WROCLAW UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF ELECTRONICS

FIELD: Electronics
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Optimization Methods: Nonlinear Equations

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Contents

Chapter 1

Solution to the given problems

Problem 1: Show that the following system of nonlinear equations:

$$F_1(\mathbf{x}) = x_1^3 + 3x_1^2 + 3x_1 - x_2 = 0,$$
$$F_2(\mathbf{x}) = x_1^2 + 2x_1 - x_2 + 1 = 0,$$

has the global minimum at $x^* = [0.46557 \quad 2.1479]^T$, the local minimum at $x_1 = [-1 \quad -0.5]^T$, and the saddle point at $x_2 = \begin{bmatrix} -\frac{1}{3} & -\frac{7}{54} \end{bmatrix}^T$.

In general, the unconstrained nonlinear least squares problem is equivalent to:

$$\min_x \sum_{i=1}^{N} F_i^2(x)$$

with the objective function:

$$f(x) = \frac{1}{2}\sum_{i=1}^{N} F_i^2(x)$$

In the case of the following task, objective function will be

$$\begin{aligned}
f(x) &= \frac{1}{2}(F_1^2 + F_2^2) \\
&= \frac{1}{2}[(x_1^6 + 6x_1^5 + 15x_1^4 - 2x_1^3x_2 + 18x_1^3 - 6x_1^2x_2 + 9x_1^2 - 6x_1x_2 + x_2^2) \\
+ &\quad (x_1^4 + 4x_1^3 - 2x_1^2x_2 + 6x_1^2 - 4x_1x_2 + 4x_1 + x_2^2 - 2x_2 + 1)] \\
&= \frac{1}{2}(x_1^6 + 6x_1^5 + 16x_1^4 - 2x_1^3x_2 + 22x_1^3 - 8x_1^2x_2 + 15x_1^2 - 10x_1x_2 + 4x_1 + 2x_2^2 - 2x_2 + 1)
\end{aligned}$$

And the point to check is:
$x = [x_1 \ x_2]^T = [1 \ 1]^T$

The First order optimality condition is:

$$\nabla f(x^*) = 0$$

$$\begin{aligned}\frac{\delta f(x)}{\delta x_2} &= 400x_1^3 + 2x_1 - 2 - 400x_2x_1 \\ &= 400 + 2 - 2 - 400 = 0\end{aligned}$$

$$\begin{aligned}\frac{\delta f(x)}{\delta x_1} &= 200x_2 - 200x_1^2 \\ &= 200 - 200 = 0\end{aligned}$$

In given point x = [1 1] the first–order optimality condition is fulfilled.

The Second order optimality condition is:

$$\begin{aligned}\nabla f(x^*) &= 0 \ (calculated\ in\ previous\ step) \\ \nabla^2 f(x^*) \ &= positive\ semi-definite\ matrix\end{aligned}$$

$$\begin{aligned}\nabla^2 f(x^*) \ &= \begin{bmatrix} \frac{\delta^2 f(x)}{\delta x_1^2} & \frac{\delta^2 f(x)}{\delta x_1 x_2} \\ \frac{\delta^2 f(x)}{\delta x_1 x_2} & \frac{\delta^2 f(x)}{\delta x_2^2} \end{bmatrix} \\ &= \begin{bmatrix} 1200x_1^2 + 2 - 400x_2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix} \\ for\ point\ [1\ 1] &= \begin{bmatrix} 1200 + 2 - 400 & -400 \\ -400 & 200 \end{bmatrix} \\ H &= \begin{bmatrix} 802 & -400 \\ -400 & 200 \end{bmatrix}\end{aligned}$$

As it can be noticed, all principal minors are positive ($H_{1,1}$ *and* $H_{2,2}$). Therefore the $H(x_*)$ is positive semi-definite.

Octave code below was written to plot contour for this task.

```
pkg load symbolic

syms x1 x2
f = @(x1,x2) 100.*(x2 - x1.^2).^2 + (1 - x1).^2;

ezcontour(f,[-3,3])
```

has the global minimum at $x^* = [0.46557 \quad 2.1479]^T$, the local minimum at $x_1 = [-1 \quad -0.5]^T$, and the saddle point at $x_2 = \left[-\frac{1}{3} \quad -\frac{7}{54}\right]^T$.

Figure 1.1 Contour plot of given function.

Chapter 2

Listings of algorithms

2.1 Coded selected algorithms

Algorithm 1 - Simplex algorithm

Algorithm 2 - Revised Simplex algorithm

Bibliography

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