WROCLAW UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF ELECTRONICS

FIELD: Electronics
SPECIALITY: Advanced Applied Electronics

Optimization Methods: Linear programming

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Chapter 1

Solution to the given problems

Problem 1: Check the first- and second-order optimality conditions in the point: $x = [1 \quad 1]^T$ of the Rosenbrock's function: $f(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$. Draw a contour plot of this function.

The first step will be expanding the given function, so further we can calculate the derivatives for gradient:

$$\begin{aligned} f(x) &= 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \\ &= 100(x_2^2 - 2x_2x_1^2 + x_1^4) + (1 - 2x_1 + x_1^2) \\ &= 100x_2^2 - 200x_2x_1^2 + 100x_1^4 + 1 - 2x_1 + x_1^2 \\ &= 100x_1^4 + x_1^2 - 2x_1 + 100x_2^2 - 200x_2x_1^2 + 1 \end{aligned}$$

And the point to check is:
$x = [x_1 \ x_2]^T = [1 \ 1]^T$

The First order optimality condition is:

$$\nabla f(x^*) = 0$$

$$\begin{aligned} \frac{\delta f(x)}{\delta x_2} &= 400x_1^3 + 2x_1 - 2 - 400x_2x_1 \\ &= 400 + 2 - 2 - 400 = 0 \end{aligned}$$

$$\begin{aligned} \frac{\delta f(x)}{\delta x_1} &= 200x_2 - 200x_1^2 \\ &= 200 - 200 = 0 \end{aligned}$$

In given point x = [1 1] the first–order optimality condition is fulfilled.

The Second order optimality condition is:

$$\begin{aligned} \nabla f(x^*) &= 0 \; (calculated\ in\ previous\ step) \\ \nabla^2 f(x^*) &= positive\ semi-definite\ matrix \end{aligned}$$

$$\begin{aligned} \nabla^2 f(x^*) &= \begin{bmatrix} \frac{\delta^2 f(x)}{\delta x_1^2} & \frac{\delta^2 f(x)}{\delta x_1 x_2} \\ \frac{\delta^2 f(x)}{\delta x_1 x_2} & \frac{\delta^2 f(x)}{\delta x_2^2} \end{bmatrix} \\ &= \begin{bmatrix} 1200x_1^2 + 2 - 400x_2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix} \\ for\ point\ [1\ 1] &= \begin{bmatrix} 1200 + 2 - 400 & -400 \\ -400 & 200 \end{bmatrix} \\ H &= \begin{bmatrix} 802 & -400 \\ -400 & 200 \end{bmatrix} \end{aligned}$$

As it can be noticed, all principal minors are positive ($H_{1,1}$ *and* $H_{2,2}$). Therefore the $H(x_*)$ is positive semi-definite.

Octave code below was written to plot contour for this task.

```
pkg load symbolic

syms x1 x2
f = @(x1,x2) 100.*(x2 - x1.^2).^2 + (1 - x1).^2;

ezcontour(f,[-3,3])
```

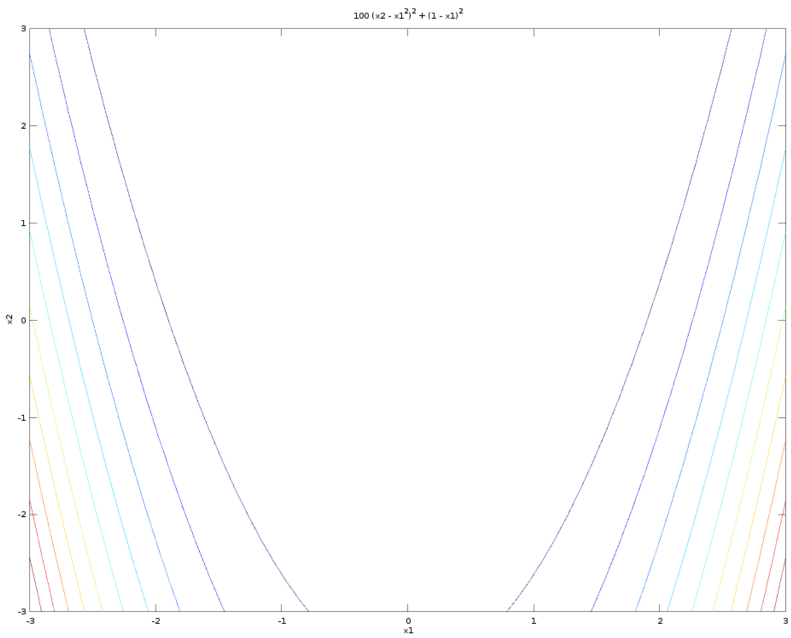

Figure 1.1 Contour plot of given function.

Chapter 2

Listings of algorithms

2.1 Coded selected algorithms

Algorithm 1 - Simplex algorithm

Algorithm 2 - Revised Simplex algorithm

Bibliography

[1] Luenberger, D. G., & Ye, Y. (2015). Linear and nonlinear programming (Vol. 228). Springer.

[2] Zdunek R., Optimization Methods - lecture slides.