The Three Body Decay $\pi_0 \to \gamma\chi\chi$

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Abstract

Set of notes describing the kinematics for the two subsequent on-shell two-body decays $\pi_0 \to \gamma V$, $V \to \chi\chi$. The kinematics constrain the masses to lie in the ranges $2m_\chi < m_V < m_\pi$.

Contents

1 The First Decay

The first decay in the process involves $\pi_0 \to \gamma V$ where V is some dark vector mediator that kinetically mixes with the standard model photon. In the situation of interest, we have a distribution of pion energies which we wish to translate into a distribution of dark matter energies. Let p^π_μ be the 4-momentum of the pion in the lab frame. We choose our co-ordinate frame such that the pion is propagating along the z-axis. In the lab frame, this 4-momenta is given by;

$$p^\pi_\mu = (E_\pi, 0, 0, \sqrt{E_\pi^2 - m_\pi^2}) \tag{1.1}$$

whilst in the rest frame of the pion we have,

$$p^{\pi,c}_\mu = (m_\pi, \mathbf{0}) \tag{1.2}$$

In the rest frame, we can consider the decay as shown in Figure 1.1. The angles θ and ϕ parametrise the decay and should be sampled from suitable distributions. In the implementation of what follows, we take $\phi \in U[0, 2\pi]$ and $\cos\theta \in U[-1, 1]$. Note that this of course does not mean the angles are uniformly distributed in the lab frame since they are boosted in the direction of the pion.

1.1 Energy and Momentum Conservation

Conserving energy and momentum in this frame;

$$\sqrt{\left|\mathbf{p}^c_V\right|^2 + m_V^2} + \left|\mathbf{p}^c_\gamma\right| = m_\pi \tag{1.3}$$

$$\mathbf{p}^c_V = -\mathbf{p}^c_\gamma \tag{1.4}$$

Here the superscript indicates that the quantities are measured in the rest frame of the pion. Substituting the second relation into the first,

$$\sqrt{\left|\mathbf{p}^c_V\right|^2 + m_V^2} + \left|\mathbf{p}^c_V\right| = m_\pi \tag{1.5}$$

Solving this for $|\mathbf{p}^c_V|$, we find,

$$|\mathbf{p}^c_V| := p_V = \frac{m_\pi^2 - m_V^2}{2m_\pi} \tag{1.6}$$

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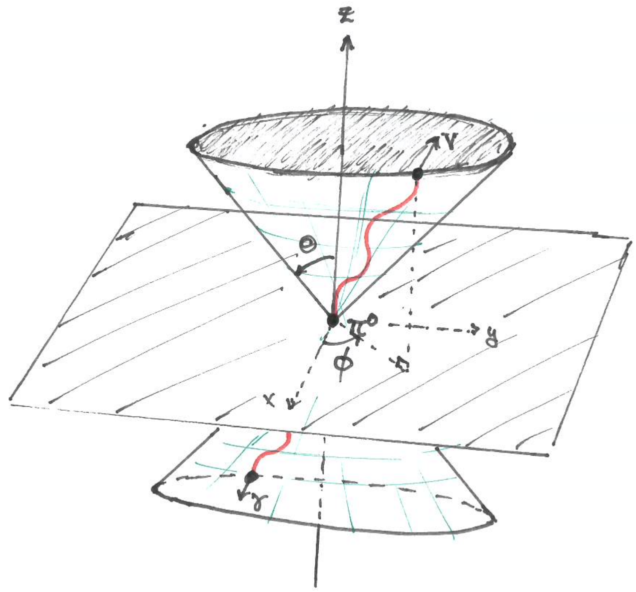

Figure 1.1 *We parametrise the decay of the π_0 using two angles, θ and ϕ which ar distributed accordingly.*

1.2 Going Back to the Lab Frame

We now have the 4-momentum for the massive mediator in the pion rest frame via the parametrisation,

$$p_\mu^{V,c} = (\sqrt{p_V^2 + m_V^2}, p_V \sin\theta\cos\phi, p_V \sin\theta \sin\phi, p_V \cos\theta) \tag{1.7}$$

We can then boost to the lab frame by making a Lorentz transformation with respect to the pion 3-momentum. This leads to a Lorentz transformation of the form,

$$\Lambda_1 = \begin{pmatrix} \gamma_1 & 0 & 0 & \gamma_1\beta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma_1\beta_1 & 0 & 0 & \gamma_1 \end{pmatrix} \tag{1.8}$$

where the γ and β factors are given as;

$$\gamma_1 = \frac{E_\pi}{m_\pi}, \; \beta_1 = \left(1 - \frac{1}{\gamma^2}\right)^{\frac{1}{2}} \tag{1.9}$$

The 4-momentum of the mediator is then given via $p_\mu^V := (E^V, \mathbf{p}^V) = (\Lambda_1 p^{V,c})_\mu$;

$$E^V = \gamma_1\sqrt{p_V^2 + m_V^2} + \gamma_1\beta_1 p_V \cos\theta \tag{1.10}$$

$$p_x^V = p_V \sin\theta\cos\phi \tag{1.11}$$

$$p_y^V = p_V \sin\theta\sin\phi \tag{1.12}$$

$$p_z^V = \gamma_1\beta_1\sqrt{p_V^2 + m_V^2} + \gamma_1 p_V \cos\theta \tag{1.13}$$

where p_V, γ_1 and β_1 are as defined in (1.6) and (1.9).

2 The Second Decay

We need to repeat the process above but for the decay $V \to \chi\chi$. There will be two differences. Firstly, both the products of the decay have the same mass. Secondly, the mediator V no longer has a 3-momentum along the z axis so the form of the Lorentz boost will change. We construct an identical diagram to Figure 1.1 except we relabel the angles to be θ^V and ϕ^V. We let the 4-momentum of the dark matter particles in the rest frame of V be;

$$p_\mu^{1,c} = (E_1^c, \mathbf{p}_1^c),\ p_\mu^{2,c} = (E_1^c, -\mathbf{p}_1^c) \tag{2.1}$$

Now conservation of energy in the rest frame of V gives;

$$2\sqrt{|p_1^c|^2 + m_\chi^2} = m_V \Rightarrow |\mathbf{p}_1^c| := p_\chi = \sqrt{\frac{1}{4}m_V^2 - m_\chi^2} \tag{2.2}$$

With the parameterisation given, the 4-momentum of the dark matter particles in the rest frame of V is now given by;

$$E_1^c = \sqrt{p_\chi^2 + m_\chi^2}, \mathbf{p}_1^c = (p_\chi \sin\theta^V \cos\phi^V, \ldots) \tag{2.3}$$

2.1 Boosting to the Lab Frame

The final step is to boost back to the lab frame using a suitable Lorentz transformation. This is determined by the 3-momentum of the mediator. The Lorentz matrix then takes the form;

$$\Lambda_2 = \begin{pmatrix} \gamma_2 & \gamma_2\beta_2\frac{p_x^V}{q_V} & \gamma_2\beta_2\frac{p_y^V}{q_V} & \gamma_2\beta_2\frac{p_z^V}{q_V} \\ \gamma_2\beta_2\frac{p_x^V}{q_V} & 1+(\gamma_2-1)\frac{(p_x^V)^2}{q_V^2} & (\gamma_2-1)\frac{p_x^V p_y^V}{q_V^2} & (\gamma_2-1)\frac{p_x^V p_z^V}{q_V^2} \\ \gamma_2\beta_2\frac{p_y^V}{q_V} & (\gamma_2-1)\frac{p_y^V p_x^V}{q_V^2} & 1+(\gamma_2-1)\frac{(p_y^V)^2}{q_V^2} & (\gamma_2-1)\frac{p_y^V p_z^V}{q_V^2} \\ \gamma_2\beta_2\frac{p_z^V}{q_V} & (\gamma_2-1)\frac{p_z^V p_x^V}{q_V^2} & (\gamma_2-1)\frac{p_z^V p_y^V}{q_V^2} & 1+(\gamma_2-1)\frac{(p_x^V)^2}{q_V^2} \end{pmatrix} \tag{2.4}$$

where we have,

$$\gamma_2 = \frac{E^V}{m_V},\ \beta_2 = \left(1 - \frac{1}{\gamma_2^2}\right)^{\frac{1}{2}},\ q_V = \sqrt{(p_x^V)^2 + (p_y^V)^2 + (p_z^V)^2} \tag{2.5}$$

The energies and 3-momenta of the dark matter particles, in the lab frame of the original π_0 are then obtained via;

$$p_\mu(\chi_1) = (\Lambda_2 p^{1,c})_\mu,\ p_\mu(\chi_2) = (\Lambda_2 p^{2,c})_\mu \tag{2.6}$$

The resulting energies, $E_\chi^{1,2}$ can be obtained from the zeroth components, as well as the kinetic energies $T_\chi^{1,2} = E_\chi^{1,2} - m_\chi$. Explicitly, we find;

$$E_\chi^{1,2} = \gamma_2\sqrt{p_\chi^2 + m_\chi^2} \pm \frac{\gamma_2\beta_2 p_\chi}{q_V}\left(p_x^V \sin\theta^V\cos\phi^V + p_y^V \sin\theta^V \sin\phi^V + p_z^V\cos\theta^V\right) \tag{2.7}$$

where p_χ is defined in (2.2), $p_V^{x,y,z}$ in (1.11) - (1.13), and q_V, γ_2, β_2 in (2.5).