# Lab 2: System Properties and Convolution

## A. Impulse Response

**Problem A.1:** Use MATLAB command poly to generate the characteristic polynomial from the characteristic values specified by lambda.

Code:

```
R = [1e4, 1e4, 1e4];
C = [1e-9, 1e-6];

A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];

lambda = roots(A);
```

Results:

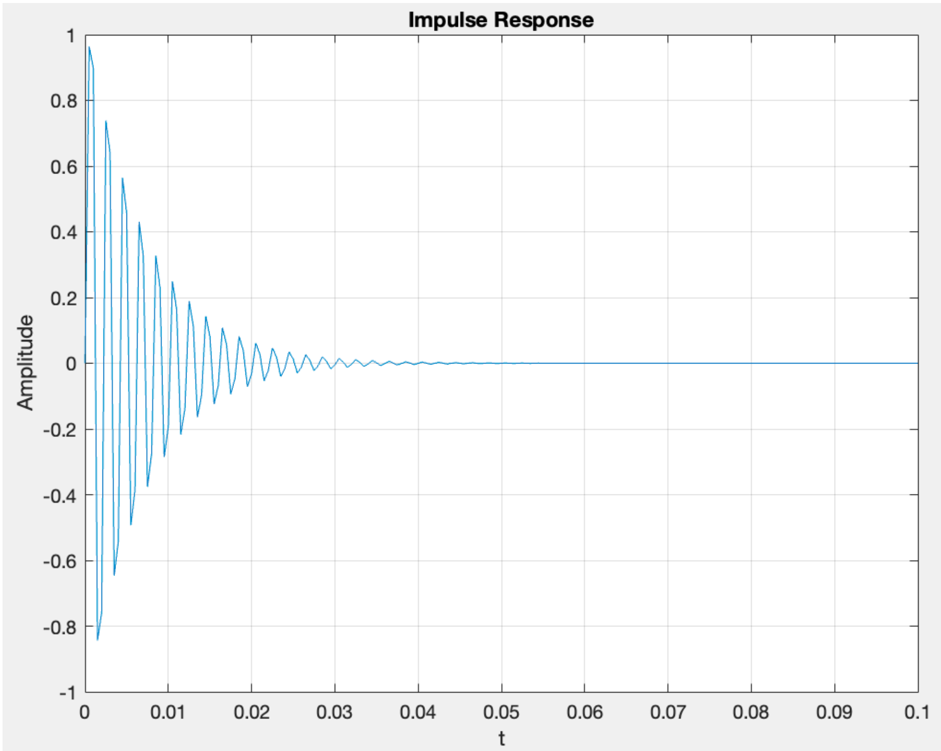
```
>> CH2MP1
>> lambda

lambda =

   1.0e+03 *

  -0.1500 + 3.1587i
  -0.1500 - 3.1587i

>> poly(A)

ans =

   1.0e+09 *

    0.0000   -0.0100    3.0100   -3.0000
```

**Problem A.2:** Plot the impulse response of the system in Problem A.1 for t = [0:0.0005:0.1].

Code:

```
R = [1e4, 1e4, 1e4];
C = [1e-9, 1e-6];

A0 = 1;
A1 = (1/R(1)+1/R(2)+1/R(3))/C(2);
A2 = 1/(R(1)*R(2)*C(1)*C(2));
A = [A0 A1 A2];

lambda = roots(A);

H = tf(A2, A);
t = 0:0.0005:0.1;
u = (t == 0);
y = lsim(H,u,t);
figure; plot(t,y);grid;
xlabel('t');ylabel('Amplitude');title('Impulse Response');
```

Plot:




**Problem A.3:** Complete Lathi, Section 2.7-2 Function M-Files, page 214.

Code:

```
function [lambda] = CH2MP2(R,C)
% CH2MP2.m : Chapter 2, MATLAB Program 2
% Function M-file finds characteristic roots of op-amp circuit.
% INPUTS:    R = length-3 vector of resistances
%            C = length-2 vector of capacitances
% OUTPUTS:  lambda = characteristic roots
% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A);
```

Results:

```
>> lambda = CH2MP2([1e4, 1e4, 1e4],[1e-9, 1e-6])

lambda =

   1.0e+03 *

  -0.1500 + 3.1587i
  -0.1500 - 3.1587i
```

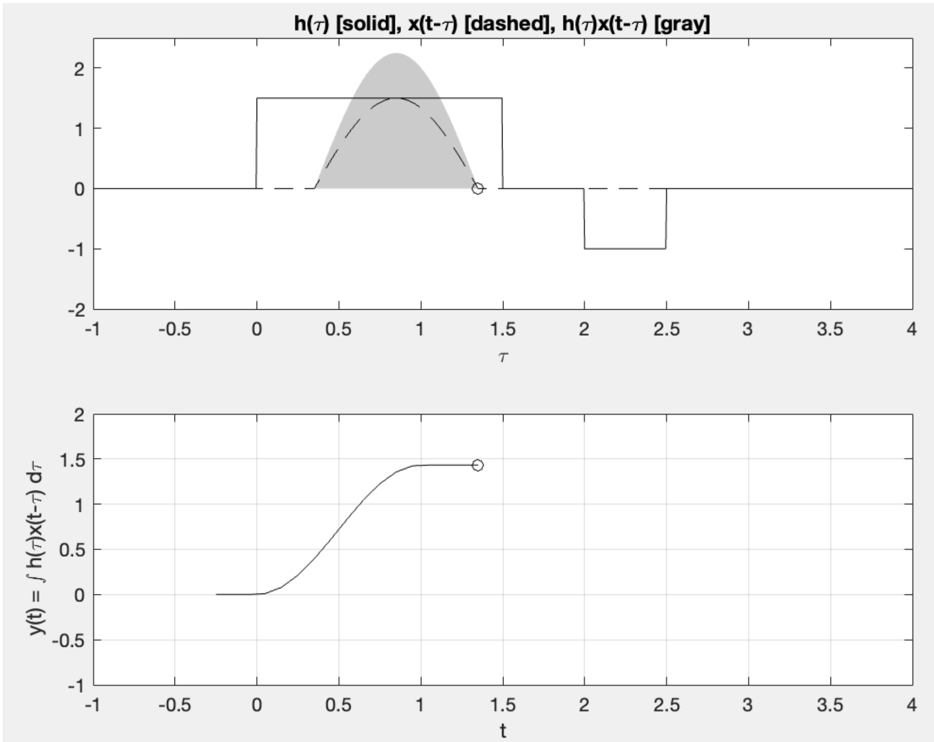
## B. Convolution

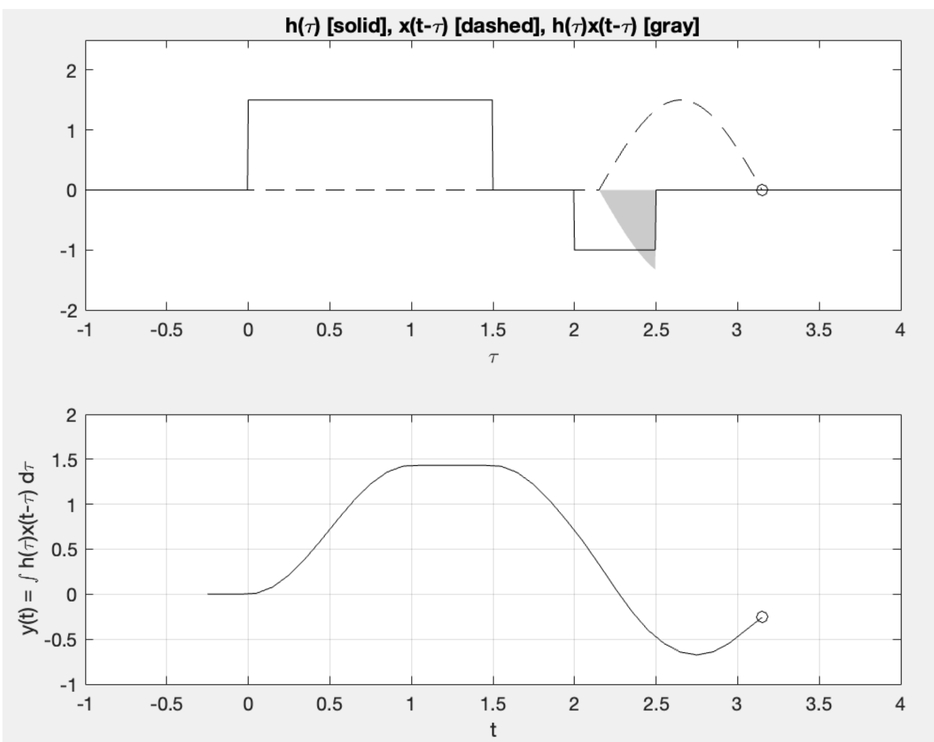
**Problem B.1:** Plot y(t) at step t = 2.25 as shown in Figure 2.28. Use the MATLAB command pause instead of drawnow to observe the steps of the convolution operation slowly.

Code:

```
figure(1) % Create figure window
u = @(t) 1.0*(t>=0);
x = @(t) 1.5*sin(pi*t).*(u(t)-u(t-1));
h = @(t) 1.5*(u(t)-u(t-1.5))-u(t-2)+u(t-2.5);
dtau = 0.005; tau = -1:dtau:4;
ti = 0; tvec = -.25:.1:3.75;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec,
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau); lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

Plots:


h(τ) [solid], x(t-τ) [dashed], h(τ)x(t-τ) [gray]
τ
y(t) = ∫ h(τ)x(t-τ) dτ
t

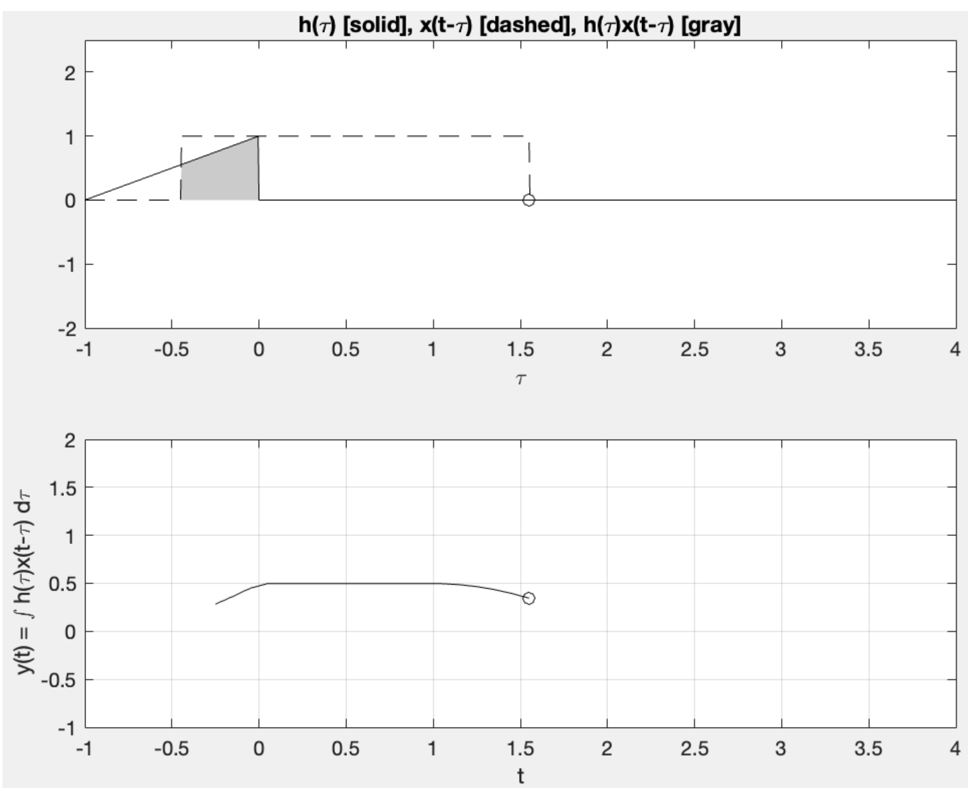

h(τ) [solid], x(t-τ) [dashed], h(τ)x(t-τ) [gray]
τ
y(t) = ∫ h(τ)x(t-τ) dτ
t

**Problem B.2:** Perform the convolution of the signal x(t) in Figure P2.4-28 (a) with h(t) in Figure P2.4-30. Plot all signals and results.

Code:

```
figure(2)
u = @(t) 1.0*(t>=0);
x = @(t) u(t)-u(t-2);
h = @(t) (t+1).*(u(t+1)-u(t));
dtau = 0.005; tau = -1:dtau:4;
ti = 0; tvec = -.25:.1:3.75;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau); lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

Plots:

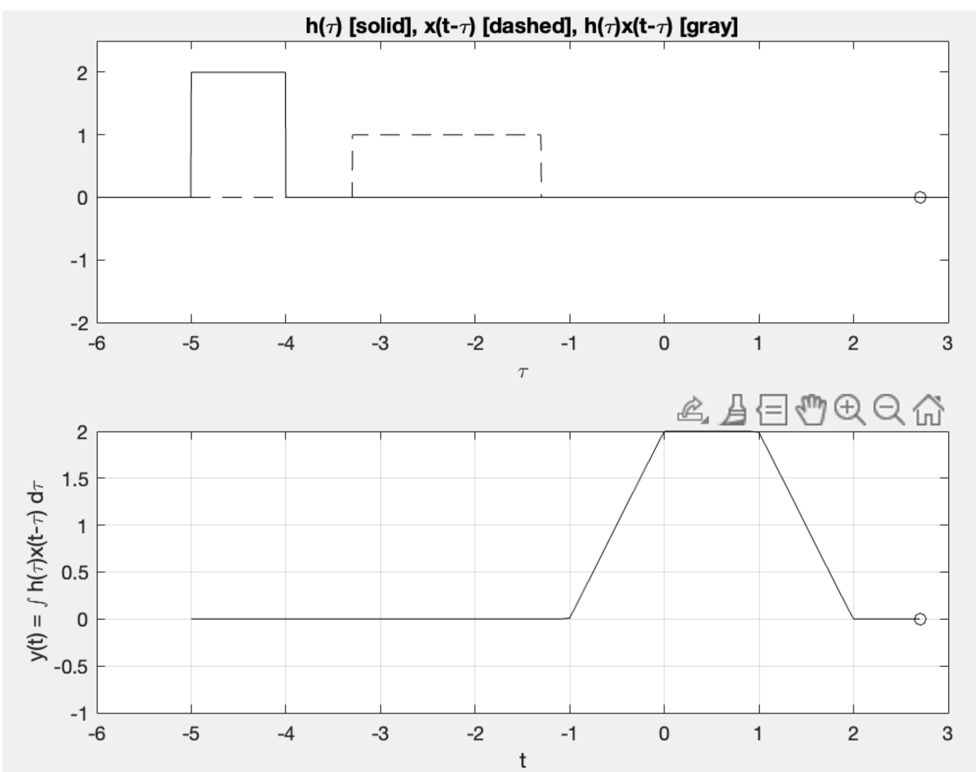


**Problem B.3:** Perform the convolution of the signal x1(t) and x2(t) in Figure P2.4-27(a), (b) and (h). Plot all signals and results.

Code: (a)

```
figure(3)
u = @(t) 1.0*(t>=0);
A = 1; B = 2; %Assumption
x = @(t) A*(u(t-4)-u(t-6));
h = @(t) B*(u(t+5)-u(t+4));
dtau = 0.005; tau = -6:dtau:3;
ti = 0; tvec = -5:.1:5;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau); lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

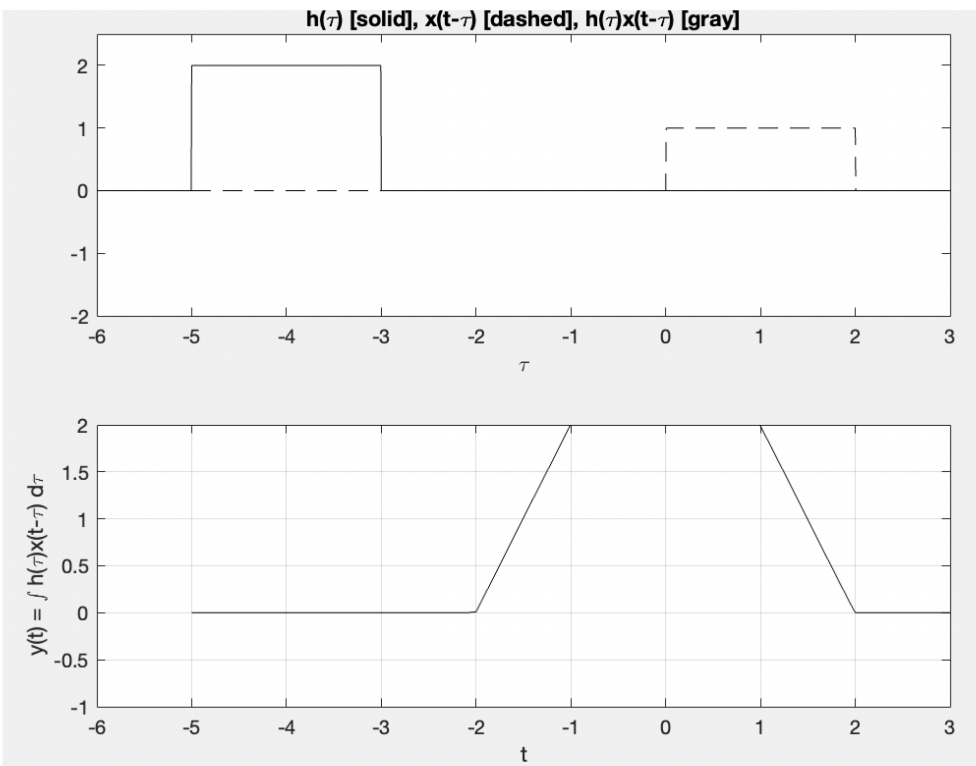
Plot: (a)



Code: (b)

```
figure(4)
u = @(t) 1.0*(t>=0);
A = 1; B = 2; %Assumption
x = @(t) A*(u(t-3)-u(t-5));
h = @(t) B*(u(t+5)-u(t+3));
dtau = 0.005; tau = -6:dtau:3;
ti = 0; tvec = -5:.1:5;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau); lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

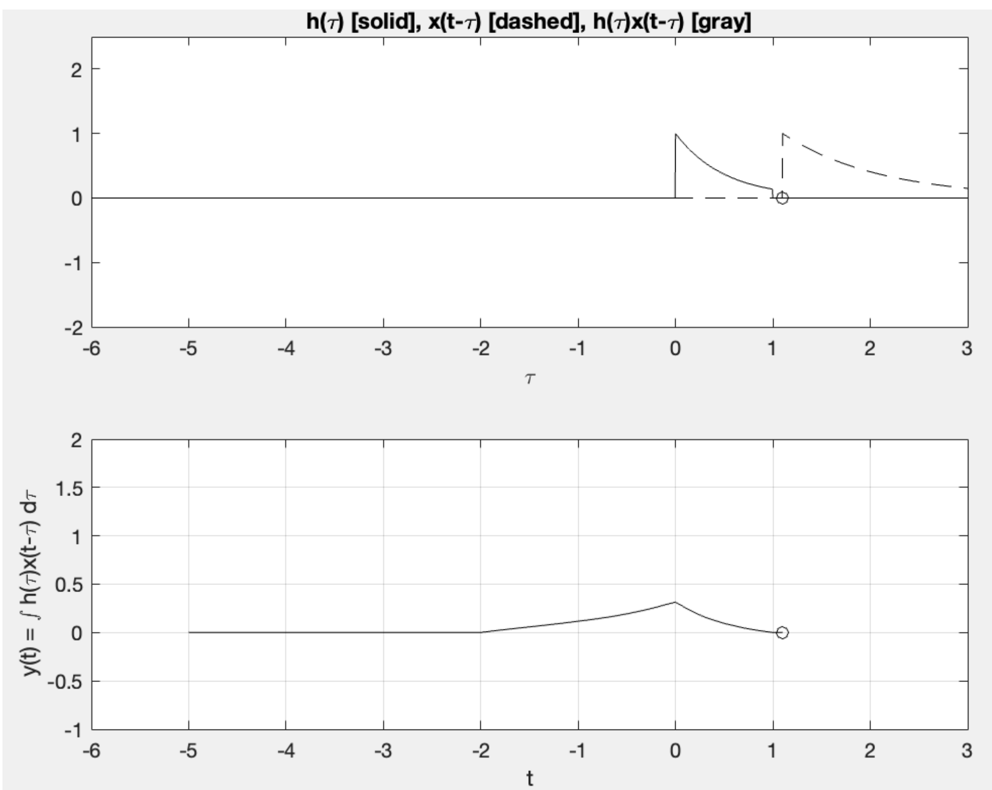
Plot: (b)



Code: (h)

```
figure(5)
u = @(t) 1.0*(t>=0);
x = @(t) exp(t).*(u(t+2)-u(t));
h = @(t) exp(-2*t).*(u(t)-u(t-1));
dtau = 0.005; tau = -6:dtau:3;
ti = 0; tvec = -5:.1:5;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau); lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

Plot: (h)



## C. System Behavior and Stability

**Problem C.1:** Consider the LTI systems S1, S2, S3 and S4 represented by their respective unit impulse response functions given as follow:

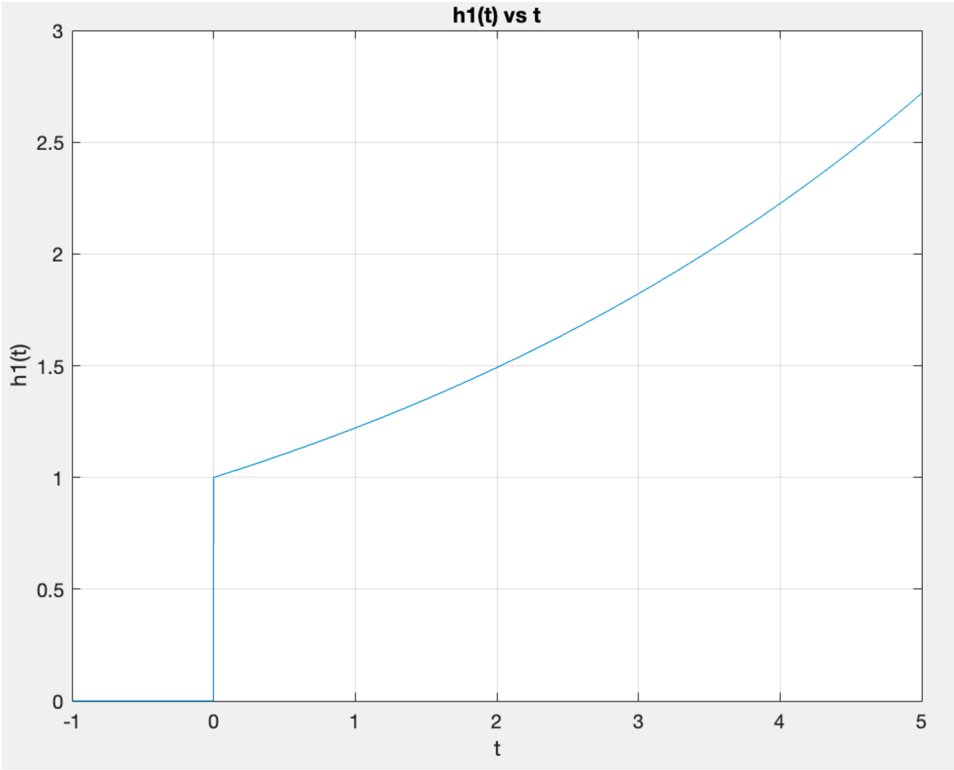
$$h_1(t) = e^{\frac{t}{5}}u(t); \tag{3}$$
$$h_2(t) = 4e^{-\frac{t}{5}}u(t); \tag{4}$$
$$h_3(t) = 4e^{-t}u(t); \tag{5}$$
$$h_4(t) = 4\left(e^{-\frac{t}{5}} - e^{-t}\right)u(t); \tag{6}$$

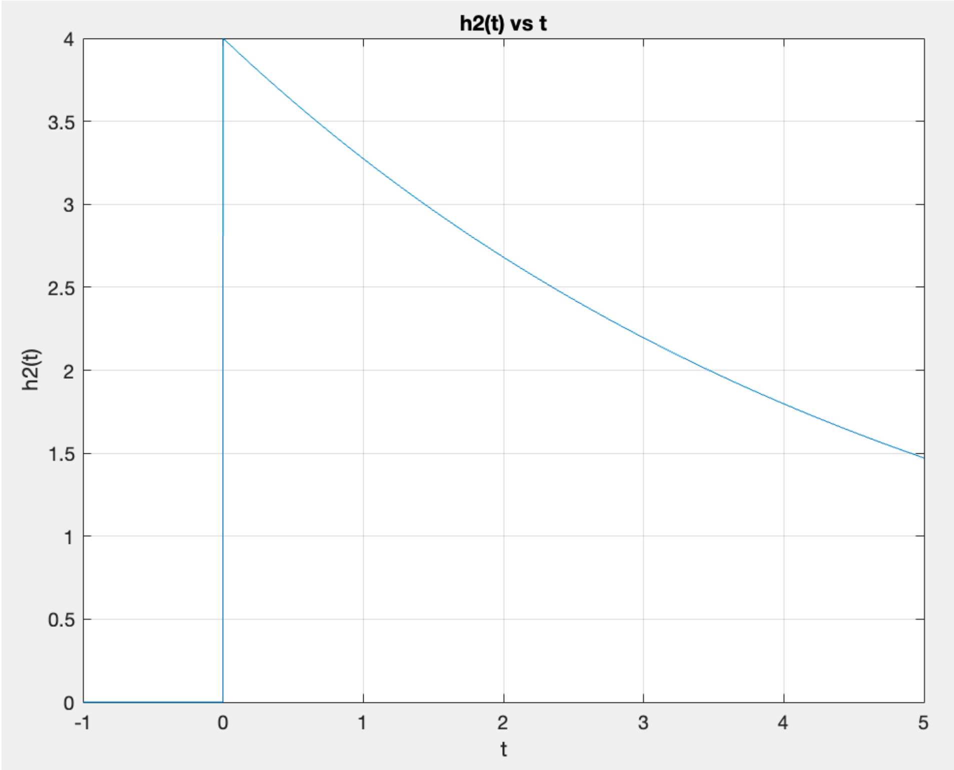
Plot each unit response function for t = [-1:0.001:5].

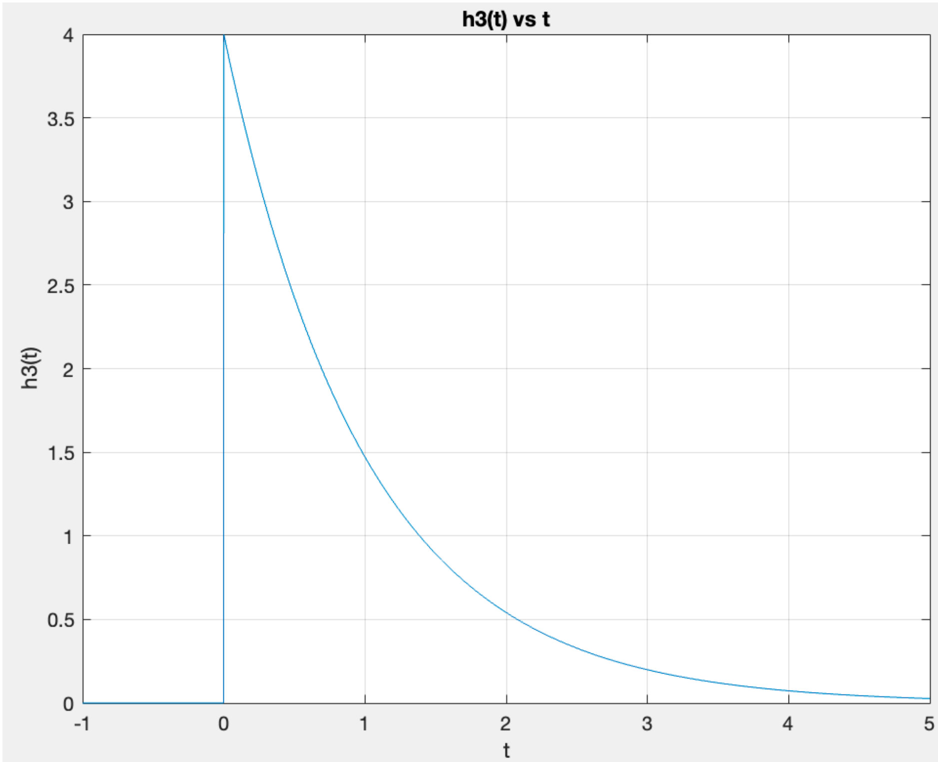
Code:

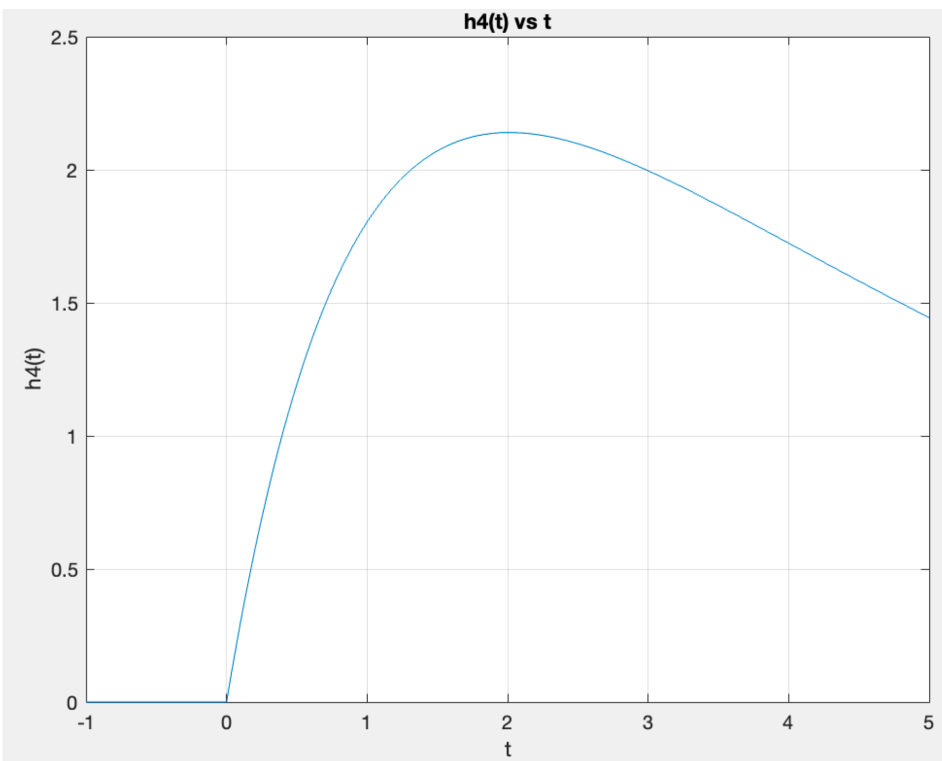
```
u = @(t) 1.0.* (t >= 0);
h1 = @(t) exp(t/5).*u(t);
h2 = @(t) 4*exp(-t/5).*u(t);
h3 = @(t) 4*exp(-t).*u(t);
h4 = @(t) 4*(exp(-t/5) - exp(-t)).*u(t);

t = [-1:0.001:5];

plot(t, h1(t));grid;
xlabel('t'); ylabel('h1(t)');title('h1(t) vs t');
plot(t, h2(t));grid;
xlabel('t'); ylabel('h2(t)');title('h2(t) vs t');
plot(t, h3(t));grid;
xlabel('t'); ylabel('h3(t)');title('h3(t) vs t');
plot(t, h4(t));grid;
xlabel('t'); ylabel('h4(t)');title('h4(t) vs t');

plot(t, h1(t));
xlabel('t');ylabel('h(t)');title('h(t) vs t');
hold on;
plot(t, h2(t));
plot(t, h3(t));
plot(t, h4(t));
hold off;
```

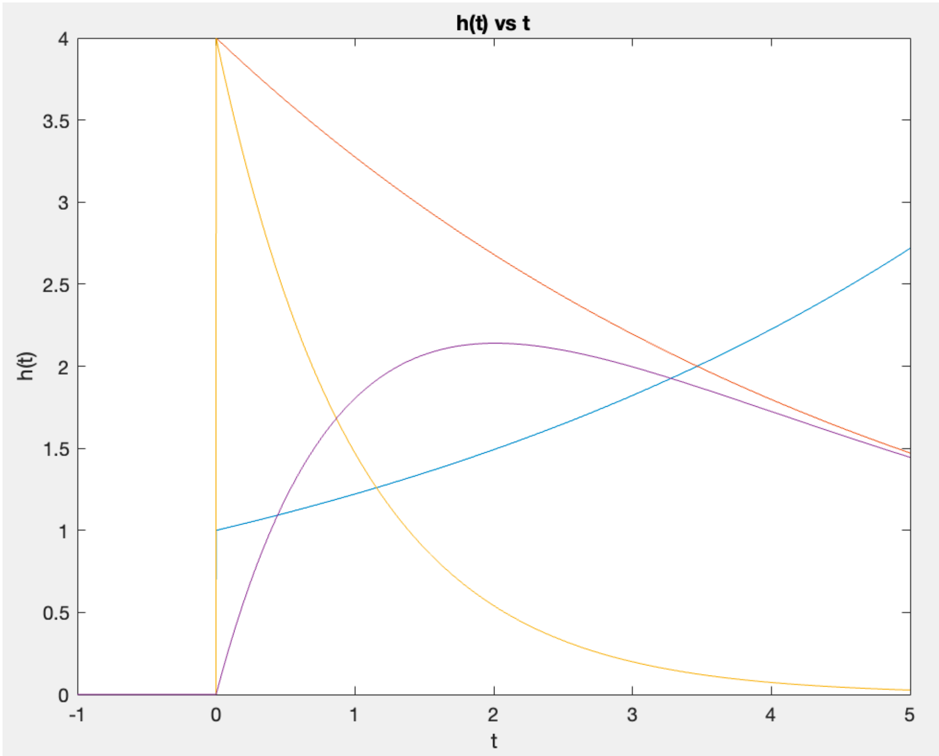
Plots:


h1(t) vs t
h1(t)
t
3
2.5
2
1.5
1
0.5
0
-1
0
1
2
3
4
5


h2(t) vs t
h2(t)
t
4
3.5
3
2.5
2
1.5
1
0.5
0
-1
0
1
2
3
4
5


h3(t) vs t
h3(t)
t
4
3.5
3
2.5
2
1.5
1
0.5
0
-1
0
1
2
3
4
5


h4(t) vs t
h4(t)
t
2.5
2
1.5
1
0.5
0
-1
0
1
2
3
4
5




**Problem C.2:** Determine the characteristic values (eigenvalues) of systems S1-S4.

Characteristic values (Eigenvalues) of systems S1-S4

S1: $\lambda 1 = 1/5$

S2: $\lambda 1 = -1/5$

S3: $\lambda 1 = -1$

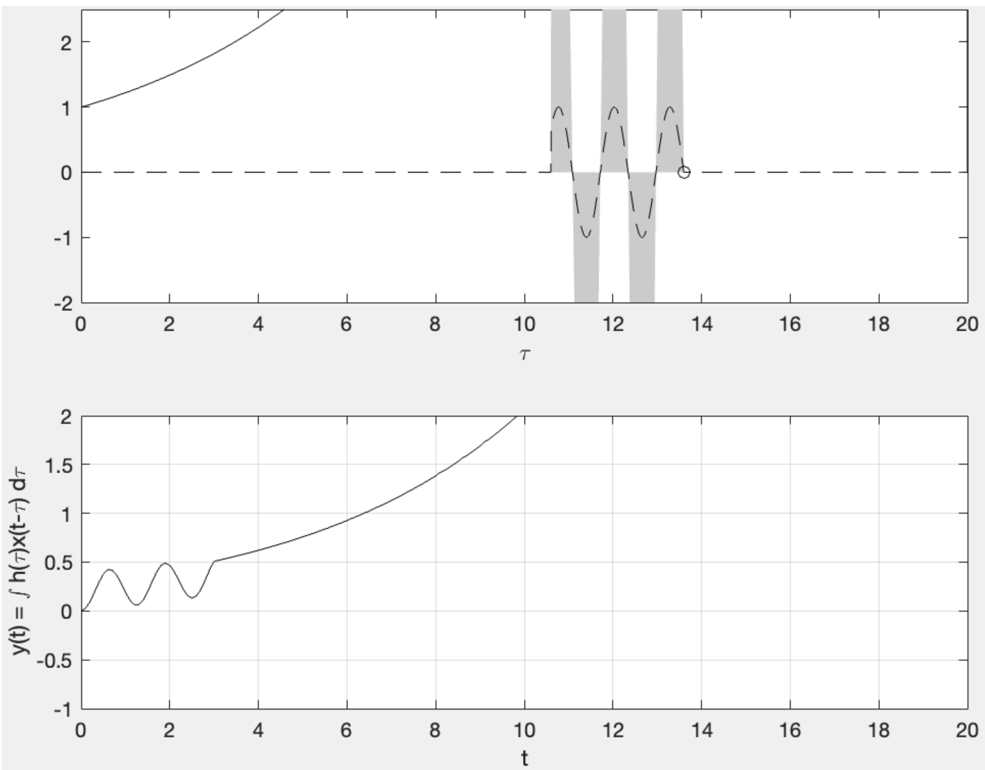
S4: $\lambda 1 = -1/5$, $\lambda 2 = -1$

**Problem C.3:** Truncate the impulse response functions h1(t)….h4(t) such that they are nonzero only for 0 <= t <= 20. Determine the convolution of the truncated impulse response functions with the input signal x(t) = [u(t) – u(t – 3)]sin5t using the M-file in Problem B.1 with the following changes tau = [0:dtau:20] and tvec = [0:0.1:20]. Plot the output of each system. State and explain your observations. Is there any relationship between the outputs of systems S2, S3 and S4? Explain.

Code: (h1)

```
figure(6)
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) exp(t/5).*(u(t)-u(t-20));
dtau = 0.005; tau = 0:dtau:20;
ti = 0; tvec = 0:.1:20;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau); lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h1(t) and x(t)');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

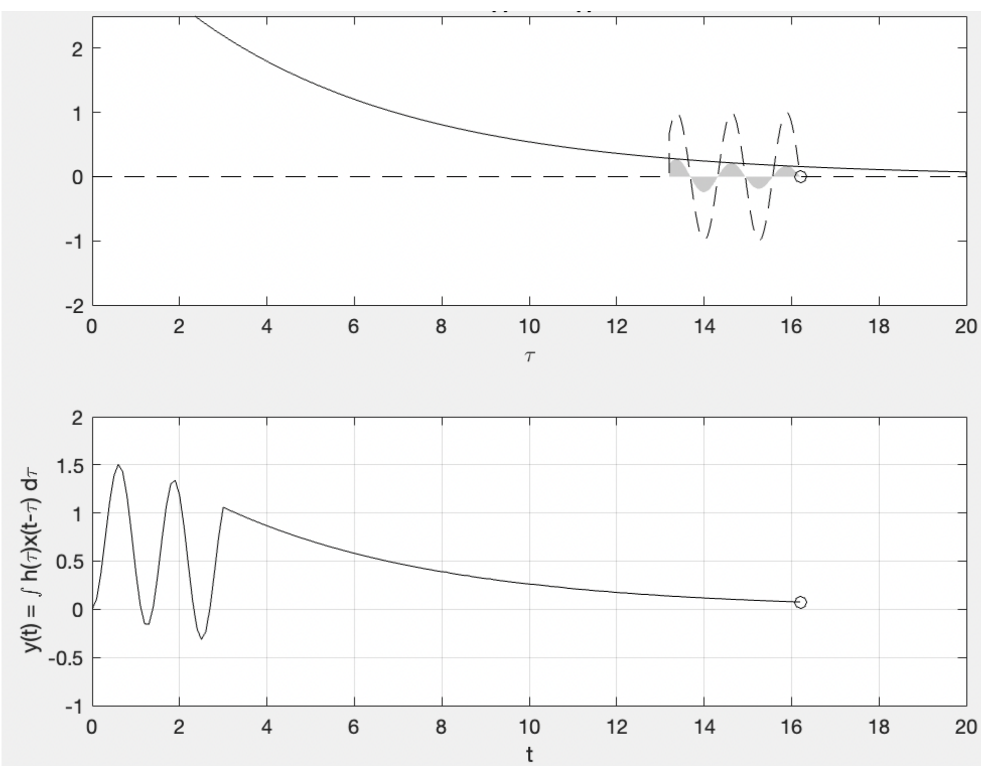
Plot: (h1)



Code: (h2)

```
figure(7)
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) 4*exp(-t/5).*(u(t)-u(t-20));
dtau = 0.005; tau = 0:dtau:20;
ti = 0; tvec = 0:.1:20;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau); lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h2(t) and x(t)');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

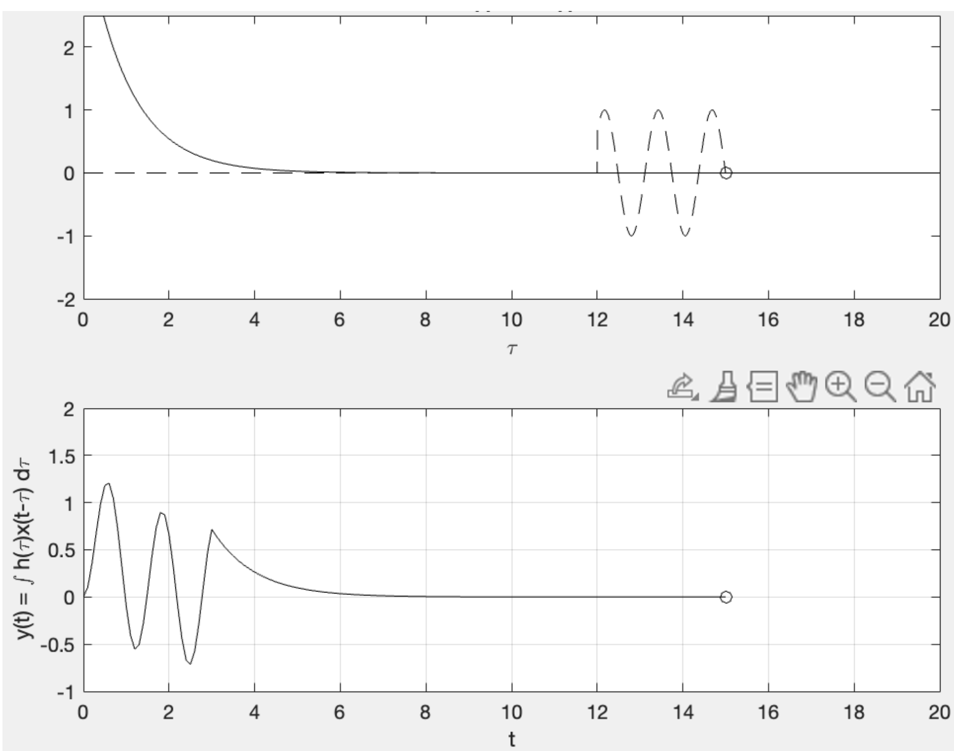
Plot: (h2)



Code: (h3)

```matlab
figure(8)
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) 4*exp(-t).*(u(t)-u(t-20));
dtau = 0.005; tau = 0:dtau:20;
ti = 0; tvec = 0:.1:20;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau); lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h3(t) and x(t)');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

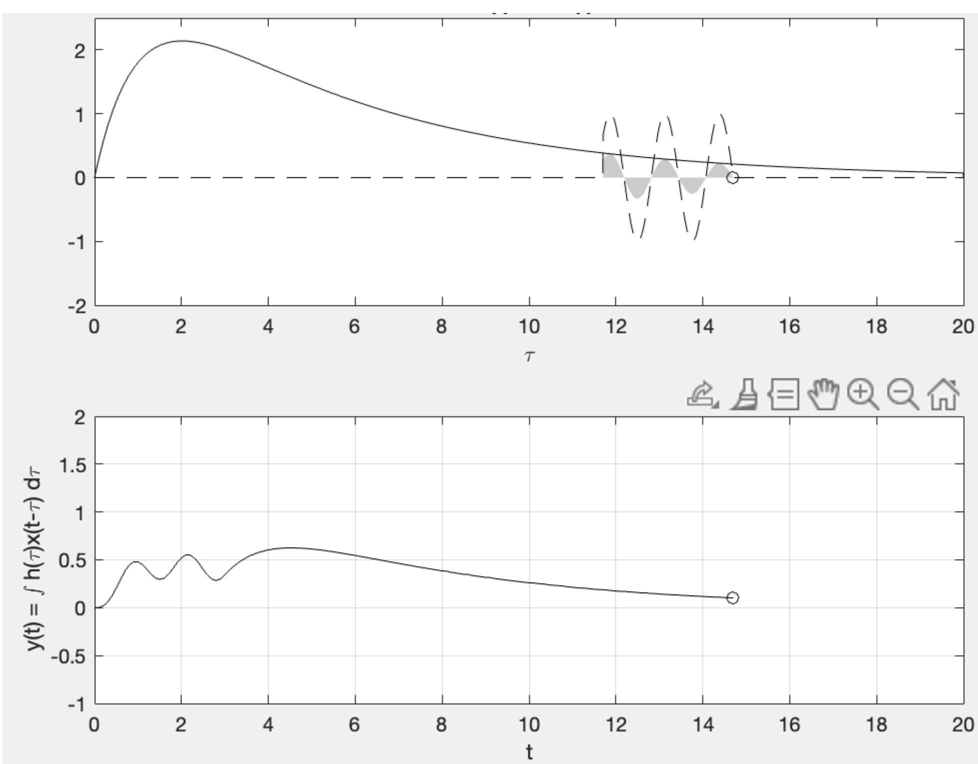
Plot: (h3)



Code: (h4)

```
figure(9)
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) 4*(exp(-t/5)-exp(-t)).*(u(t)-u(t-20));
dtau = 0.005; tau = 0:dtau:20;
ti = 0; tvec = 0:.1:20;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau); lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau'); title('h4(t) and x(t)');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

Plot: (h4)

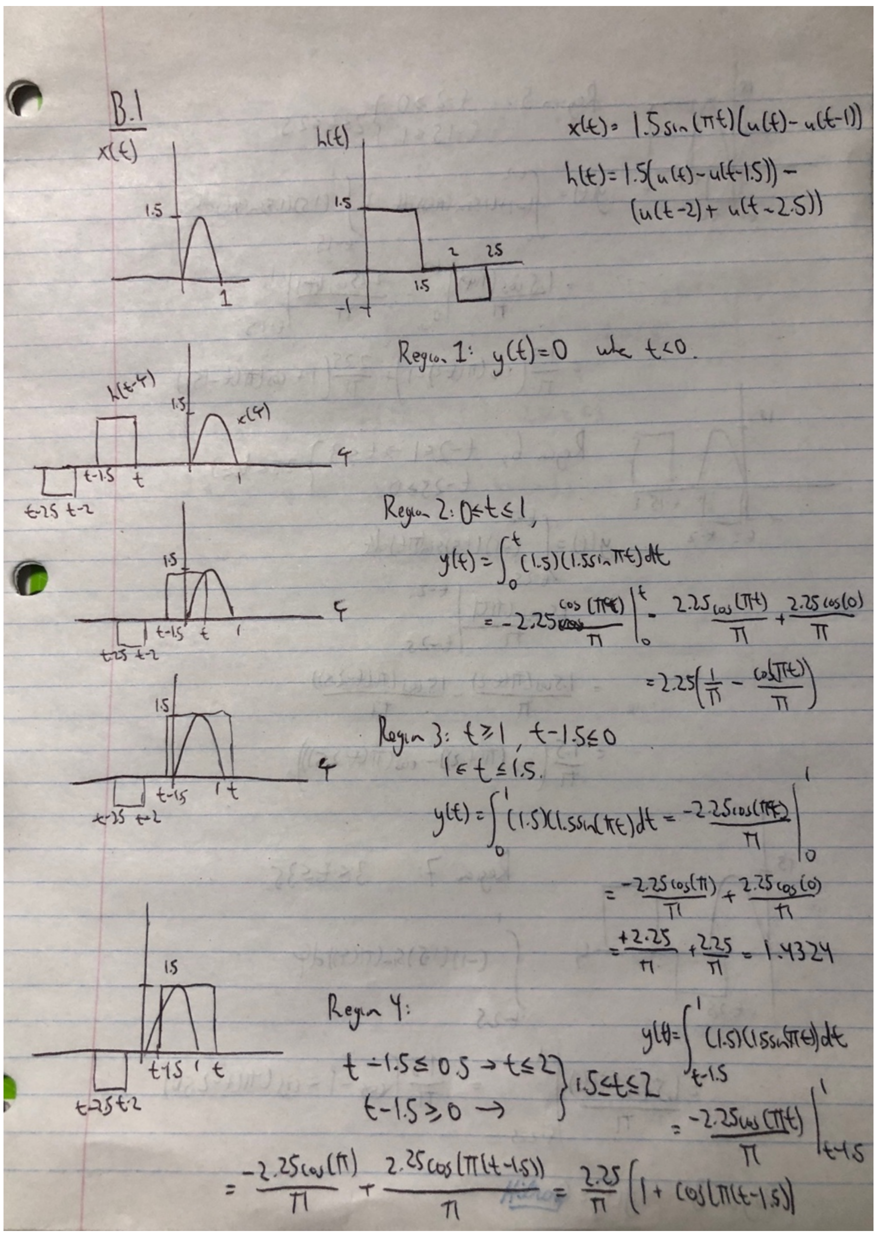


I observed that all the convolutions done in this section had a part of the graph where it closely resembled a sine function. In terms of outputs S2, S3 and S4. S2 and S3 have similar convolutions in terms of its shape while S4 has kind of the same shape but it differs in the way it slopes.

## D. Discussion

**Problem D.1:** Calculate the results of Problems B.1, B.2 and B.3 above by hand and compare to those obtained with your MATLAB code.

B.1:



B.1

$x(t) = 1.5\sin(\pi t)(u(t) - u(t-1))$

$h(t) = 1.5(u(t) - u(t-1.5)) - (u(t-2) + u(t-2.5))$

Region 1: $y(t) = 0$ when $t < 0$.

Region 2: $0 \le t \le 1$,

$y(t) = \int_0^t (1.5)(1.5\sin \pi\tau)\,d\tau = -2.25\frac{\cos(\pi\tau)}{\pi}\Big|_0^t = -\frac{2.25\cos(\pi t)}{\pi} + \frac{2.25\cos(0)}{\pi}$

$= 2.25\left(\frac{1}{\pi} - \frac{\cos(\pi t)}{\pi}\right)$

Region 3: $t \ge 1$, $t - 1.5 \le 0$

$1 \le t \le 1.5$.

$y(t) = \int_0^1 (1.5)(1.5\sin(\pi\tau))\,d\tau = \frac{-2.25\cos(\pi\tau)}{\pi}\Big|_0^1$

$= \frac{-2.25\cos(\pi)}{\pi} + \frac{2.25\cos(0)}{\pi}$

$= \frac{+2.25}{\pi} + \frac{2.25}{\pi} = 1.4324$

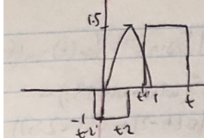
Region 4:

$t - 1.5 \le 0.5 \rightarrow t \le 2$
$t - 1.5 \ge 0 \rightarrow$ $\Big\}\ 1.5 \le t \le 2$

$y(t) = \int_{t-1.5}^1 (1.5)(1.5\sin(\pi\tau))\,d\tau$

$= \frac{-2.25\cos(\pi\tau)}{\pi}\Big|_{t-1.5}^1$

$= \frac{-2.25\cos(\pi)}{\pi} + \frac{2.25\cos(\pi(t-1.5))}{\pi} = \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5))\right)$

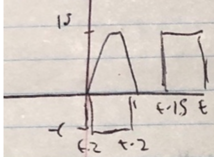



Region 5: $\left.\begin{matrix} t-2 \geq 0 \\ t-1.5 \leq 1 \end{matrix}\right\} 2 \leq t \leq 2.5$

$$y(t) = \int_0^{t-2} (-1)(1.5\sin(\pi t))\,dt + \int_{t-1.5}^{1} (1.5)(1.5\sin(\pi t))\,dt$$

$$= \frac{1.5\cos(\pi t)}{\pi}\Big|_0^{t-2} - \frac{2.25\cos(\pi t)}{\pi}\Big|_{t-1.5}^{1}$$

$$= \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - 1\right) + \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5))\right)$$



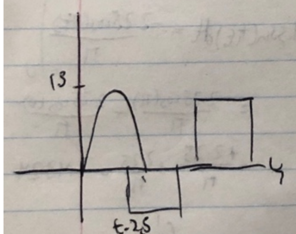

Region 6; $\left.\begin{matrix} t-2 \leq 1 \rightarrow t \leq 3 \\ t-2.5 \geq 0 \end{matrix}\right\} 2.5 \leq t \leq 3$

$$y(t) = \int_{t-2.5}^{t-2} (-1)(1.5)\sin(\pi t)\,dt$$

$$= \frac{1.5\cos(\pi t)}{\pi}\Big|_{t-2.5}^{t-2}$$

$$= \frac{1.5\cos(\pi(t-2))}{\pi} - \frac{1.5\cos(\pi(t-2.5))}{\pi}$$

$$= \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - \cos(\pi(t-2.5))\right)$$




Region 7: $3 \leq t \leq 3.5$

$$\int_{t-2.5}^{1} (-1)(1.5)\sin(\pi(\varphi))\,d\varphi$$

$$= \frac{1.5\cos(\pi t)}{\pi}\Big|_{t-2.5}^{1} = \frac{1.5}{\pi}\left(-1 - \cos(\pi(t-2.5))\right)$$

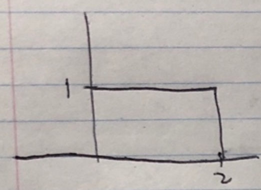
Region 8: $y(t) = 0$

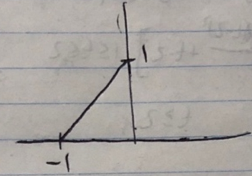
$$y(t) = \begin{cases} 0 & t \leq 0 \\ 2.25\left(\frac{1}{\pi} - \frac{\cos(\pi t)}{\pi}\right) & 0 \leq t < 1 \\ 1.4329 & 1 \leq t \leq 1.5 \\ \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5))\right) & 1.5 \leq t \leq 2 \\ \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - 1\right) + \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5))\right) & 2 \leq t \leq 2.5 \\ \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - \cos(\pi(t-2.5))\right) & 2.5 \leq t \leq 3 \\ \frac{1.5}{\pi}\left(-1 - \cos(\pi(t-2.5))\right) & 3 \leq t \leq 3.5 \\ 0 & t \geq 3.5 \end{cases}$$

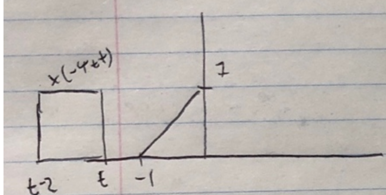
B.2:

B.2

$x(t) = u(t) - u(t-2)$ $\qquad$ $h(t) = (t+1)(u(t+1) - u(t))$

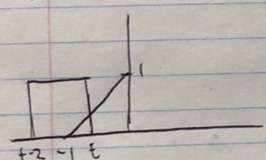



Region 1: $y(t) = 0$

$t \leq -1$

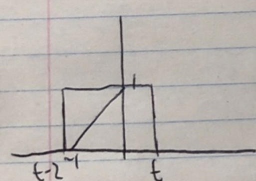



Region 2: $y(t) = \int_{-1}^{t} (1)(\tau+1)\,d\tau$

$-1 \leq t \leq 0$

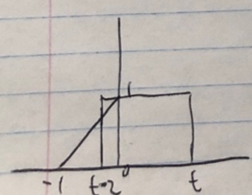
$= \tau\left(\frac{\tau}{2}+1\right)\Big|_{-1}^{t} = \left(\frac{t^2}{2}+t\right) - \left(\frac{1}{2}-1\right)$

$= \frac{t^2}{2} + t + \frac{1}{2}.$




Region 3: $\left.\begin{matrix} t-2 \leq -1 \\ t \geq 0 \end{matrix}\right\} 0 \leq t \leq 1$

$\int_{-1}^{0} (\tau+1)\,d\tau = \frac{\tau^2}{2} + \tau \Big|_{-1}^{0} = \frac{1}{2} - 1 = -0.5$




Region 4: $\left.\begin{matrix} t-2 \leq 0 \\ t-2 \geq -1 \end{matrix}\right\} 1 \leq t \leq 2.$

$\int_{t-2}^{0} (1)(\tau+1)\,d\tau = \frac{\tau^2}{2} + t \Big|_{t-2}^{0} = -\left[\frac{(t-2)^2}{2} + t - 2\right]$

Region 5: $y(t) = 0 \quad t-2 \geq 0 \rightarrow t \geq 2.$

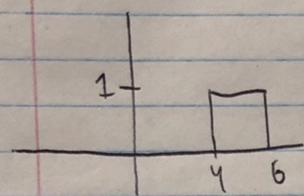
$$y(t) = \begin{cases} 0 & t \leq -1 \\ \frac{t^2}{2} + t + \frac{1}{2} & -1 \leq t \leq 0 \\ -0.5 & 0 \leq t \leq 1 \\ -\left[\frac{(t-2)^2}{2} + t - 2\right] & 1 \leq t \leq 2 \\ 0 & t \geq 2 \end{cases}$$

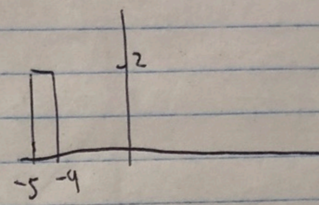
B.3:

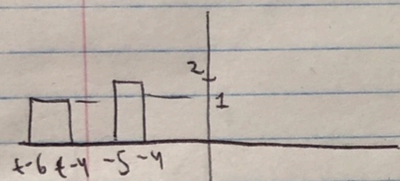
**B.3(a)** Assume $A = 1,\ B = 2$

$x(t) = u(t-4) - u(t-6)$

$h(t) = 2\left(u(t+5) - u(t+4)\right)$

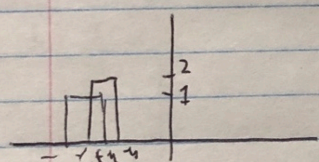


Region 1: $y(t) = 0$

$t-4 \le -5,\quad t \le -1$



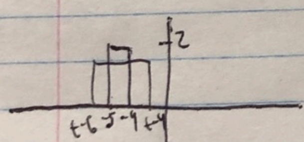
Region 2: $y(t) = \int_{-5}^{t-4} (1)(2)\,d\tau = 2\tau \Big|_{-5}^{t-4}$

$-5 \le t-4 \le -4$

$-1 \le t \le 0$

$= 2(t-4) - (-10)$

$= 2t - 8 + 10 = 2t + 2$

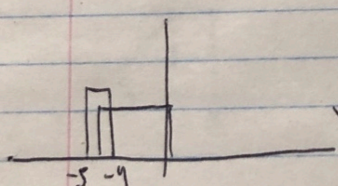


Region 3:

$\left.\begin{array}{l} t-4 \ge -4 \\ t-6 \le -5 \end{array}\right\} 0 \le t \le 1$

$y(t) = \int_{-5}^{-4} 2\,d\tau = 2\tau \Big|_{-5}^{-4} = (-8) - (-10)$

$= 2$



Region 4: $y(t) = \int_{t-6}^{-4} 2\,d\tau = 2\tau \Big|_{t-6}^{-4}$

$1 \le t \le 2$

$= -8 - (2t - 12)$

$= -2t + 4$

Region 5: $y(t) = 0$

$t - 6 \ge -4$

$t \ge 2$

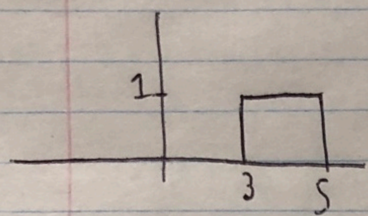
$$y(t) = \begin{cases} 0 & t \le -1 \\ 2t+2 & -1 \le t \le 0 \\ 2 & 0 \le t \le 1 \\ -2t+4 & 1 \le t \le 2 \\ 0 & t \ge 2 \end{cases}$$

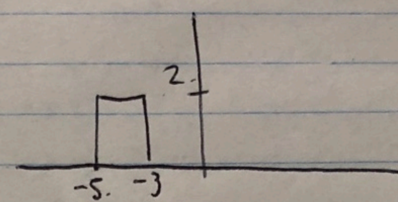
## B.3(b)

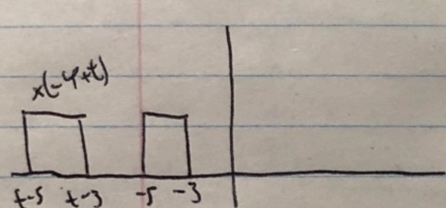
Assume $A=1$, $B=2$

$x(t) = u(t+4-3) - u(t-5)$

$h(t) = 2[u(t+5) - u(t+3)]$

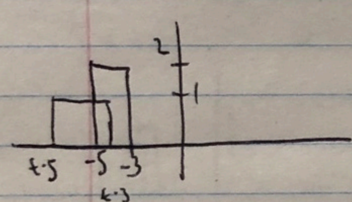



Region 1: $y(t)=0$

$t-3 \le -5 \rightarrow t \le -2.$




Region 2:

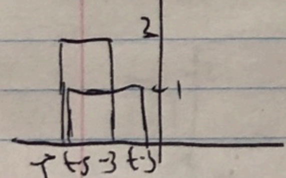
$\left.\begin{array}{l} t-3 \ge -5 \\ t-3 \le -3 \end{array}\right\} -2 \le t \le 0$

$y(t) = \int_{-5}^{t-3} 2\, d\tau = 2\tau \Big|_{-5}^{t-3}$

$= 2(t-3) - (-10)$

$= 2t - 6 + 10$

$= 2t + 4$




Region 3:

$\left.\begin{array}{l} t-3 \ge -3 \\ t-5 \le -3 \end{array}\right\} \quad 0 \le t \le 2$

$y(t) = \int_{t-5}^{-3} 2\, d\tau = 2\tau \Big|_{t-5}^{-3}$

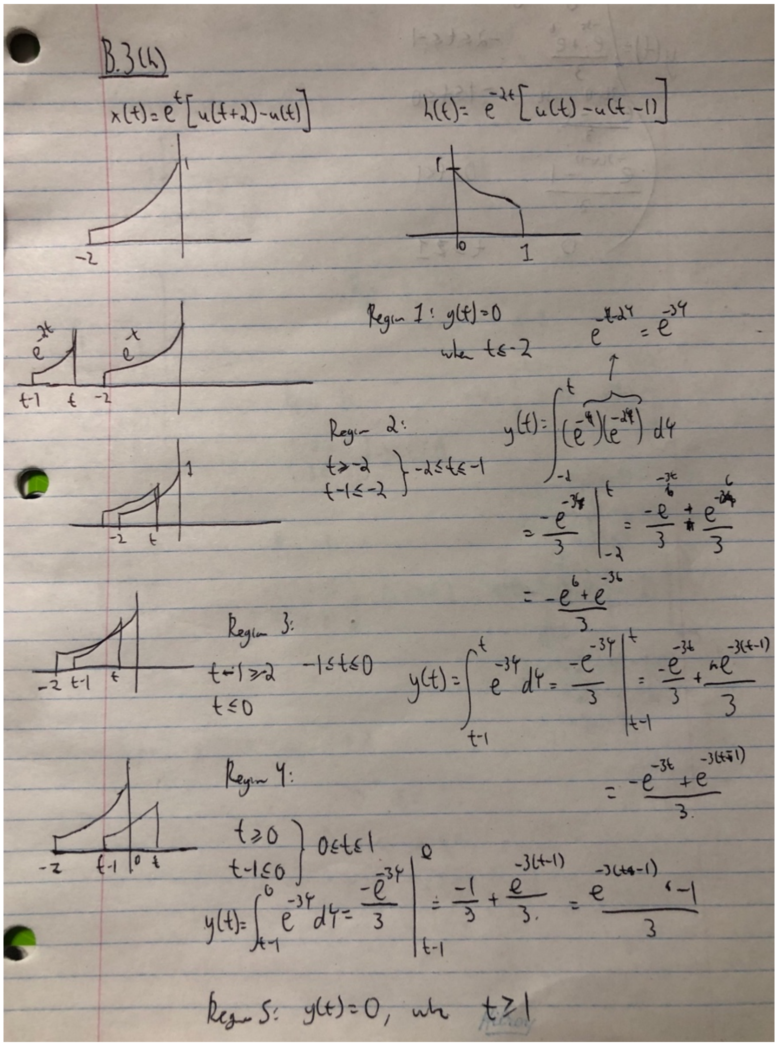
$= 2(t-5) - (-6)$

$= 2t - 10 + 6$

$= 2t - 4$

Region 4: $y(t) = 0$ when $t \ge 2$

$$y(t) = \begin{cases} 0 & t \le -2 \\ 2t+4 & -2 \le t \le 0 \\ 2t-4 & 0 \le t \le 2 \\ 0 & t \ge 2 \end{cases}$$




$$y(t)=\begin{cases} 0 & t\le -2 \\ \dfrac{e^{-3t}+e^{6}}{3} & -2\le t\le -1 \\ \dfrac{e^{-3(t-1)}-e^{-3t}}{3} & -1\le t\le 0 \\ \dfrac{e^{-3(t-1)}-1}{3} & 0\le t<1 \\ 0 & t\ge 1 \end{cases}$$

Overall, the results from the convolutions done by MATLAB and those that were done manually yielded similar results.

**Problem D.2:** What can you say about the width/duration of the signal resulting from the convolution of two signals?

The width/duration of the convoluted signal is equal to the sum of the durations of each signal x(t) and h(t).