination node, the SNR and the rate are given in Eq.(13) and Eq.(15), respectively. The achievable capacity from source node S the secondary node SU_k depends on the bottleneck link of two hops, i.e.

$$C_k = \min\{C_{sr}, C^i_{rsu,k}, \} \tag{0.2}$$

From Eq.(12) and Eq. (14), the rate of the first hop depends on the transmitting power P_s. It is assumed that the second hop is the bottleneck link in this paper, because we consider the donwlink transmission and the power of base station, P_s, is usually not constrained. The constraint $C_{sr} > C^i_{rsu,k}$ can be satisfied by setting P_s. Hence, we focus on the resource allocation of second hop in the relay network. Actually, the fixed power also affects the resource allocation of second hop, because it is included in the total power consumption as shown in Eq.(16), which is the denominator of Energy Efficiency in Eq.(18).

The corresponding parts mentioned in the revision are given as follows:

"As shown in Fig. 5, we consider a two-hop cooperative network consisting of one source node named S, one relay node named R, one primary user PU and K secondary users (SU_k, k=1, 2, ..., K). The OFDMA technique is adopted to implement transmission from the relay to SU_k and the total bandwidth W is equally divided into N orthogonal subcarriers. It is assumed that each node is equipped with only one single antenna and the relay operates in HD mode assisted DF protocol. The decode-and-forward(DF) relay system is considered. The first hop is from the source node relay node, the second hop is from the relay node to destination node. There is no direct transmission link between S and SU_k."

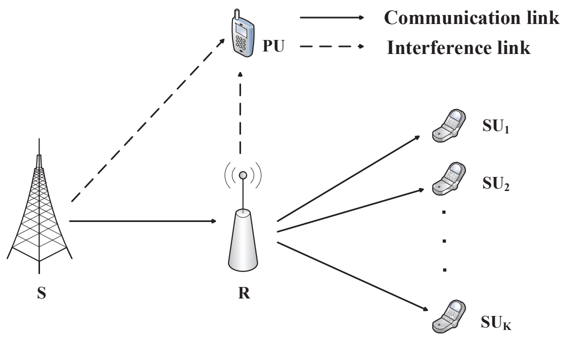

Figure 1: System model.

4. **Question**: In Section III, the problem (27) may be infeasible. How to determine that the problem is feasible? How to derive the problem (47) is unclear? The convergence of the

proposed Algorithm 1 is unclear. The complexity of Algorithm 1 is unclear; the worst-case solution may underestimate the system performance, is there any other solution to deal with the uncertain CSI?

Answer: Thanks for the reviewer's constructive comments!

In this system, the considered channel gain in the communication link is imperfect, and there is always channel estimation error in practice. Moreover, it is difficult to determine the channel estimation error immediately. So the worst-case method is introduced to solve this problem. In the worst-case method, we need to consider the worst-case estimation error. The conversion from $OP1$ to $OP2$ is to transform the objective function and constraints into a form that can still meet the minimum requirements in the worst-case condition. In addition, the channel gains of the communication link and the interference link are converted according to the worst case. Specially, considering the boundaries of channel gains, three variables, $C_{rsu,k}^{i*}, SNR_{rsu,k}^{i*}, P_h^*$, are introduced as shown in Eq.(23), Eq.(25), Eq.(26), respectively. By replacing the constrains $C9 - C12$, the optimization problem $OP3$ is achieved, which is problem (27). The above conversion follows the rules of the worst-case method and it is derived in reasonable steps.

Further, the subsequent transformation and solving process are given in the next subsection, "C Problem solution". The main ideas to solve the problem are as follows. Since the fractional form of the objective function is the ratio of two functions with variables P and ρ_k^{i*}, the optimization problem is non-convex. First, the objective function of the original problem is transformed into an equivalent function in the form of subtraction by fractional programming. Secondly, the proposed problem is reconstructed into two convex sub-problems. Next, the Karush-Kuhn-Tucker (KKT) condition is used to determine the optimal power allocation and power allocation ratio for given subcarrier allocation β_k^i, and the optimal solution is determined by Lagrangian dual decomposition. Finally, the optimal β_k^i is determined based on the power and PS ratio obtained in the previous step. Therefore, the problem is solvable.

In the derivation before problem (47), we use the worst-case method to convert the original problem with uncertainty into a certainty problem. After a series of derivations, the non-convex problem is transformed into a convex problem. After mutual iteration, the optimal $P_{r,k}^i$ and ρ_k^i are obtained. In order to avoid co-channel interference at this time, we substitute $P_{r,k}^{i*}$ and ρ_k^{i*} into equations (35) and (36) to obtain problem (47). Problem (47) is obtained by sorting, that is, a portion that requires subcarrier allocation and another portion that does not, are added together. In this way, it only needs to solve the part that requires subcarrier allocation. We have revised the original expression and the revised part is marked in red.

Now we prove the convergence of the algorithm. Since the transformed problem is a standard convex optimization problem, the optimal value can be determined when the partial derivative of the Lagrangian function $\frac{\partial \mathcal{L}_1}{\partial P^i_{r,k}} = 0$ and $\frac{\partial \mathcal{L}_2}{\partial \rho^i_k} = 0$. Hence,we have

$$\begin{aligned}\frac{\partial \mathcal{L}_1}{\partial P^i_{r,k}} &= \frac{W\beta^i_k\rho^i_k H^i_{rsu,k}}{2(P^i_{r,k}\rho^i_k Hrsu,k^i + \sigma^{i2}_{rsu,k})\ln 2} - q(\beta^i_k - \eta\beta^i_k(1-\rho^i_k)H^i_{rsu,k}) \\ &+ \frac{\mu_k W\rho^i_k H^i_{rsu,k}}{2(P^i_{r,k}\rho^i_k H^i_{rsu,k} + \sigma^{i2}_{rsu,k})\ln 2} + \frac{\theta_k\rho^i_k H^i_{rsu,k}}{\sigma^{i2}_{rsu,k}} - \gamma\beta^i_k G^i_{rpu,k} \\ &- \lambda\beta^i_k + \nu_k\eta(1-\rho^i_k)H^i_{rsu,k} = 0,\end{aligned} \tag{0.3}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_2}{\partial \rho^i_k} &= \frac{W\beta^i_k P^i_{r,k} H^i_{rsu,k}}{2(P^i_{r,k}\rho^i_k H^i_{rsu,k} + \sigma^{i2}_{rsu,k})\ln 2} - q\eta\beta^i_k P^i_{r,k} H^i_{rsu,k} \\ &+ \frac{\mu_k W P^i_{r,k} H^i_{rsu,k}}{2(P^i_{r,k}\rho^i_k H^i_{rsu,k} + \sigma^{i2}_{rsu,k})\ln 2} + \frac{\theta_k P^i_{r,k} H^i_{rsu,k}}{\sigma^{i2}_{rsu,k}} - \nu_k\eta P^i_{r,k} H^i_{rsu,k} - \omega_k = 0.\end{aligned} \tag{0.4}$$

By the aforementioned equation, we can obtain

$$\begin{aligned}P^i_{r,k} &= \left[\frac{W(\beta^i_k + \mu_k)}{(q(\beta^i_k - \eta H^i_{rsu,k}(1-\rho^i_k)) - \frac{\theta_k\rho^i_k H^i_{rsu,k}}{\sigma^{i2}_{rsu,k}} + \gamma\beta^i_k G^i_{rpu,k} + \lambda\beta^i_k + \nu_k\eta(1-\rho^i_k)H^i_{rsu,k})2\ln 2} - \frac{\sigma^{i2}_{rsu,k}}{\rho^i_k H^i_{rsu,k}}\right]^+ \\ &= \frac{A_1}{A_2\rho^i_k + A_3} - \frac{A_4}{\rho^i_k},\end{aligned} \tag{0.5}$$

where $A_1 = W(\beta^i_k + \mu_k)$, $A_2 = (q\eta H^i_{rsu,k} - \frac{\theta_k H^i_{rsu,k}}{\sigma^{i2}_{rsu,k}} - \nu_k\eta H^i_{rsu,k})2\ln 2$, $A_3 = (q\beta^i_k - q\eta H^i_{rsu,k} + \gamma\beta^i_k G^i_{rpu,k} + \lambda\beta^i_k + \nu_k\eta H^i_{rsu,k})2\ln 2$, and $A_4 = \frac{\sigma^{i2}_{rsu,k}}{H^i_{rsu,k}}$.

$$\begin{aligned}\rho^i_k &= \frac{W(\beta^i_k + \mu_k)}{(q\eta\beta^i_k P^i_{r,k} H^i_{rsu,k} - \frac{\theta_k P^i_{r,k} H^i_{rsu,k}}{\sigma^{i2}_{rsu,k}} + \nu_k\eta P^i_{r,k} H^i_{rsu,k})2\ln 2} - \frac{\sigma^{i2}_{rsu,k}}{P^i_{r,k} H^i_{rsu,k}} \\ &= (\frac{A_1}{B_1} - A_4)\frac{1}{P^i_{r,k}} \geq 0,\end{aligned} \tag{0.6}$$

where $B_1 = (q\eta\beta^i_k P^i_{r,k} H^i_{rsu,k} - \frac{\theta_k P^i_{r,k} H^i_{rsu,k}}{\sigma^{i2}_{rsu,k}} + \nu_k\eta P^i_{r,k} H^i_{rsu,k})2\ln 2$. We can obtain,

$$C_1 = (\frac{A_1}{B_1} - A_4) \geq 0, \tag{0.7}$$

Substituting (0.6) into (0.5), we can obtain

$$P_{r,k}^i = \frac{A_1}{A_2(\frac{A_1}{B_1} - A_4)\frac{1}{P_{r,k}^i} + A_3} - \frac{A_4}{(\frac{A_1}{B_1} - A_4)\frac{1}{P_{r,k}^i}}, \tag{0.8}$$

then it holds,

$$[1 + \frac{A_4}{(\frac{A_1}{B_1} - A_4)}]P_{r,k}^i = \frac{A_1}{A_2(\frac{A_1}{B_1} - A_4)\frac{1}{P_{r,k}^i} + A_3}. \tag{0.9}$$

The problem we are constructing is a continuous convex function, so we can take the critical value of ρ_k^i. When $\rho_k^i \to 0$,

$$P_{r,k}^i = \frac{A_1}{A_3} - \frac{A_4}{\rho_k^i} \geq 0. \tag{0.10}$$

In order to satisfy $P_{r,k}^i > 0$, it holds $A_3 > 0$ and $A_3 \to 0$.

When $\rho_k^i = 1$,

$$P_{r,k}^i = \frac{A_1}{A_2 + A_3} - A_4 \geq 0, \tag{0.11}$$

then we can obtain $A_2 > 0$ and $A_2 \gg A_3$. From (0.7), we obtain

$$C_3 = A_2(\frac{A_1}{B_1} - A_4) \geq 0, \tag{0.12}$$

and

$$C_2 = 1 + \frac{A_4}{(\frac{A_1}{B_1} - A_4)} > 0. \tag{0.13}$$

Then we can obtain

$$P_{r,k}^i = \frac{\frac{A_1}{C_2}}{C_3\frac{1}{P_{r,k}^i} + \frac{A_3}{C_2}} > 0. \tag{0.14}$$

The right-hand side of (0.42) can be rewritten in the following form:

$$I(P_{r,k}^i) = \frac{\frac{A_1}{C_2}}{C_3\frac{1}{P_{r,k}^i} + \frac{A_3}{C_2}}. \tag{0.15}$$

Standard interference functions guarantee convergence to a unique fixed-point from any initial $P_{r,k}^i(0)$ after certain iterations through $P_{r,k}^i(t+1) = I(P_{r,k}^i(t))$[6]. To prove that (0.43) is a standard interference function, the following three properties (i.e., positivity, monotonicity, and scalability) should be satisfied.

1)Positivity follows from (0.42).

2)Letting $P^i_{r,k} = \alpha Q^i_{r,k}$ with $\alpha \geq 1$, monotonicity follows from

$$I(P^i_{r,k}) = \frac{\frac{A_1}{C_2}}{C_3 \frac{1}{\alpha Q^i_{r,k}} + \frac{A_3}{C_2}} \geq \frac{\frac{A_1}{C_2}}{C_3 \frac{1}{Q^i_{r,k}} + \frac{A_3}{C_2}} = I(Q^i_{r,k}). \tag{0.16}$$

3)Letting $\alpha > 1$, scalability follows from

$$\alpha I(P^i_{r,k}) = \frac{\frac{A_1}{C_2}}{C_3 \frac{1}{\alpha P^i_{r,k}} + \frac{A_3}{\alpha C_2}} > \frac{\frac{A_1}{C_2}}{C_3 \frac{1}{\alpha P^i_{r,k}} + \frac{A_3}{C_2}} = I(\alpha P^i_{r,k}). \tag{0.17}$$

Hence, the constructed interference function can be converged to a fixed point, which is equivalent to $\frac{\partial \mathcal{L}_1}{\partial P^i_{r,k}} = 0$ and $\frac{\partial \mathcal{L}_2}{\partial \rho^i_k} = 0$ in the original iteration. Therefore, our proposed OPA algorithm can be converged.

As for the complexity of Algorithm 1, the worst case time complexity of the Hungarian algorithm is $O(n^3)$, where n represents the number of subcarriers. In the objective function fraction, the numerator and denominator contain J variables. In the worst case, the optimal q^* can be obtained with the maximum iterations Z. The time complexity of the Dinkelbach method is $O(\log(JZ))$ in the worst case. Thus, the complexity of the proposed algorithm is $O(\log(JZ)n^3)$.

In general, there are three common ways to deal with imperfect wireless channels, i.e., Bayesian, worst-case and probability constraint approaches. The Bayesian approach assumes that the channel gain is a random variable and guarantees the constraints on the average. The worst-case approach assumes that the actual channel gain is bounded to a closed set called the uncertain set, and guarantees the performance level for any channel realization in the uncertain set. The probability constraint approach can deal with the cases that the wireless channel gain is a random quantity with some probability density function or the wireless channel gain is bounded by an uncertain region with some probability threshold. There are many existing works considering the uncertain channel. To sum up, if the probability density of the actual channel gains are known, the Bayesian and probability constraint approaches can be used to solve the robust optimization problem. If the channel gains fluctuate within a closed uncertain set, the worst-case and probability constraint approaches can be applied to solve the robust problem. As pointed by the reviewer, the worst case method is conservative scheme, because not all scenes are the worst case. However, in a dynamic communication environment, it is very difficult to get the instantaneous channel gain. It is also hard to know exact distribution that the channel gain obeys. Therefore, the worst-case method which is simple and convenient to operate has been widely used in practical application. In fact, similar to some other literature which adopts the worst case method, the upper bound of error is achieved by statistical method. As long as the system can ensure normal communication in extreme case, the system can communicate normally in other cases.

[6] R. D. Yates. A Framework for Uplink Power Control in Cellular Radio Systems. *IEEE Journal on Selected Areas in Communications*, 13(7): 1341-1347, 1995.

5. **Question**: In Section IV, two naive reference algorithms, FSA and APA are used. Some more advanced reference algorithms in existing literature are suggested to be used.

   Answer: Thanks for the reviewer's suggestions! We add comparison with the channel ordering algorithm proposed in [7]. In Fig. 2, the black polyline indicates the Channel ordering method. In this method, the channel frequency response with the highest magnitude is searched among the subcarriers. The corresponding subcarrier is assigned to user 1. Out of $N-1$ remaining subcarriers, the same search is repeated. This process loops until all subcarriers are allocated. From the figure we can see that the energy efficiency of OPA scheme has always been higher than other algorithm. In Fig. 3, we compare the system energy efficiency between the OPA scheme and the Channel ordering scheme with different minimum rate requirements. It shows that the system energy efficiency of the OPA algorithm is also higher than the channel ordering algorithm. Fig. 4 shows the system average energy efficiency when different numbers of secondary users are considered. From the simulation results, it is found that the proposed OPA shows superiority in aspect of energy efficiency improvement, because the optimal transmission power, power splitting ratio and subcarrier allocation are achieved in the resource allocation algorithm.

[7] Y. Otani, S. Ohno, K. D. Teo and T. Hinamoto. Subcarrier Allocation for multi-user OFDM system. In *Asia-Pacific Conference on Communications*, 2015: 1073-1077.

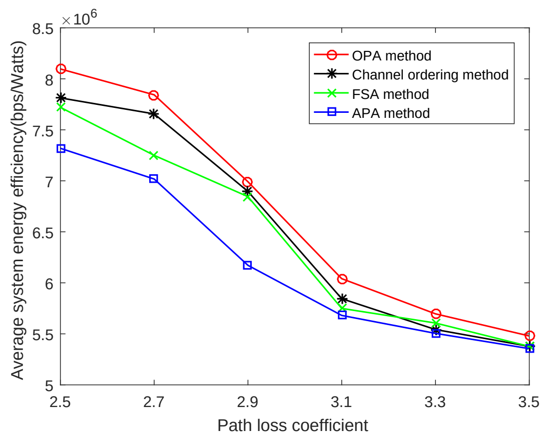

Figure 2: Energy efficiencies comparison with different path loss coefficient.

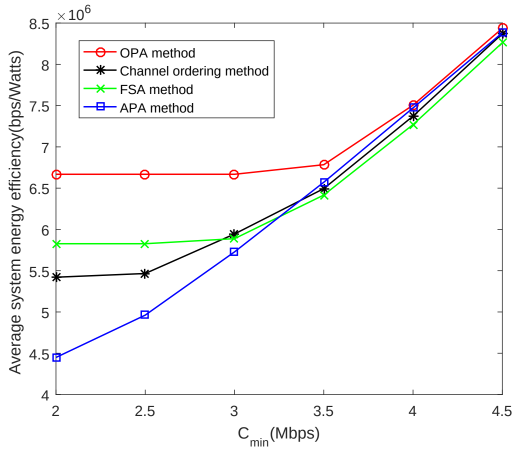

Figure 3: Energy efficiency comparison with different minimum system rate requirement.

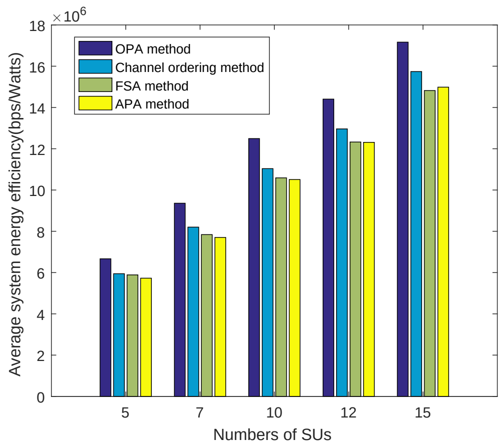

Figure 4: Energy efficiency comparison with different topologies.

Finally, the authors thank the reviewer for the comments provided, and the time and efforts he/she has spent in the review again. Without the careful comments, the paper would not reach its current quality. We hope that the above modifications have answered the reviewer's concerns.

Response to Reviewer 2

We would like to thank the reviewer for his/her careful examining work and the time spent.

Our responses to the reviewer's comments are given as follows:

1. **Question**: The authors claimed that they jointly optimize the transmission power, power splitting ratio and subcarrier paring to maximize the energy efficiency of the secondary system. The definition of the "subcarrier paring" is not correct. Normally, subcarrier paring is performed through two transmission phases relaying, which means that the subcarrier in the second phase paired with the subcarrier in the first phase. However, in this paper, "subcarrier paring" is only performed in the second phase. It would be more suitable to name it "subcarrier allocation" rather than "subcarrier paring". Thus, the first contribution of this paper shown in the introduction will be very limited.

   Answer: Thanks for the reviewer's comments. We totally agree with the reviewer's point that it would be more suitable to name it "subcarrier allocation" rather than "subcarrier paring". We have revised the original expression and the revised part is marked in red.

   For the contribution of the first part, we want to show that the joint optimization problem is formulated, which is including power control, power splitting ratio and subcarrier allocation with energy harvesting requirements. The goal is to maximize the Energy Efficiency of the secondary network. However, the established problem is non-convex and it is with the integer variables(subcarrier allocation indicator β_i^k). It is difficult to determine the optimal solutions. In this paper, a practical resource allocation algorithm is developed and the energy efficiency of the secondary system can be effectively improved. The statement of contributions in Introduction has been refined in the revision.

2. **Question**: The assumption of the system model is not reasonable. Firstly, why the primary user and secondary users have the same source node. Secondly, the transmission from the relay to the secondary users adopted OFDMA technique in the second phase. However, OFDMA is not adopted in the first phase for the transmission from the source node to relay. Thirdly, it is unrealistic to assume that the power splitting is performed in each subcarrier.

   Answer: Thanks for the reviewer's comments! Firstly, we feel very sorry for our negligence. The primary user and secondary users have different source node. In the original system model diagram, the line type of the S-to-PU interference link was incorrectly drawn as the line type of the transmission link. The revised system model is shown in Fig. 5, where only the interference links for primary user are given and the primary source node is omitted. In the cognitive radio system, the primary network and the secondary network coexist, and there are two typical working modes for the cognitive users, overlay and underlay. In this paper, the underlay mode is assumed, thus the secondary users share the same frequency resource and the caused interference to the primary user must be below the allowed level. The condition is described as

$$\sum_{k=1}^{K}\sum_{i=1}^{N}\beta_k^i P_{r,k}^i g_{rpu,k}^i \leq I_{th}, \tag{0.18}$$

and it is the constraint $C2$ in the optimization problem (19).

The corresponding explanations have been added in the revision.

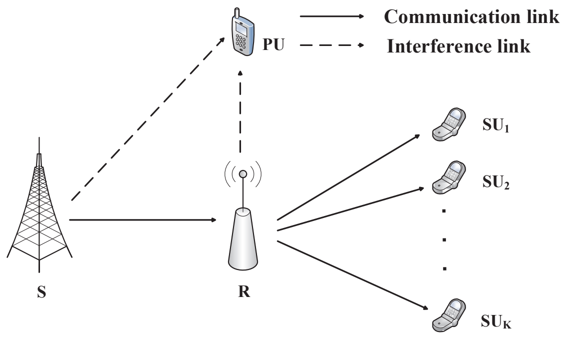

Figure 5: System model.

Secondly, in this system model, we consider a single base station node, a single relay node, and multiple secondary user nodes. In the first phase, a single source node transmits signal to a relay node. The entire channel is used for transmission at this stage, so OFDMA is not adopted for channel allocation. In the second phase, it is the transmission from a single node to multiple destinations. In order to avoid interference among multiple users in the system, OFDMA technology is introduced. At the same time, we use the Hungarian algorithm for subcarrier allocation so that secondary users are matched to the most suitable subcarrier to maximize system efficiency.

Thirdly, in this paper, the receiver is able to divide the received signal into two parts by the power splitting mechanism, one part is recorded as ρ for information decoding and the other part is $1-\rho$ for energy harvesting, where, $\rho \in (0,1)$ The harvested energy is formulated as:

$$P_h = \eta P_{r,k}^i h_{rsu,k}^i (1-\rho_k^i),$$

where η represents the efficiency of energy conversion for converting the received signal into the storage energy. The power splitting is executed at the secondary user. Moreover, it is constrained that only one subcarrier is allocated to a user, i.e. the constraint C_7 in optimization problem (19). Thus, a single-antenna SWIPT system is able to realize the energy harvesting and it is a practical working mode. The considered scenario is same as Ref. [1](refer to Fig.1 and Fig.2).

In addition, the situation of each secondary user is different. Considering factors such as distance, path loss, transmission loss, etc., the power and energy harvesting ratio required by different secondary users is not unified. Similar relay model has been adopted by some existing works. Ref. [2] considered an underlay cognitive relay network, and proposed a new wireless energy harvesting protocol for the network. Ref. [3] investigated an underlay relay-assisted cognitive radio network where a secondary relay harvests energy from a secondary source's signal.

Some necessary explanations have been added in the revision to clarify the assumption.

[1] R. Morsi, V. Jamali, A. Hagelauer, D. W. K. Ng, and R. Schober, "Conditional Capacity and Transmit Signal Design for SWIPT Systems With Multiple Nonlinear Energy Harvesting Receivers, *IEEE Transactions on Communications*, 68(1), pp.582-601, Jan. 2020.

[2] Y. Liu, S. A. Mousavifar, Y. Deng, C. Leung, and M. Elkashlan. "Wireless Energy Harvesting in a Cognitive Relay Network". *IEEE Transactions on Wireless Communications*, 15(4): 2498-2508, 2016.

[3] G. Im, and J. H. Lee. "Wireless Energy Harvesting in a Cognitive Relay Network". In *2015 IEEE 82nd Vehicular Technology Conference (VTC2015-Fall)*, 2015: 1-5.

3. **Question**: SUs did not consume any energy, why need to give energy harvest as a supplement.

   Answer: Thanks for the reviewer's comments! Usually, the terminal device in wireless communication network is energy-limited. So the energy saving is a key issue in wireless resource management. At the secondary users, the power splitting method is used for energy harvesting. The harvested energy can provide an energy supply for decoding information. In this paper, only the downlink communication is considered. The signal is sent by the base station via the relay node, and then relayed by the user SU. However, the communication is two-way. If uplink communication is considered, secondary users need to consume energy to transmit signals. The harvested energy can also be supplemented at this time.

4. **Question**: What do P_s and P_h mean? How much are they?

   Answer: Thanks for the reviewer's comments! It is assumed that the relay node operates in half-duplex mode. In half-duplex mode, the entire transmission period is divided into two equal time slots. In the first time slot, the base station S transmits information to the relay R. Ps is the power from node S to relay R. In this paper, the decode-and-forward(DF) relay system is considered. The first hop is from the source node relay node, the SNR and the rate are given in Eq.(12) and Eq.(14), respectively. The second hop is from the relay node to destination node, the SNR and the rate are given in Eq.(13) and Eq.(15), respectively. The achievable capacity from source node S the secondary node SU_k depends on the bottleneck link of two hops, i.e.

$$C_k = \min\{C_{sr}, C^i_{rsu,k},\}$$

From Eq.(12) and Eq. (14), the rate of the first hop depends on the transmitting power P_s. It is assumed that the second hop is the bottleneck link in this paper, because we consider the donwlink transmission and the power of base station, P_s, is usually not constrained. The constraint $C_{sr} > C^i_{rsu,k}$ can be satisfied by setting P_s. In the simulation, the value of P_s is 0.3Watt.

In the second time slot, the relay R transmits signals to the secondary user SU_k. Each secondary user is equipped with a SWIPT architecture that is capable of decoding information as well as harvesting energy from the received signal according to PS scheme. P_h is the harvested energy. It depends on the variables $P^i_{r,k}$ and ρ^i_k. The harvested energy P_h is formulated as follows, which is Eq.(2) in the manuscript.

$$P_h = \eta P^i_{r,k} h^i_{rsu,k}(1-\rho^i_k).$$

5. **Question**: OP4 is decomposed into two subproblems SP1 and SP2. However, the authors did not explain why it can be decomposed. Moreover, they did not show the performance gap between two problems.

   Answer: Thanks for the reviewer's comments! In this paper, the original objective function is a non-convex function in the fractional form containing two variables $P^i_{r,k}$ and ρ^i_k. To solve this non-convex problem, we first use the Dinkelbach method to transform it into a subtraction form. However, the transformed objective function is still a non-convex function that couples two variables. In order to transform it into a convex problem that is easy to solve, we separate two variables $P^i_{r,k}$ and ρ^i_k. The objective function is decomposed into two sub-problems namely SP1 in Eq.(0.19) and SP2 in Eq.(0.20).

$$\begin{aligned} &\mathit{SP1}: \max_{P^i_{r,k}} \; C_{rsu} - qP_{con} \\ &\mathit{subject\ to} \quad C_3, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}. \end{aligned} \tag{0.19}$$

$$\begin{aligned} &\mathit{SP2}: \max_{\rho^i_k} \; C_{rsu} - qP_{con} \\ &\mathit{subject\ to} \quad C_6, C_7, C_8, C_9, C_{11}, C_{12}. \end{aligned} \tag{0.20}$$

$$
\begin{aligned}
\mathcal{L}_1(P_{r,k}^i, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\theta}, \gamma, \lambda, \boldsymbol{\nu}) &= \sum_{k=1}^{K}\sum_{i=1}^{N} \frac{W}{2}\beta_k^i \log_2(1+\frac{P_{r,k}^i \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}}) \\
&- q(P_c + P_s + \sum_{k=1}^{K}\sum_{i=1}^{N}\beta_k^i P_{r,k}^i - \eta \sum_{k=1}^{K}\sum_{i=1}^{N}\beta_k^i P_{r,k}^i H_{rsu,k}^i (1-\rho_k^i)) \\
&+ \mu_k(\frac{W}{2}\log_2(1+\frac{P_{r,k}^i \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}}) - C^{\min}) + \theta_k(\frac{P_{r,k}^i \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}} - SNR_{th}) \\
&- \gamma(\sum_{k=1}^{K}\sum_{i=1}^{N}\beta_k^i P_{r,k}^i G_{rpu,k}^i - I_{th}) - \lambda(\sum_{k=1}^{K}\sum_{i=1}^{N}\beta_k^i P_{r,k}^i - P_{th}) \\
&+ \nu_k(\eta P_{r,k}^i H_{rsu,k}^i (1-\rho_k^i) - P_h^{\min}).
\end{aligned} \tag{0.21}
$$

$$
\begin{aligned}
\mathcal{L}_2(\rho_k^i, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\nu}, \boldsymbol{\omega}) &= \sum_{k=1}^{K}\sum_{i=1}^{N} \frac{W}{2}\beta_k^i \log_2(1+\frac{P_{r,k}^i \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}}) \\
&- q(P_c + P_s + \sum_{k=1}^{K}\sum_{i=1}^{N}\beta_k^i P_{r,k}^i - \eta \sum_{k=1}^{K}\sum_{i=1}^{N}\beta_k^i P_{r,k}^i H_{rsu,k}^i (1-\rho_k^i)) \\
&+ \mu_k(\frac{W}{2}\log_2(1+\frac{P_{r,k}^i \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}}) - C^{\min}) + \theta_k(\frac{P_{r,k}^i \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}} - SNR_{th}) \\
&+ \nu_k(\eta P_{r,k}^i H_{rsu,k}^i (1-\rho_k^i) - P_h^{\min}) - \omega_k(\rho_k^i - 1).
\end{aligned} \tag{0.22}
$$

$$
\frac{\partial^2 \mathcal{L}_1}{\partial (P_{r,k}^i)^2} = \frac{-W(\beta_k^i + \mu_k)(H_{rsu,k}^i \rho_k^i)^2}{2(P_{r,k}^i \rho_k^i Hrsu,k^i + \sigma_{rsu,k}^{i2})^2 \ln 2} \tag{0.23}
$$

$$
\frac{\partial^2 \mathcal{L}_2}{\partial (\rho_k^i)^2} = \frac{-W(\beta_k^i + \mu_k)(H_{rsu,k}^i P_{r,k}^i)^2}{2(P_{r,k}^i \rho_k^i Hrsu,k^i + \sigma_{rsu,k}^{i2})^2 \ln 2} \tag{0.24}
$$

The optimal solution can be obtained through the Lagrangian dual algorithm. The Lagrangian multipliers are formulated in Eq.(0.21) and Eq.(0.22).

Eq.(0.23) finds the second derivative of $P_{r,k}^i$, $\frac{\partial^2 \mathcal{L}_1}{\partial (P_{r,k}^i)^2} < 0$. Eq.(0.24) finds the second derivative of ρ_k^i, $\frac{\partial^2 \mathcal{L}_2}{\partial (\rho_k^i)^2} < 0$. So SP1 and SP2 are convex optimization problems. Since both subproblems are convex, they both have unique optimal solutions. This method is also used in [4-5] and shows better approximation.

[4] D. W. K. Ng, E. S. Lo, and R. Schober. Wireless Information and Power Transfer: Energy Efficiency Optimization in OFDMA Systems. *IEEE Transactions on Wireless Communications*, 12(12): 6352-6370, 2013.

[5] B. Li, W. Xu, and X. Gao. Energy-Efficient Simultaneous Information and Power Transfer in OFDM-Based CRNs. In *2015 IEEE 81st Vehicular Technology Conference (VTC Spring)*, 2015: 1-5.

6. **Question**: In simulation results, the authors should give the locations of the SUs or the distances between R and SUs.

   Answer: Thanks for the reviewer's comments! The base station is located at (0,0). The relay node is located at (25,0). The secondary users are located at (55,20), (53,10), (58,5), (60,-15), (57, -20), respectively. We have added this part in the revision, and marked in red.

Thanks for the constructive comments and time spent, and we have revised our paper and we hope that the above comments have answered the reviewer's concerns and meet her/his expectations. We will happily welcome any additional suggestions and feedback by the reviewers.

Response to Reviewer 3

We would like to thank the reviewer for spending his/her time to assess the paper, and make some very constructive and detailed informative comments provided in the review. Our responses are given as follows:

1. **Question**: The complexity of the proposed algorithm should be further discussed.

   Answer: Thanks for the reviewer's valuable comments! The worst case time complexity of the Hungarian algorithm is $O(n^3)$, where n represents the number of subcarriers. In the objective function fraction, the numerator and denominator contain J variables. In the worst case, the optimal q^* can be obtained with the maximum iterations Z. The time complexity of the Dinkelbach method is $O(\log(JZ))$ in the worst case. Thus, the complexity of the proposed algorithm is $O(\log(JZ)n^3)$.

2. **Question**: In the simulation, why $k^i_{rsu,k}$ is random variable while $k^i_{rpu,k}$ is const?

   Answer: Thanks for the reviewer's comments! In this system, it contains one primary user and k secondary users. We mainly analyze and optimize the performance of secondary systems. We set the fixed transmission loss coefficient of the secondary system to random variable in [0, 1] in order to simulate the channel environment and channel state of the subcarriers randomly ordered. There is only one interference link of the relay to the primary user, and no sequencing is required. Therefore, $k^i_{rpu,k}$ is fixed. The setting is to make the simulation process more reliable.

3. **Question**: Why the performance for APA is not presented in Fig. 7?

   Answer: Thanks for the reviewer's comments! We are very sorry for our negligence, we

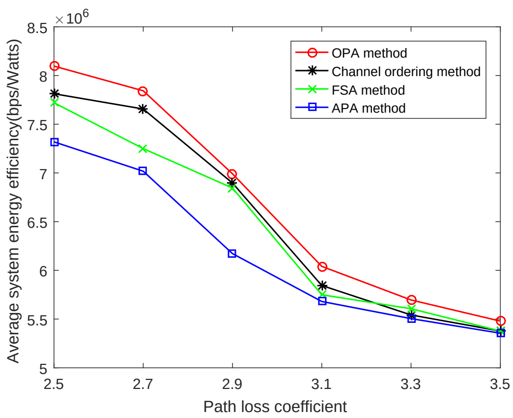

Figure 6: Energy efficiencies comparison with different path loss coefficient.

labeled the figure name as the ASA by mistake. We have corrected it as APA scheme in the revision. In addition, according to other reviewer's comments, new comparison has been added and the Figure 7 has been updated. We compare the proposed algorithm with some typical algorithms. The blue polyline represents the APA algorithm. The APA algorithm indicates that the power allocation ratio has not been optimized, that is, the ratio for harvesting energy and decoding information is 0.5. The energy efficiency of OPA algorithm is better than other algorithms.

Thanks for the constructive comments and time spent, and we have revised our paper and we hope that the above comments have answered the reviewer's concerns and meet her/his expectations. We will happily welcome any additional suggestions and feedback by the reviewers.

Response to Reviewer 4

We would like to thank the reviewer for spending his/her time to assess the paper, and make some very constructive and detailed informative comments provided in the review. Our responses are given as follows:

1. **Question**: Some of the analysis is a bit vague. For example, Dinkelback method usually requires that the considered objective is qusi-convex while there is no analysis on it. The authors may refer to the following paper to see how it can be proved.

   Answer: Thanks for the reviewer's comments! In the revision, we have referred and cited the following references to clarify the qusi-convex objective when using the Dinkelbach method.

   [1] Z. Chang, J. Gong, Y. Li, Z. Zhou, T. Ristaniemi, G. Shi, Z. Han, and Z. Niu. Energy Efficient Resource Allocation for Wireless Power Transfer Enabled Collaborative Mobile Clouds. *IEEE Journal on Selected Areas in Communications*, 34(12): 3438-3450, 2016.

   [2] Z. Chang, J. Gong, T. Ristaniemi, and Z. Niu. Energy-Efficient Resource Allocation and User Scheduling for Collaborative Mobile Clouds With Hybrid Receivers. *IEEE Transactions on Vehicular Technology*, 65(12): 9834-9846, 2016.

   In this paper, the original objective function is non-convex. we decompose it into two sub-problems namely SP1 in (0.25) and SP2 in (0.26) which are separately solved.

$$\begin{aligned} &\mathit{SP1}: \max_{P^i_{r,k}} \; C_{rsu} - qP_{con} \\ &\mathit{subject\ to} \quad C_3, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}. \end{aligned} \tag{0.25}$$

$$\begin{aligned} &\mathit{SP2}: \max_{\rho^i_k} \; C_{rsu} - qP_{con} \\ &\mathit{subject\ to} \quad C_6, C_7, C_8, C_9, C_{11}, C_{12}. \end{aligned} \tag{0.26}$$

   The optimal solution can be obtained through the Lagrangian dual algorithm. The Lagrangian multipliers are formulated in (0.27) and (0.28).

$$\begin{aligned} &\mathcal{L}_1(P^i_{r,k}, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\theta}, \gamma, \lambda, \boldsymbol{\nu}) = \sum_{k=1}^{K}\sum_{i=1}^{N}\frac{W}{2}\beta^i_k \log_2(1+\frac{P^i_{r,k}\rho^i_k H^i_{rsu,k}}{\sigma^{i2}_{rsu,k}}) \\ &- q(P_c + P_s + \sum_{k=1}^{K}\sum_{i=1}^{N}\beta^i_k P^i_{r,k} - \eta\sum_{k=1}^{K}\sum_{i=1}^{N}\beta^i_k P^i_{r,k} H^i_{rsu,k}(1-\rho^i_k)) \\ &+ \mu_k(\frac{W}{2}\log_2(1+\frac{P^i_{r,k}\rho^i_k H^i_{rsu,k}}{\sigma^{i2}_{rsu,k}}) - C^{\min}) + \theta_k(\frac{P^i_{r,k}\rho^i_k H^i_{rsu,k}}{\sigma^{i2}_{rsu,k}} - SNR_{th}) \\ &-\gamma(\sum_{k=1}^{K}\sum_{i=1}^{N}\beta^i_k P^i_{r,k} G^i_{rpu,k} - I_{th}) - \lambda(\sum_{k=1}^{K}\sum_{i=1}^{N}\beta^i_k P^i_{r,k} - P_{th}) + \nu_k(\eta P^i_{r,k} H^i_{rsu,k}(1-\rho^i_k) - P^{\min}_h). \end{aligned} \tag{0.27}$$

$$
\begin{aligned}
\mathcal{L}_2(\rho_k^i, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\nu}, \boldsymbol{\omega}) &= \sum_{k=1}^{K}\sum_{i=1}^{N} \frac{W}{2}\beta_k^i \log_2(1 + \frac{P_{r,k}^i \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}}) \\
&- q(P_c + P_s + \sum_{k=1}^{K}\sum_{i=1}^{N} \beta_k^i P_{r,k}^i - \eta \sum_{k=1}^{K}\sum_{i=1}^{N} \beta_k^i P_{r,k}^i H_{rsu,k}^i (1-\rho_k^i)) \\
&+ \mu_k(\frac{W}{2}\log_2(1 + \frac{P_{r,k}^i \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}}) - C^{\min}) + \theta_k(\frac{P_{r,k}^i \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}} - SNR_{th}) \\
&+ \nu_k(\eta P_{r,k}^i H_{rsu,k}^i (1-\rho_k^i) - P_h^{\min}) - \omega_k(\rho_k^i - 1).
\end{aligned} \tag{0.28}
$$

(0.29) finds the second derivative of $P_{r,k}^i$, $\frac{\partial^2 \mathcal{L}_1}{\partial (P_{r,k}^i)^2} < 0$. (0.30) finds the second derivative of ρ_k^i, $\frac{\partial^2 \mathcal{L}_2}{\partial (\rho_k^i)^2} < 0$. So SP1 and SP2 are convex optimization problems.

$$
\frac{\partial^2 \mathcal{L}_1}{\partial (P_{r,k}^i)^2} = \frac{-W(\beta_k^i + \mu_k)(H_{rsu,k}^i \rho_k^i)^2}{2(P_{r,k}^i \rho_k^i Hrsu, k^i + \sigma_{rsu,k}^{i2})^2 \ln 2} \tag{0.29}
$$

$$
\frac{\partial^2 \mathcal{L}_2}{\partial (\rho_k^i)^2} = \frac{-W(\beta_k^i + \mu_k)(H_{rsu,k}^i P_{r,k}^i)^2}{2(P_{r,k}^i \rho_k^i Hrsu, k^i + \sigma_{rsu,k}^{i2})^2 \ln 2} \tag{0.30}
$$

2. **Question**: There is no analysis on the convergence, complexity and optimality. In fact, the use of Dinkelback method may cause the convergence problem and the complexity of Hungarian algorithm should be examined.

   Answer: Thanks for the reviewer's comments! We first prove the convergence and optimality of the algorithm. Since the transformed problem is a standard convex optimization problem, the optimal value can be determined by addressing the partial derivative of the Lagrangian function $\frac{\partial \mathcal{L}_1}{\partial P_{r,k}^i} = 0$ and $\frac{\partial \mathcal{L}_2}{\partial \rho_k^i} = 0$. Hence, we have

$$
\begin{aligned}
\frac{\partial \mathcal{L}_1}{\partial P_{r,k}^i} &= \frac{W \beta_k^i \rho_k^i H_{rsu,k}^i}{2(P_{r,k}^i \rho_k^i Hrsu, k^i + \sigma_{rsu,k}^{i2}) \ln 2} - q(\beta_k^i - \eta \beta_k^i (1-\rho_k^i) H_{rsu,k}^i) \\
&+ \frac{\mu_k W \rho_k^i H_{rsu,k}^i}{2(P_{r,k}^i \rho_k^i H_{rsu,k}^i + \sigma_{rsu,k}^{i2}) \ln 2} + \frac{\theta_k \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}} - \gamma \beta_k^i G_{rpu,k}^i \\
&- \lambda \beta_k^i + \nu_k \eta (1-\rho_k^i) H_{rsu,k}^i = 0,
\end{aligned} \tag{0.31}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_2}{\partial \rho_k^i} &= \frac{W \beta_k^i P_{r,k}^i H_{rsu,k}^i}{2(P_{r,k}^i \rho_k^i H_{rsu,k}^i + \sigma_{rsu,k}^{i2}) \ln 2} - q\eta \beta_k^i P_{r,k}^i H_{rsu,k}^i \\
&+ \frac{\mu_k W P_{r,k}^i H_{rsu,k}^i}{2(P_{r,k}^i \rho_k^i H_{rsu,k}^i + \sigma_{rsu,k}^{i2}) \ln 2} + \frac{\theta_k P_{r,k}^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}} - \nu_k \eta P_{r,k}^i H_{rsu,k}^i - \omega_k = 0.
\end{aligned} \tag{0.32}
$$

By the aforementioned equation, we can obtain

$$\begin{aligned} P_{r,k}^i &= \left[\frac{W(\beta_k^i+\mu_k)}{(q(\beta_k^i - \eta H_{rsu,k}^i(1-\rho_k^i)) - \frac{\theta_k \rho_k^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}} + \gamma\beta_k^i G_{rpu,k}^i + \lambda\beta_k^i + \nu_k\eta(1-\rho_k^i) H_{rsu,k}^i)2\ln 2} - \frac{\sigma_{rsu,k}^{i2}}{\rho_k^i H_{rsu,k}^i} \right]^+ \\ &= \frac{A_1}{A_2\rho_k^i + A_3} - \frac{A_4}{\rho_k^i}, \end{aligned} \tag{0.33}$$

where $A_1 = W(\beta_k^i+\mu_k)$, $A_2 = (q\eta H_{rsu,k}^i - \frac{\theta_k H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}} - \nu_k \eta H_{rsu,k}^i)2\ln 2$, $A_3 = (q\beta_k^i - q\eta H_{rsu,k}^i + \gamma\beta_k^i G_{rpu,k}^i + \lambda\beta_k^i + \nu_k\eta H_{rsu,k}^i)2\ln 2$, and $A_4 = \frac{\sigma_{rsu,k}^{i2}}{H_{rsu,k}^i}$.

$$\begin{aligned} \rho_k^i &= \frac{W(\beta_k^i+\mu_k)}{(q\eta\beta_k^i P_{r,k}^i H_{rsu,k}^i - \frac{\theta_k P_{r,k}^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}} + \nu_k\eta P_{r,k}^i H_{rsu,k}^i)2\ln 2} - \frac{\sigma_{rsu,k}^{i2}}{P_{r,k}^i H_{rsu,k}^i} \\ &= (\frac{A_1}{B_1} - A_4)\frac{1}{P_{r,k}^i} \geq 0, \end{aligned} \tag{0.34}$$

where $B_1 = (q\eta\beta_k^i P_{r,k}^i H_{rsu,k}^i - \frac{\theta_k P_{r,k}^i H_{rsu,k}^i}{\sigma_{rsu,k}^{i2}} + \nu_k\eta P_{r,k}^i H_{rsu,k}^i)2\ln 2$. We can obtain,

$$C_1 = (\frac{A_1}{B_1} - A_4) \geq 0, \tag{0.35}$$

Substitute (0.34) into (0.33), we can obtain

$$P_{r,k}^i = \frac{A_1}{A_2(\frac{A_1}{B_1} - A_4)\frac{1}{P_{r,k}^i} + A_3} - \frac{A_4}{(\frac{A_1}{B_1} - A_4)\frac{1}{P_{r,k}^i}}, \tag{0.36}$$

then we can obtain,

$$[1 + \frac{A_4}{(\frac{A_1}{B_1} - A_4)}]P_{r,k}^i = \frac{A_1}{A_2(\frac{A_1}{B_1} - A_4)\frac{1}{P_{r,k}^i} + A_3}. \tag{0.37}$$

The problem we are constructing is a continuous convex function, so we can take the critical value of ρ_k^i. When $\rho_k^i \to 0$,

$$P_{r,k}^i = \frac{A_1}{A_3} - \frac{A_4}{\rho_k^i} \geq 0. \tag{0.38}$$

In order to satisfy $P_{r,k}^i > 0$, it holds $A_3 > 0$ and $A_3 \to 0$.

When $\rho_k^i = 1$,

$$P_{r,k}^i = \frac{A_1}{A_2 + A_3} - A_4 \geq 0, \tag{0.39}$$

then we can obtain $A_2 > 0$ and $A_2 \gg A_3$. From (0.35), we obtain

$$C_3 = A_2(\frac{A_1}{B_1} - A_4) \geq 0, \tag{0.40}$$

and

$$C_2 = 1 + \frac{A_4}{(\frac{A_1}{B_1} - A_4)} > 0. \tag{0.41}$$

Then we can obtain

$$P^i_{r,k} = \frac{\frac{A_1}{C_2}}{C_3 \frac{1}{P^i_{r,k}} + \frac{A_3}{C_2}} > 0. \tag{0.42}$$

The right-hand side of (0.42) can be rewritten in the following form:

$$I(P^i_{r,k}) = \frac{\frac{A_1}{C_2}}{C_3 \frac{1}{P^i_{r,k}} + \frac{A_3}{C_2}}. \tag{0.43}$$

Standard interference functions guarantee convergence to a unique fixed-point from any initial $P^i_{r,k}(0)$ after certain iterations through $P^i_{r,k}(t+1) = I(P^i_{r,k}(t))$[6]. To prove that (0.43) is a standard interference function, the following three properties (i.e., positivity, monotonicity, and scalability) should be satisfied.

1)Positivity follows from (0.42).

2)Letting $P^i_{r,k} = \alpha Q^i_{r,k}$ with $\alpha \geq 1$, monotonicity follows from

$$I(P^i_{r,k}) = \frac{\frac{A_1}{C_2}}{C_3 \frac{1}{\alpha Q^i_{r,k}} + \frac{A_3}{C_2}} \geq \frac{\frac{A_1}{C_2}}{C_3 \frac{1}{Q^i_{r,k}} + \frac{A_3}{C_2}} = I(Q^i_{r,k}). \tag{0.44}$$

3)Letting $\alpha > 1$, scalability follows from

$$\alpha I(P^i_{r,k}) = \frac{\frac{A_1}{C_2}}{C_3 \frac{1}{\alpha P^i_{r,k}} + \frac{A_3}{\alpha C_2}} > \frac{\frac{A_1}{C_2}}{C_3 \frac{1}{\alpha P^i_{r,k}} + \frac{A_3}{C_2}} = I(\alpha P^i_{r,k}). \tag{0.45}$$

Hence, the constructed interference function can be converged to a fixed point, which is equivalent to $\frac{\partial \mathcal{L}_1}{\partial P^i_{r,k}} = 0$ and $\frac{\partial \mathcal{L}_2}{\partial \rho^i_k} = 0$ in the original iteration. Therefore, our proposed OPA algorithm can be converged.

The worst case time complexity of the Hungarian algorithm is $O(n^3)$, where n represents the number of subcarriers. In the objective function fraction, the numerator and denominator contain J variables. In the worst case, the optimal q^* can be obtained with the maximum iterations Z. The time complexity of the Dinkelbach method is $O(\log(JZ))$ in the worst case. Thus, the complexity of the proposed algorithm is $O(\log(JZ)n^3)$.

[3] R. D. Yates. A Framework for Uplink Power Control in Cellular Radio Systems. *IEEE Journal on Selected Areas in Communications*, 13(7): 1341-1347, 1995.

3. **Question**: The presentation is not smooth. Please revise it.

   Answer: Thanks for the reviewer's comments! We have checked and revised the original expression. More details please refer to the revision.

4. **Question**: In the simulations, the figure of convergence shows a really fast convergence speed (within 2 iterations). Please explain.

   Answer: Thanks for the reviewer's comments! We analyzed the convergence and complexity of the algorithm in the above question. From the above analysis, it can be seen that the algorithm has good convergence and low complexity. The convergence process is related to the initial value of the parameter and the step size. If the relevant parameters are not selected properly, the convergence process will be affected. In the simulation of this work, suitable parameter values have been selected. In addition, we decompose the optimization problem. As shown in Algorithm 1, there are two loops. The inner loop is used to update the transmission power and the power splitting ratio, the outer loop is used to update subcarrier allocation. The original complex problem is decomposed into simple sub-problems, and the separation of variables also reduces the complexity of the analysis. It makes iterative results quickly approach the convergence value.

5. **Question**: In Fig. 7, it shows that when path loss factor is bigger, there is no difference among all three schemes. Please give some illustrations.

   Answer: Thanks for the reviewer's comments! The fading is caused by the radiation spread of the transmission power and the propagation characteristics of the channel. It is generally considered that for the same communication environment, the path loss is the same. In Fig. 7, as the path loss coefficient increases, the attenuation increases at this time. The energy efficiency in OPA, FSA and APA schemes will be greatly reduced. When the attenuation is too large, it is difficult for all three schemes to maintain normal communication. The advantages of the OPA solution are no longer so prominent. However, in the normal communication range, the performance of the OPA scheme is still better than other schemes.

Finally, the authors thank the reviewer for the comments provided, and the time and efforts he/she has spent in the review again. Without the careful comments, the paper would not reach its current quality. We hope that the above modifications have answered the reviewer's concerns.

We look forward to hearing from you regarding our submission. We would be glad to respond to any further questions and comments that you may have.