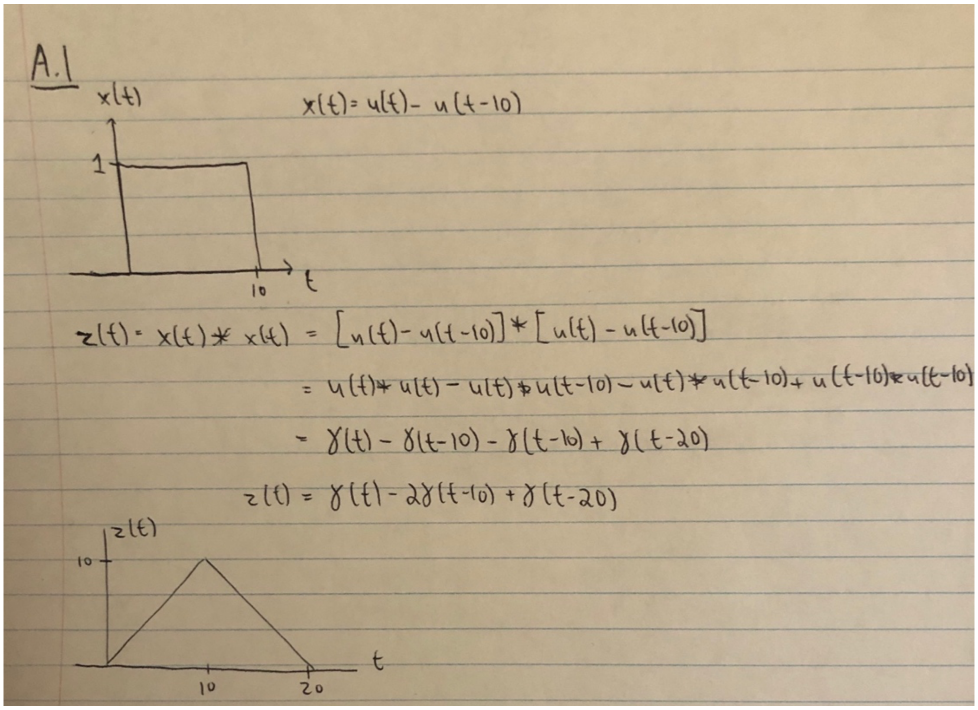
Lab 4: The Fourier Transform: Properties and Applications

A. The Fourier Transform and its Properties

Problem A.1: For the signal x(t) shown in Figure (1), compute and plot z(t) = x(t) * x(t).



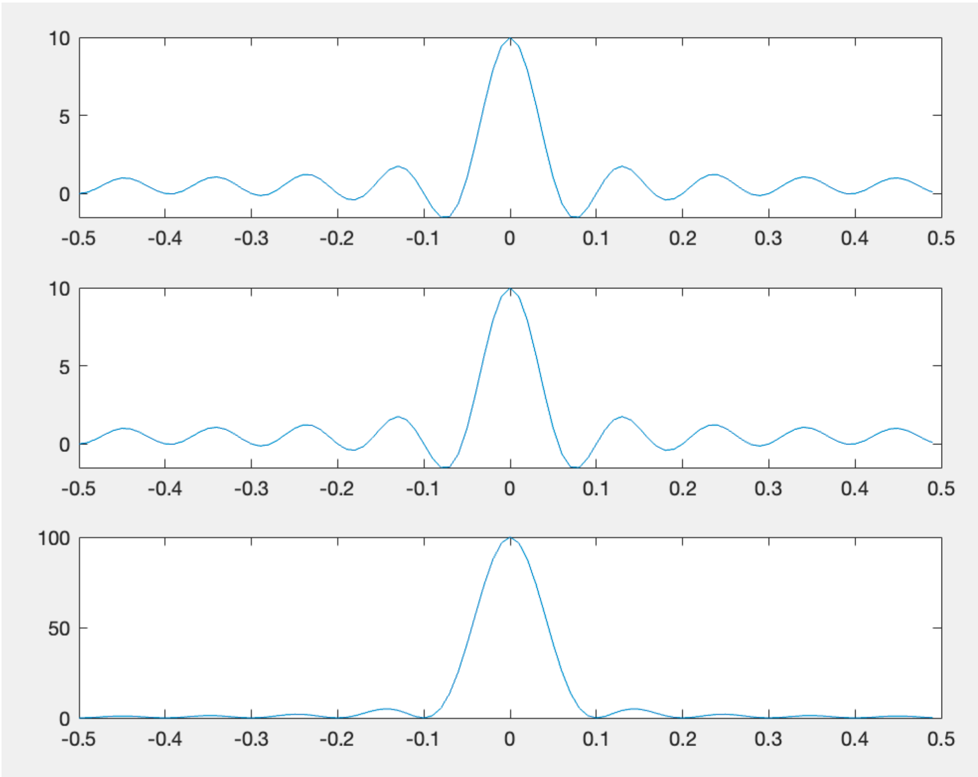
Problem A.2: Using MATLAB, calculate Z(ω) = X(ω)X(ω).

Code:

```
N = 100;
PulseWidth = 10;
t = [0:1:(N-1)];
x = [ones(1, PulseWidth), zeros(1, N-PulseWidth)];

Xf = fft(x);
f = [-(N/2):1:(N/2)-1]*(1/N);

Zf = abs((Xf).^2);

subplot(311);plot(f,fftshift(Xf));
subplot(312);plot(f,fftshift(Xf));
subplot(313);plot(f,fftshift(Zf));
```

Plot:

- X(ω) is shown in the top and middle graph while Z(ω) is shown in the bottom graph



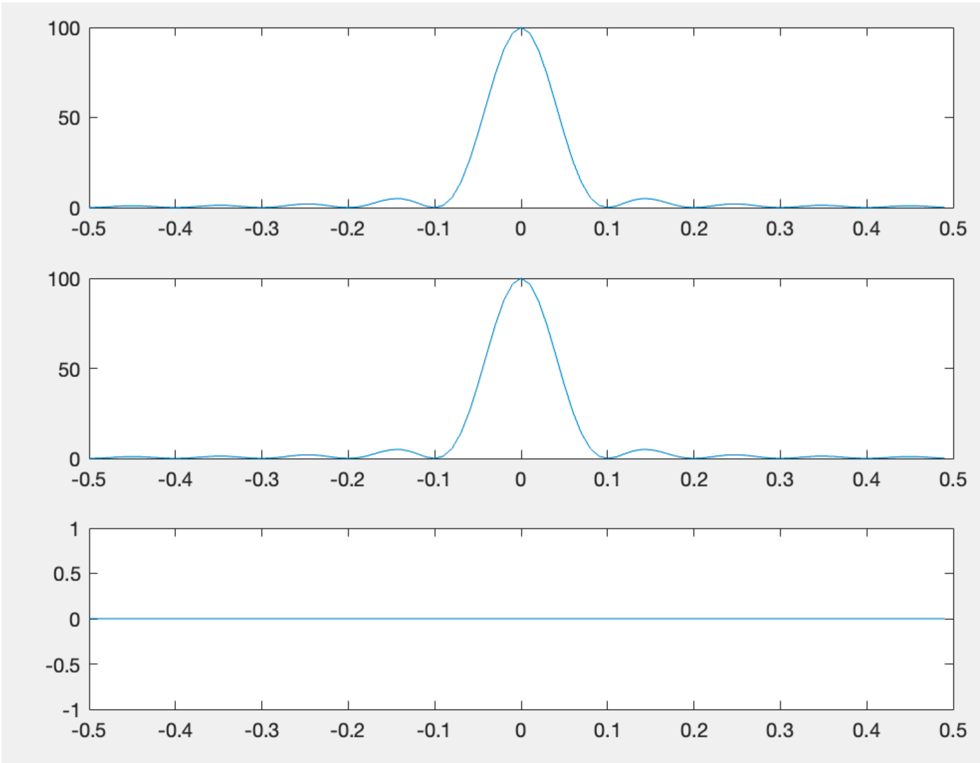
Problem A.3: Plot the magnitude and phase spectra of z(t).

Code:

```
N = 100;
PulseWidth = 10;
t = [0:1:(N-1)];
x = [ones(1, PulseWidth), zeros(1, N-PulseWidth)];

Xf = fft(x);
f = [-(N/2):1:(N/2)-1]*(1/N);

Zf = abs((Xf).^2);

subplot(311);plot(f,fftshift(Zf));
subplot(312);plot(f,fftshift(abs(Zf)));
subplot(313);plot(f,fftshift(angle(Zf)));
```

Plot:

- $Z(\omega)$ is shown in the top graph, $|Z(\omega)|$ is shown in the middle graph and $\angle Z(\omega)$ is shown in the bottom graph

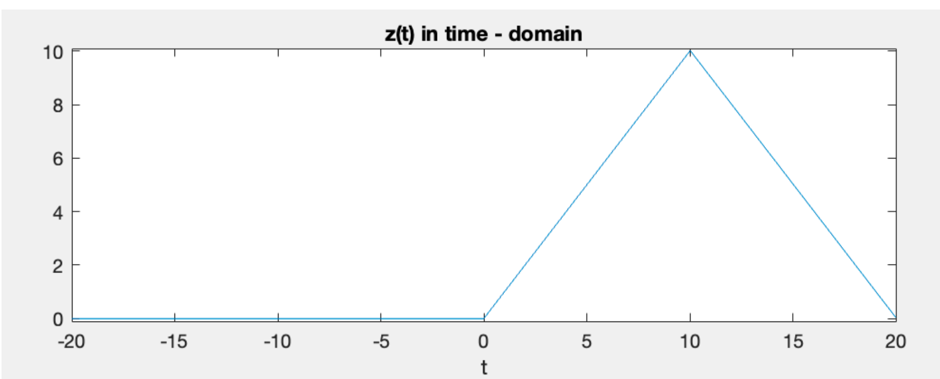


Problem A.4: Compute z(t) using time-domain and frequency-domain operations implemented in MATLAB. Plot both results and compare with the analytic result you determined in Problem A.1. Determine the appropriate time indices for proper labelling of the time-domain plots of z(t). How do the results you generate in MATLAB using time- and frequency-domain operations compare with the analytic result you computed in Problem A.1? Explain which property of the Fourier Transform you have demonstrated.

Code:

```
t1 = -20;
t2 = 20;
N = 2000;
Delta_t = (t2 - t1)/N;
t = [t1:Delta_t:t2];

x = zeros(size(t));
x(t >= 0 & t <= 10) = 1;

x1 = x*Delta_t;
z = conv(x,x1);

subplot(2,1,1);
plot(t,z(1000:3000));
axis([t1 t2 -0.1 10.1]);
title('z(t) in time - domain');
xlabel('t');
```

Plot:



- The results generated from MATLAB and those computed in Problem A.1 are identical. The convolution property of the Fourier transform is demonstrated in this problem, since z(t) = x(t) * x(t) and Z(ω) = X(ω)X(ω) holds true.

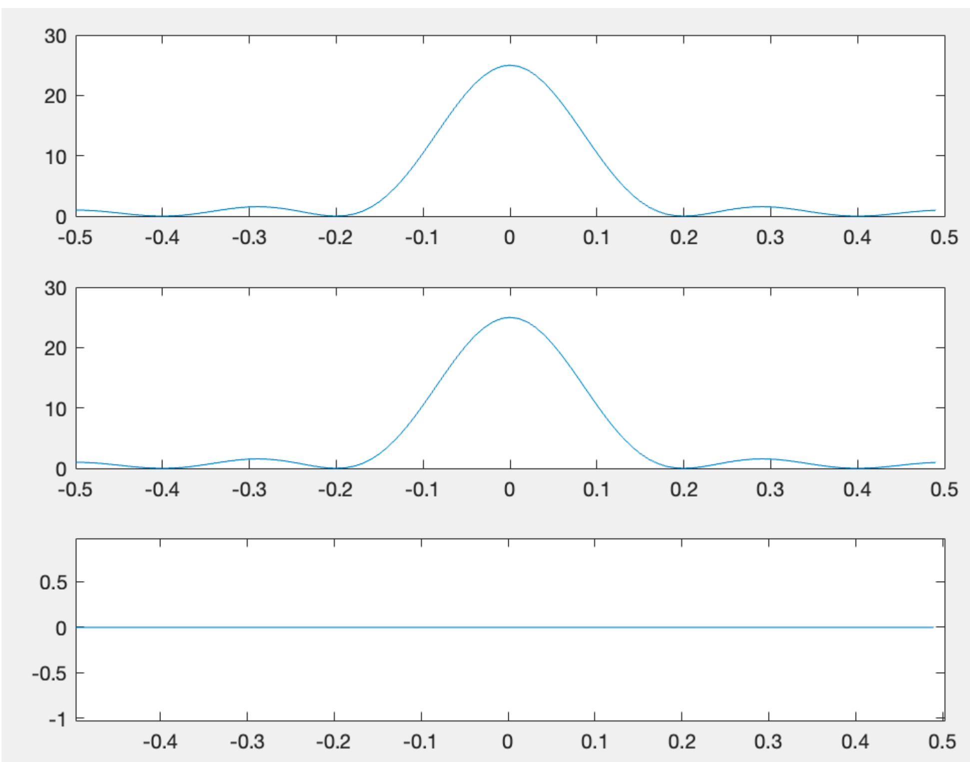
Problem A.5: Change the width of the pulse x(t) to 5 while keeping the total length at N = 100. Compute the Fourier Transform of the narrower pulse and plot the corresponding magnitude- and phase spectra. Repeat for a pulse width of 25. Explain the observed differences from the comparison of the frequency spectra generated by the three pulses with different pulse-widths. Explain which property of the Fourier Transform you have demonstrated.

Code: **(Pulse Width = 5)**

```
N = 100;
PulseWidth = 5;
t = [0:1:(N-1)];
x = [ones(1, PulseWidth), zeros(1, N-PulseWidth)];

Xf = fft(x);
f = [-(N/2):1:(N/2)-1]*(1/N);

Zf = abs((Xf).^2);

subplot(311);plot(f,fftshift(Zf));
subplot(312);plot(f,fftshift(abs(Zf)));
subplot(313);plot(f,fftshift(angle(Zf)));
```

Plot: **(Pulse Width = 5)**

- $Z(\omega)$ is shown in the top graph, $|Z(\omega)|$ is shown in the middle graph and $\angle Z(\omega)$ is shown in the bottom graph

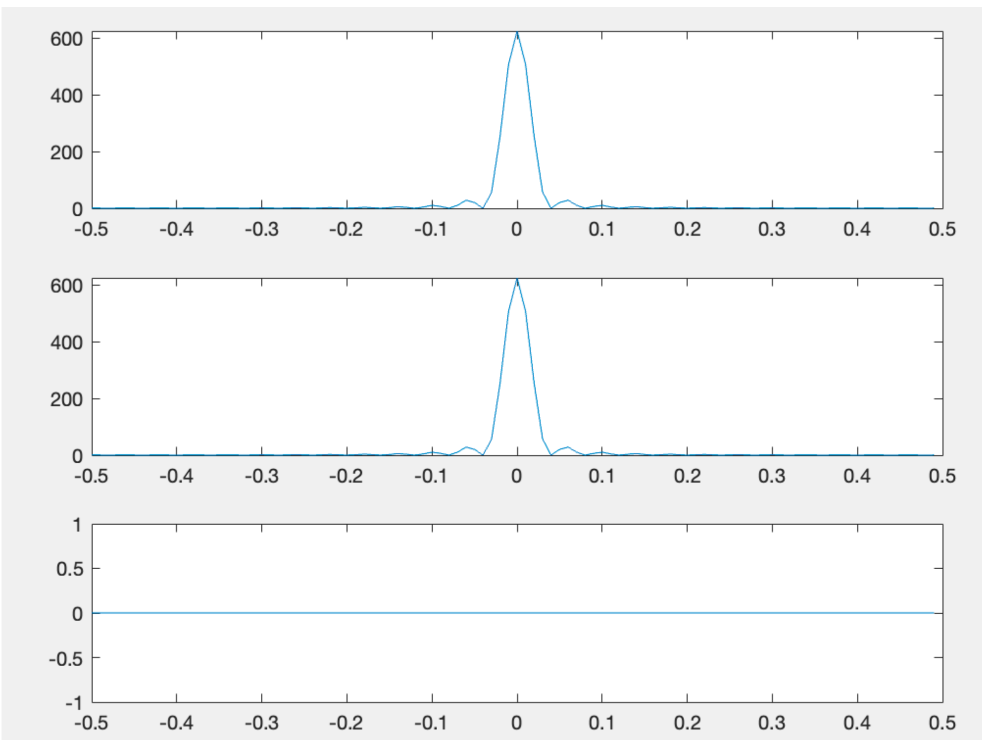


Code: **(PulseWidth = 25)**

```
N = 100;
PulseWidth = 25;
t = [0:1:(N-1)];
x = [ones(1, PulseWidth), zeros(1, N-PulseWidth)];

Xf = fft(x);
f = [-(N/2):1:(N/2)-1]*(1/N);

Zf = abs((Xf).^2);

subplot(311);plot(f,fftshift(Zf));
subplot(312);plot(f,fftshift(abs(Zf)));
subplot(313);plot(f,fftshift(angle(Zf)));
```

Plot: **(PulseWidth = 25)**

- Z(ω) is shown in the top graph, |Z(ω)| is shown in the middle graph and ∠ Z(ω) is shown in the bottom graph



Observed Differences: When the pulse width decreases and increases, the amplitude of the Fourier transform also decreases and increases respectively, this reflects the time-scaling property.

Problem A.6: Let $w_+(t) = x(t)e^{j\left(\frac{\pi}{3}\right)t}$ where x(t) is the original pulse of pulse-width 10 shown in Figure (1). Using MATLAB compute and plot the magnitude- and phase-spectra of $w_+(t)$. Compare the frequency spectra result with those you generated in Problem A.3 and comment on the observed differences. Repeat for $w_-(t) = x(t)e^{-j\left(\frac{\pi}{3}\right)t}$ and $w_c(t) = x(t)\cos\left(\frac{\pi}{3}\right)t$. Explain which property of the Fourier Transform you have demonstrated.

Code: **w_+(t)**

```
N = 100;
PulseWidth = 10;
t = [0:1:(N-1)];
x = [ones(1,PulseWidth), zeros(1,N-PulseWidth)];
wplus = x.*exp(1j.*(pi/3).*t)
Xf = fft(wplus);
f = [-(N/2):1:(N/2)-1]*(1/N);
figure();
subplot(311);
plot(f,fftshift(Xf));
title('X(w)');
xlabel('w');
subplot(312);
plot(f,fftshift(abs(Xf)));
title('|X(w)|');
xlabel('w');
subplot(313);
plot(f,fftshift(angle(Xf)));
title('angle X(w)');
xlabel('w');
```

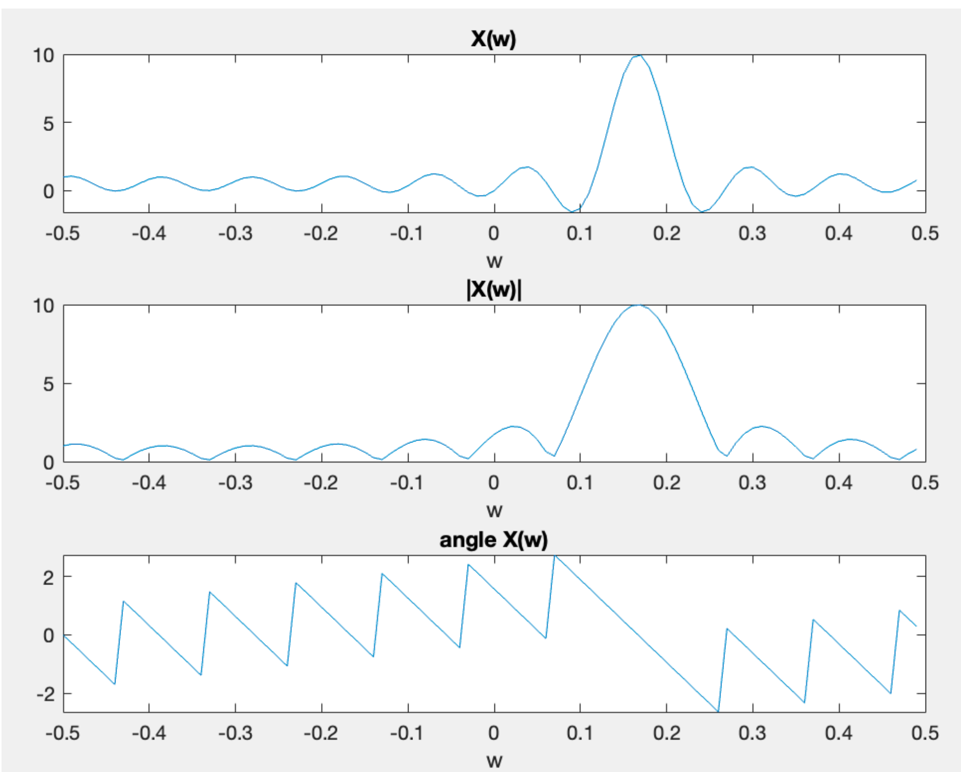
Code: **w_-(t)**

```
N = 100;
PulseWidth = 10;
t = [0:1:(N-1)];
x = [ones(1,PulseWidth), zeros(1,N-PulseWidth)];
wminus = x.*exp(-1j.*(pi/3).*t);
Xf = fft(wminus);
f = [-(N/2):1:(N/2)-1]*(1/N);
figure();
subplot(311);
plot(f,fftshift(Xf));
title('X(w)');
xlabel('w');
subplot(312);
plot(f,fftshift(abs(Xf)));
title('|X(w)|');
xlabel('w');
subplot(313);
plot(f,fftshift(angle(Xf)));
title('angle X(w)');
xlabel('w');
```

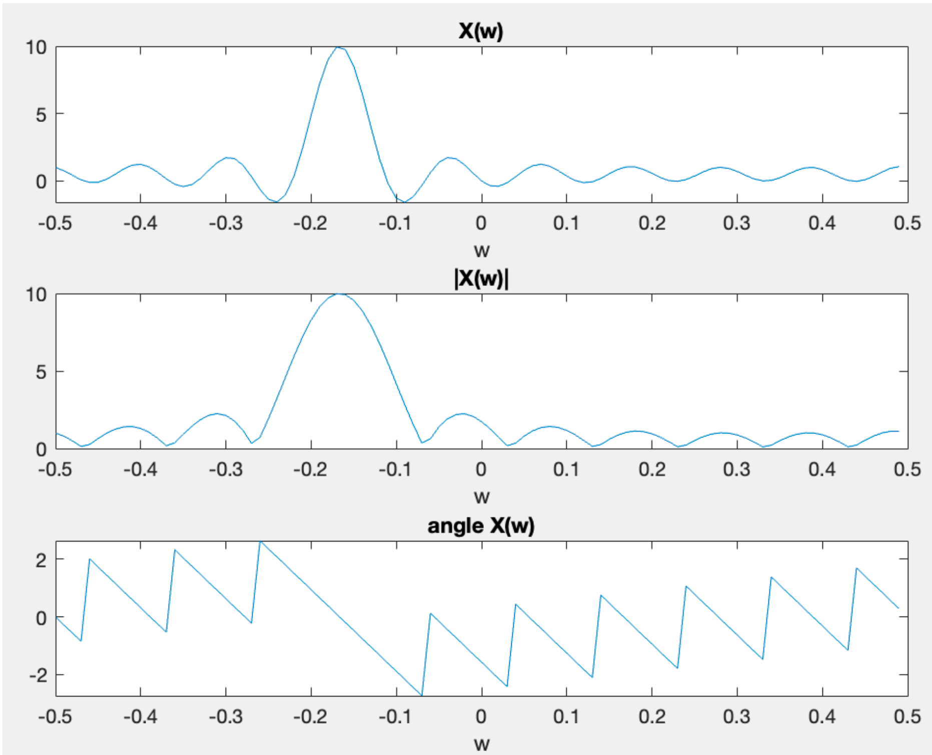
Code: $\mathbf{w_c(t)}$

```
N = 100;
PulseWidth = 10;
t = [0:1:(N-1)];
x = [ones(1,PulseWidth), zeros(1,N-PulseWidth)];
wc = x.*cos((pi/3).*t);
Xf = fft(wc);
f = [-(N/2):1:(N/2)-1]*(1/N);
figure();
subplot(311);
plot(f,fftshift(Xf));
title('X(w)');
xlabel('w');
subplot(312);
plot(f,fftshift(abs(Xf)));
title('|X(w)|');
xlabel('w');
subplot(313);
plot(f,fftshift(angle(Xf)));
title('angle X(w)');
xlabel('w');
```

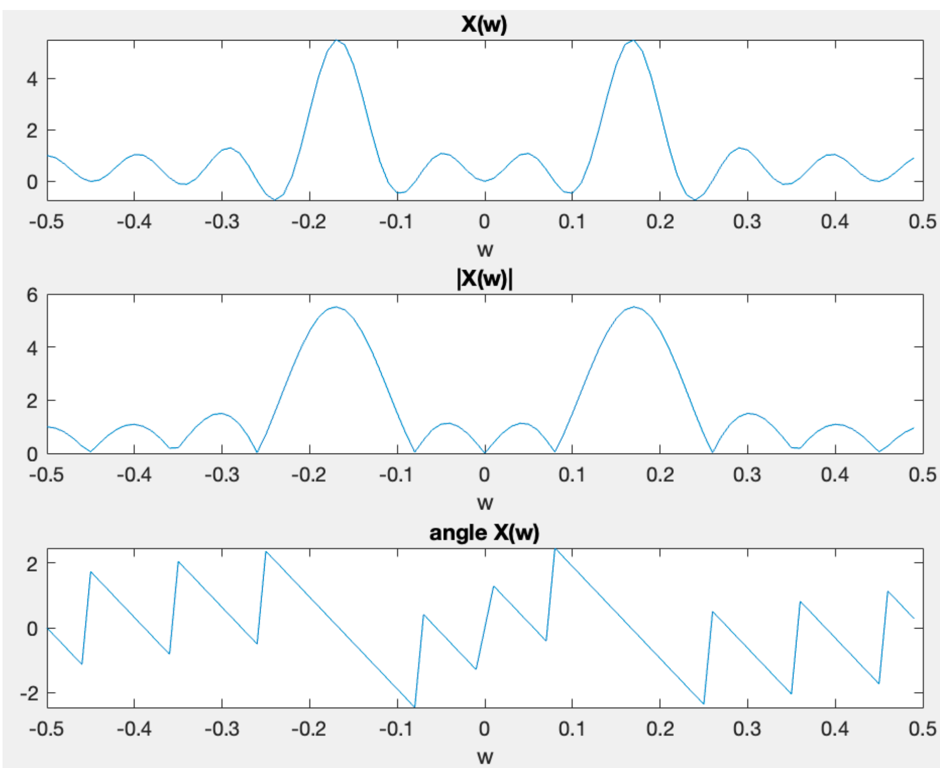
Plot: $\mathbf{w_+(t)}$



Plot: $\mathbf{w_{-}(t)}$



Plot: $\mathbf{w_c(t)}$



Observed Differences: One key difference is that the multiplication of a complex exponential to the original function creates a shift in the frequency domain. This reflects the frequency shifting property of the Fourier Transform. The $w_c(t)$ function is a cosine function which can be split into two complex exponentials, this is shown in the resulting plot as two identical waveforms with the amplitude cut in half.