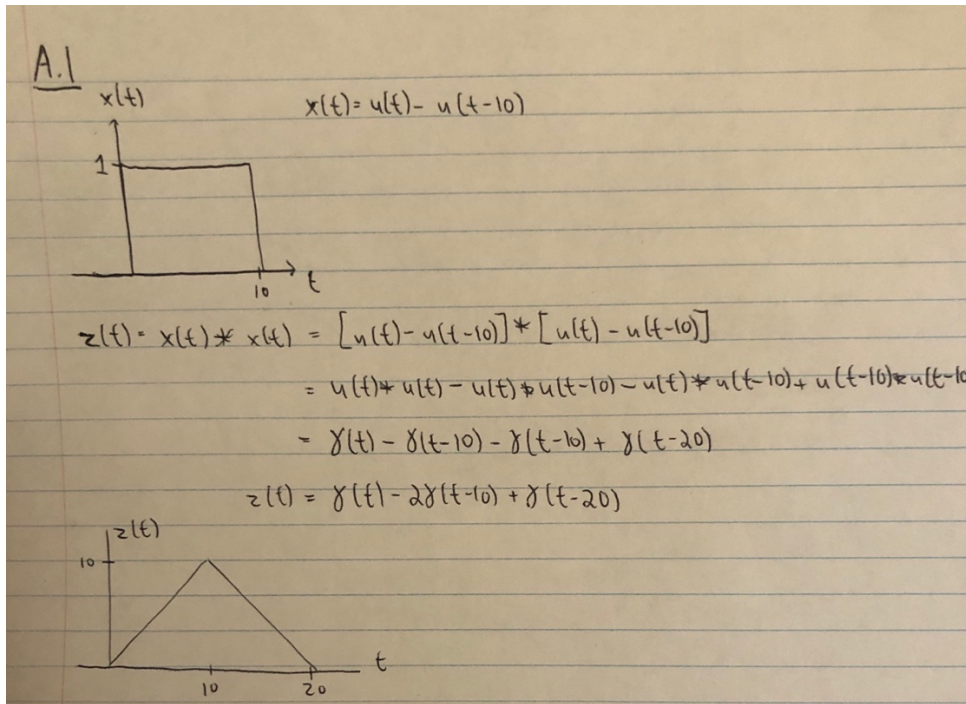


Lab 4: The Fourier Transform: Properties and Applications

A. The Fourier Transform and its Properties

Problem A.1: For the signal $x(t)$ shown in Figure (1), compute and plot $z(t) = x(t) * x(t)$.



Problem A.2: Using MATLAB, calculate $Z(\omega) = X(\omega)X(\omega)$.

Code:

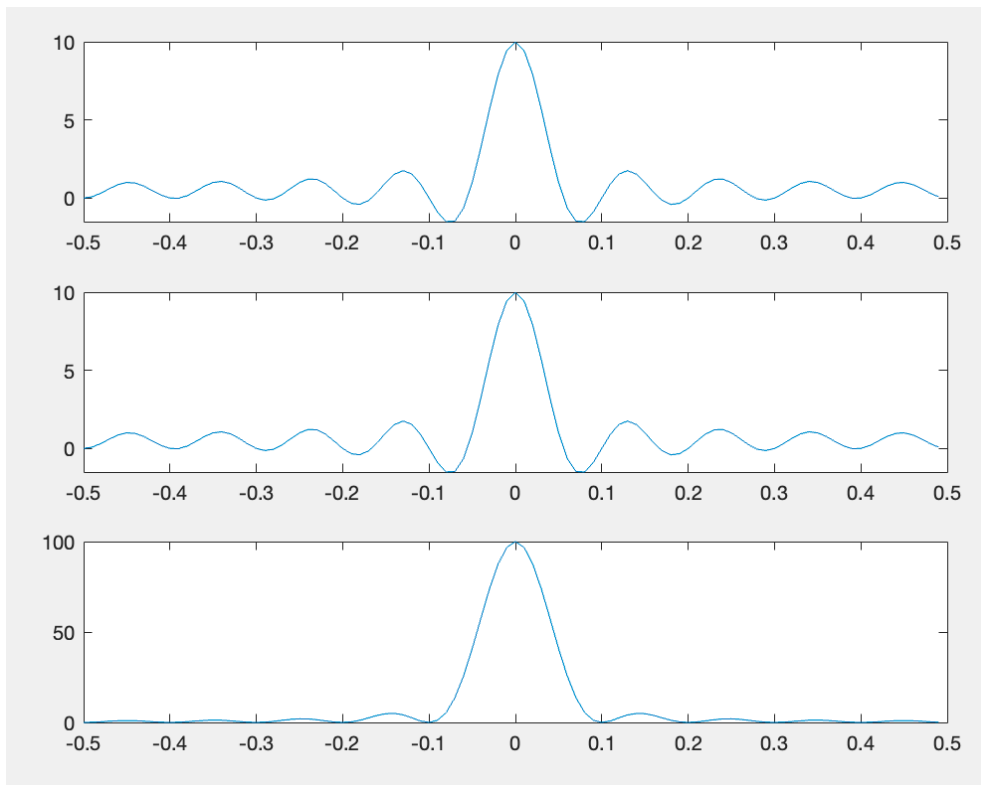
```

2      N = 100;
3      PulseWidth = 10;
4      t = [0:1:(N-1)];
5      x = [ones(1, PulseWidth), zeros(1, N-PulseWidth)];
6
7      Xf = fft(x);
8      f = [-(N/2):1:(N/2)-1]*(1/N);
9
10     Zf = abs((Xf).^2);
11
12     subplot(311); plot(f, fftshift(Xf));
13     subplot(312); plot(f, fftshift(Xf));
14     subplot(313); plot(f, fftshift(Zf));

```

Plot:

- $X(\omega)$ is shown in the top and middle graph while $Z(\omega)$ is shown in the bottom graph



Problem A.3: Plot the magnitude and phase spectra of $z(t)$.

Code:

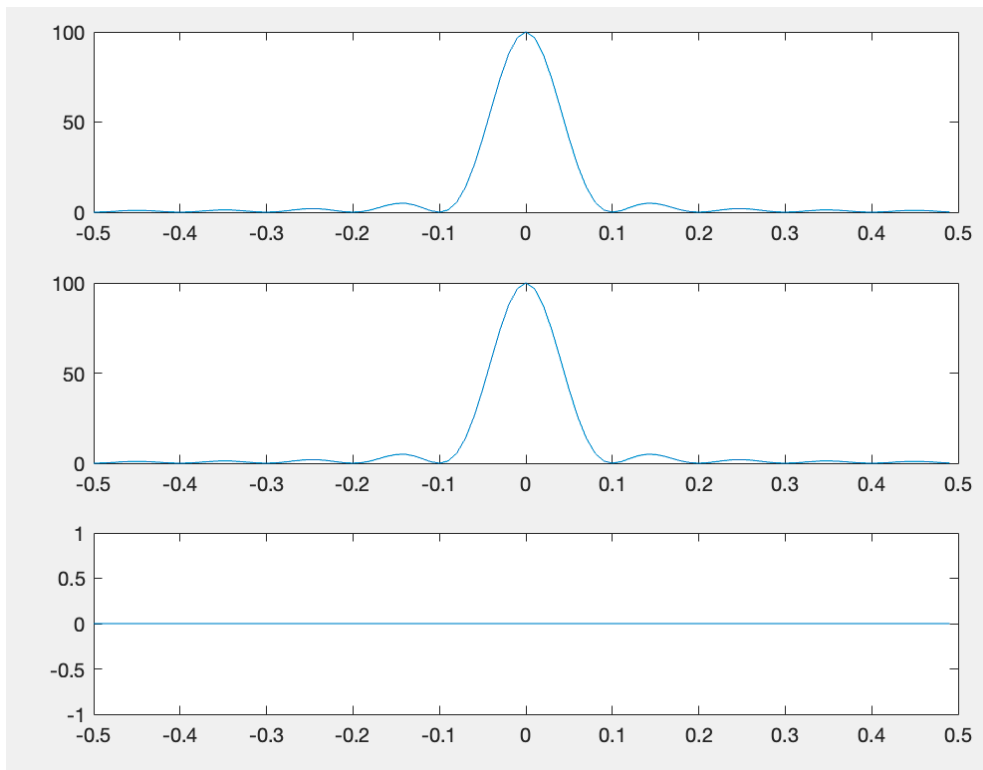
```

16 N = 100;
17 PulseWidth = 10;
18 t = [0:1:(N-1)];
19 x = [ones(1, PulseWidth), zeros(1, N-PulseWidth)];
20
21 Xf = fft(x);
22 f = [-(N/2):1:(N/2)-1]*(1/N);
23
24 Zf = abs((Xf).^2);
25
26 subplot(311);plot(f,fftshift(Zf));
27 subplot(312);plot(f,fftshift(abs(Zf)));
28 subplot(313);plot(f,fftshift(angle(Zf)));

```

Plot:

- $Z(\omega)$ is shown in the top graph, $|Z(\omega)|$ is shown in the middle graph and $\angle Z(\omega)$ is shown in the bottom graph



Problem A.4: Compute $z(t)$ using time-domain and frequency-domain operations implemented in MATLAB. Plot both results and compare with the analytic result you determined in Problem A.1. Determine the appropriate time indices for proper labelling of the time-domain plots of $z(t)$. How do the results you generate in MATLAB using time- and frequency-domain operations compare with the analytic result you computed in Problem A.1? Explain which property of the Fourier Transform you have demonstrated.

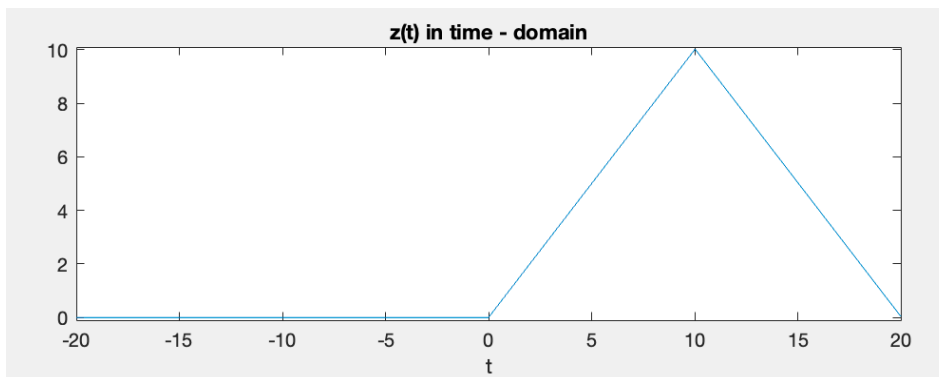
Code:

```

30     t1 = -20;
31     t2 = 20;
32     N = 2000;
33     Delta_t = (t2 - t1)/N;
34     t = [t1:Delta_t:t2];
35
36     x = zeros(size(t));
37     x(t >= 0 & t <= 10) = 1;
38
39     x1 = x*Delta_t;
40     z = conv(x,x1);
41
42     subplot(2,1,1);
43     plot(t,z(1000:3000));
44     axis([t1 t2 -0.1 10.1]);
45     title('z(t) in time - domain');
46     xlabel('t');

```

Plot:



- The results generated from MATLAB and those computed in Problem A.1 are identical. The convolution property of the Fourier transform is demonstrated in this problem, since $z(t) = x(t) * x(t)$ and $Z(\omega) = X(\omega)X(\omega)$ holds true.

Problem A.5: Change the width of the pulse $x(t)$ to 5 while keeping the total length at $N = 100$.

Compute the Fourier Transform of the narrower pulse and plot the corresponding magnitude- and phase spectra. Repeat for a pulse width of 25. Explain the observed differences from the comparison of the frequency spectra generated by the three pulses with different pulse-widths. Explain which property of the Fourier Transform you have demonstrated.

Code: **(Pulse Width = 5)**

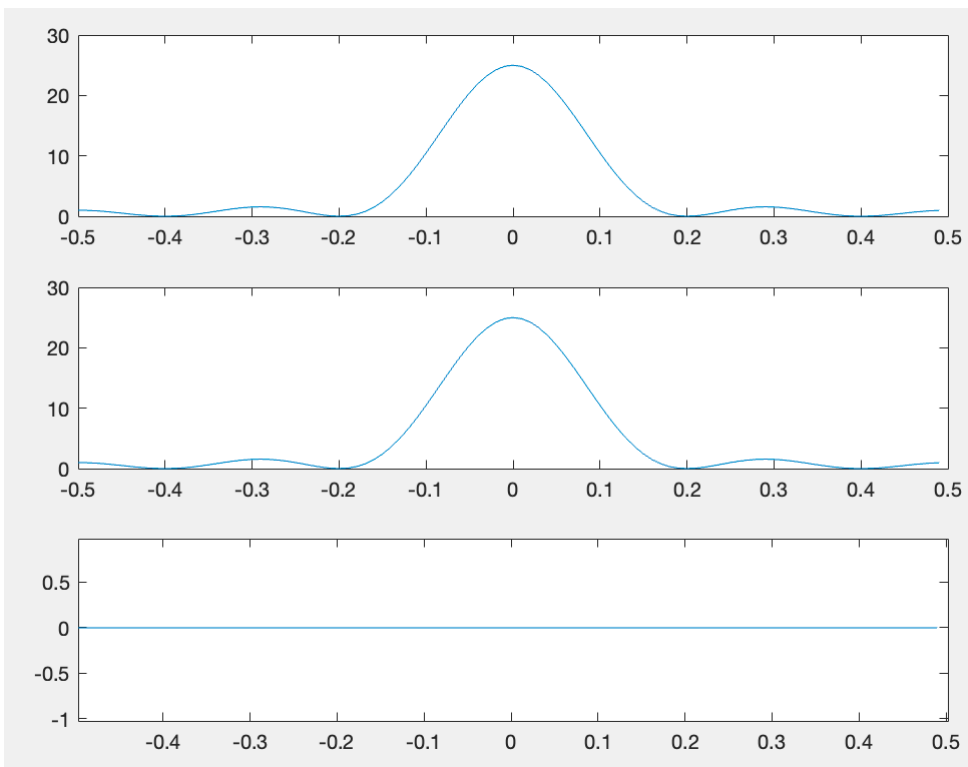
```

48 N = 100;
49 PulseWidth = 5;
50 t = [0:1:(N-1)];
51 x = [ones(1, PulseWidth), zeros(1, N-PulseWidth)];
52
53 Xf = fft(x);
54 f = [-(N/2):1:(N/2)-1]*(1/N);
55
56 Zf = abs((Xf).^2);
57
58 subplot(311);plot(f,fftshift(Zf));
59 subplot(312);plot(f,fftshift(abs(Zf)));
60 subplot(313);plot(f,fftshift(angle(Zf)));

```

Plot: **(Pulse Width = 5)**

- $Z(\omega)$ is shown in the top graph, $|Z(\omega)|$ is shown in the middle graph and $\angle Z(\omega)$ is shown in the bottom graph



Code: (**PulseWidth = 25**)

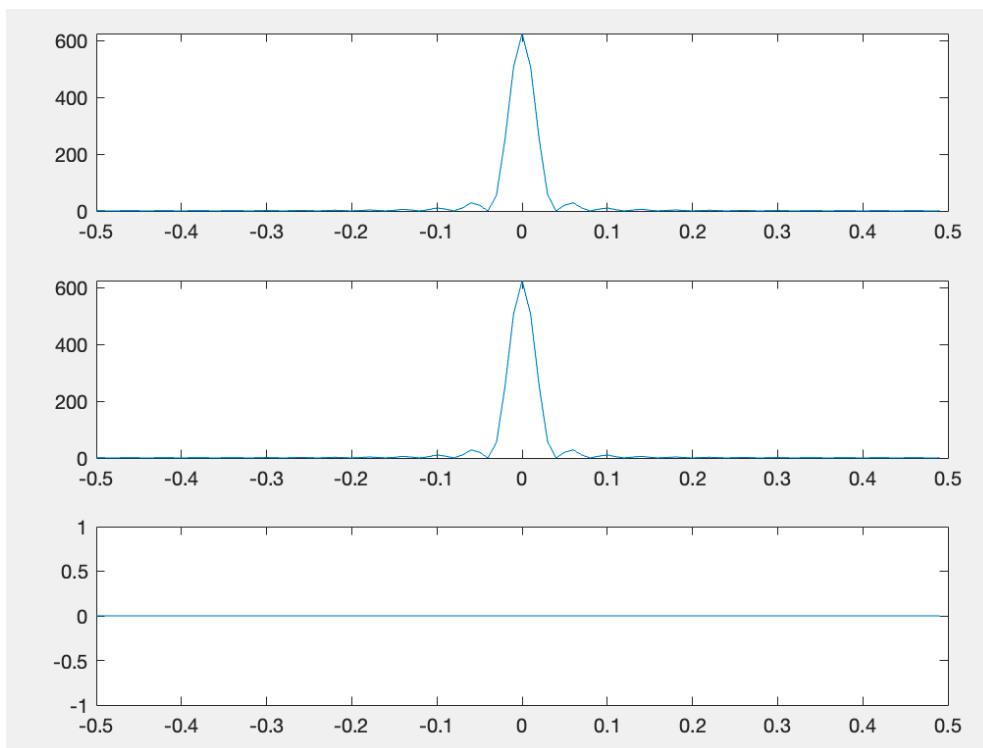
```

63     N = 100;
64     PulseWidth = 25;
65     t = [0:1:(N-1)];
66     x = [ones(1, PulseWidth), zeros(1, N-PulseWidth)];
67
68     Xf = fft(x);
69     f = [-(N/2):1:(N/2)-1]*(1/N);
70
71     Zf = abs((Xf).^2);
72
73     subplot(311);plot(f,fftshift(Zf));
74     subplot(312);plot(f,fftshift(abs(Zf)));
75     subplot(313);plot(f,fftshift(angle(Zf)));

```

Plot: (**PulseWidth = 25**)

- $Z(\omega)$ is shown in the top graph, $|Z(\omega)|$ is shown in the middle graph and $\angle Z(\omega)$ is shown in the bottom graph



Observed Differences: When the pulse width decreases and increases, the amplitude of the Fourier transform also decreases and increases respectively, this reflects the time-scaling property.

Problem A.6: Let $w_+(t) = x(t)e^{j(\frac{\pi}{3})t}$ where $x(t)$ is the original pulse of pulse-width 10 shown in Figure (1). Using MATLAB compute and plot the magnitude- and phase-spectra of $w_+(t)$.

Compare the frequency spectra result with those you generated in Problem A.3 and comment on the observed differences. Repeat for $w_-(t) = x(t)e^{-j(\frac{\pi}{3})t}$ and $w_c(t) = x(t) \cos\left(\frac{\pi}{3}\right)t$. Explain which property of the Fourier Transform you have demonstrated.

Code: $w_+(t)$

```

78     N = 100;
79     PulseWidth = 10;
80     t = [0:1:(N-1)];
81     x = [ones(1,PulseWidth), zeros(1,N-PulseWidth)];
82     wplus = x.*exp(1j.*(pi/3).*t)
83     Xf = fft(wplus);
84     f = [-(N/2):1:(N/2)-1]*(1/N);
85     figure();
86     subplot(311);
87     plot(f,fftshift(Xf));
88     title('X(w)');
89     xlabel('w');
90     subplot(312);
91     plot(f,fftshift(abs(Xf)));
92     title('|X(w)|');
93     xlabel('w');
94     subplot(313);
95     plot(f,fftshift(angle(Xf)));
96     title('angle X(w)');
97     xlabel('w');
```

Code: $w_-(t)$

```

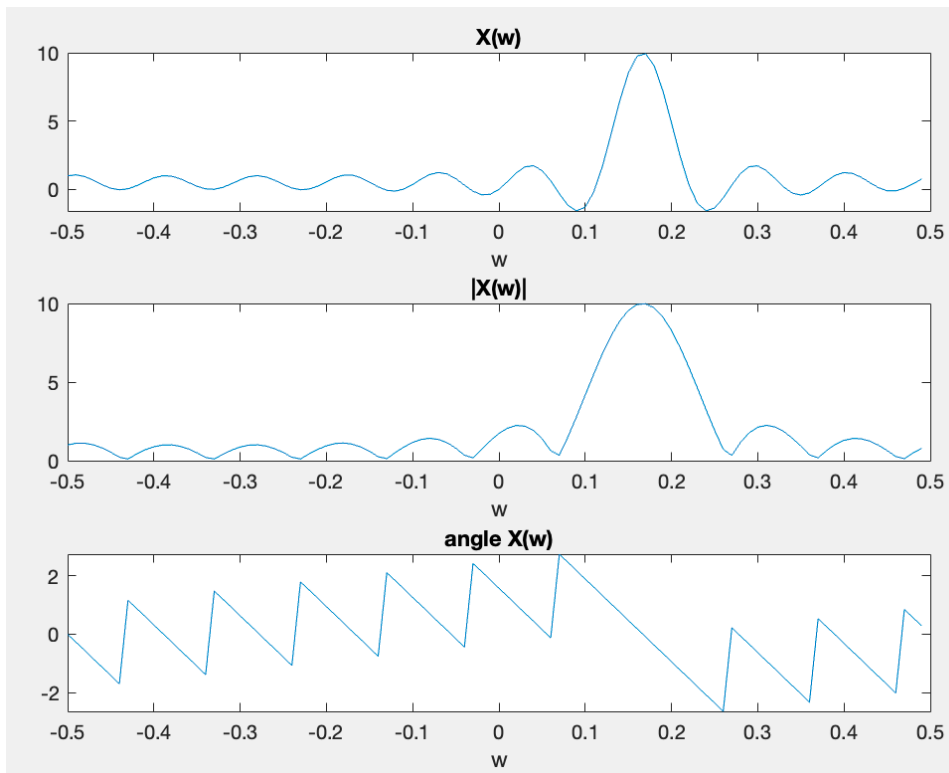
101    N = 100;
102    PulseWidth = 10;
103    t = [0:1:(N-1)];
104    x = [ones(1,PulseWidth), zeros(1,N-PulseWidth)];
105    wminus = x.*exp(-1j.*(pi/3).*t);
106    Xf = fft(wminus);
107    f = [-(N/2):1:(N/2)-1]*(1/N);
108    figure();
109    subplot(311);
110    plot(f,fftshift(Xf));
111    title('X(w)');
112    xlabel('w');
113    subplot(312);
114    plot(f,fftshift(abs(Xf)));
115    title('|X(w)|');
116    xlabel('w');
117    subplot(313);
118    plot(f,fftshift(angle(Xf)));
119    title('angle X(w)');
120    xlabel('w');
```

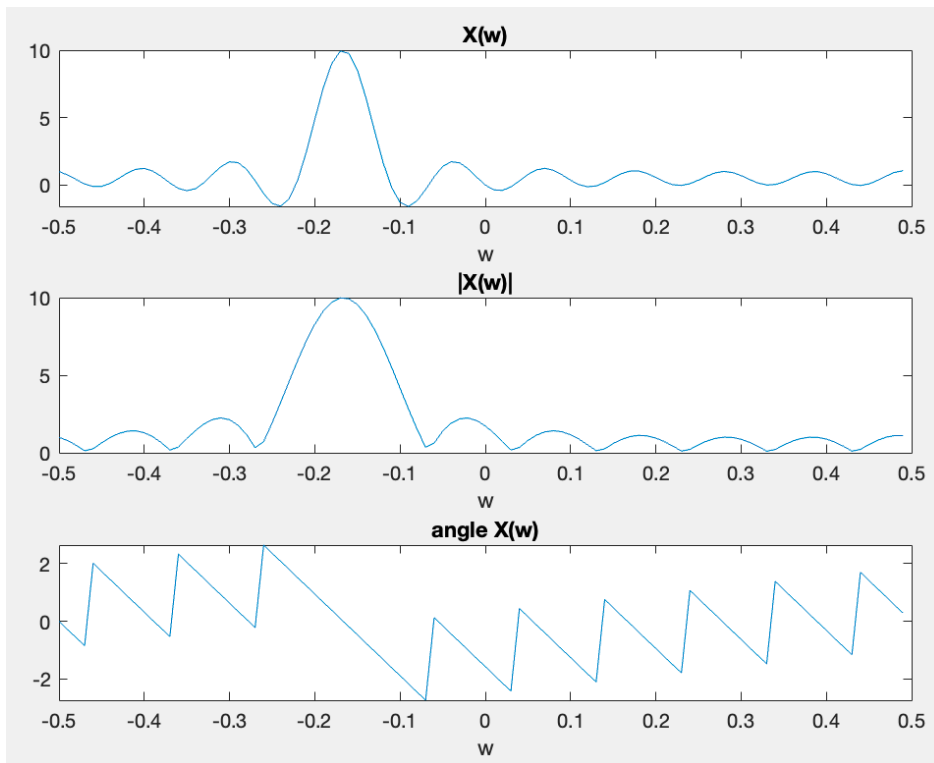
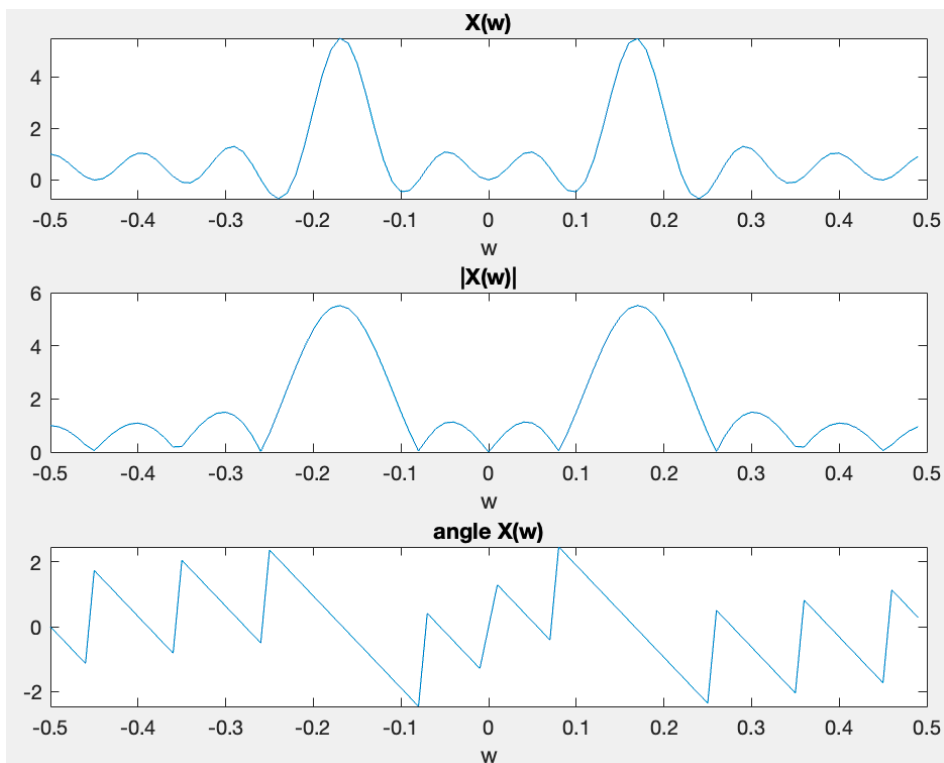
Code: $w_c(t)$

```

124 N = 100;
125 PulseWidth = 10;
126 t = [0:1:(N-1)];
127 x = [ones(1,PulseWidth), zeros(1,N-PulseWidth)];
128 wc = x.*cos((pi/3).*t);
129 Xf = fft(wc);
130 f = [-(N/2):1:(N/2)-1]*(1/N);
131 figure();
132 subplot(311);
133 plot(f,fftshift(Xf));
134 title('X(w)');
135 xlabel('w');
136 subplot(312);
137 plot(f,fftshift(abs(Xf)));
138 title('|X(w)|');
139 xlabel('w');
140 subplot(313);
141 plot(f,fftshift(angle(Xf)));
142 title('angle X(w)');
143 xlabel('w');

```

Plot: $w_c(t)$ 

Plot: $w_c(t)$ Plot: $w_e(t)$ 

Observed Differences: One key difference is that the multiplication of a complex exponential to the original function creates a shift in the frequency domain. This reflects the frequency shifting property of the Fourier Transform. The $w_c(t)$ function is a cosine function which can be split into two complex exponentials, this is shown in the resulting plot as two identical waveforms with the amplitude cut in half.