WROCLAW UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF ELECTRONICS

Field:	Electronics
Speciality:	Advanced Applied Electronics

Optimization Methods: Constrained Optimization

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Chapter 1

Solution to the given problems

Problem 1: Find numbers x_1 and x_2 that minimize the sum $(x_1 + x_2)$ subject to the constraints

$$x_1^2 + x_2^2 = 2.$$

Draw the feasible set on $\Re^2$ and find all the points that satisfy the KKT conditions. Find the optimal solution that satisfy the second-order necessary conditions. Illustrate the problem geometrically.

According to the task description, we will show the formula of the *c(x)*:

$$c(x) = x_1^2 + x_2^2 - 2 = 0$$

The lagrangian function is (λ is the lagrangian multiplier):

$$L(x, \lambda) = f(x) - \lambda c(x) = x_1 + x_2 - \lambda(x_1^2 + x_2^2 - 2)$$

Now we will formulate the KKT conditions for the given system:

$$\frac{\delta L(x,\lambda)}{\delta x_1} = 1 - \lambda x_1 \triangleq 0,\ then\ x_1 = \frac{1}{2\lambda}$$
$$\frac{\delta L(x,\lambda)}{\delta x_2} = 1 - \lambda x_1 \triangleq 0,\ then\ x_2 = \frac{1}{2\lambda}$$

The c(x) function, subject to calculated x, shall be zero:

$$x_1^2 + x_2^2 - 2 = 0$$
$$(\frac{1}{2\lambda})^2 + (\frac{1}{2\lambda})^2 - 2 = 0$$

$$\lambda^2 = \frac{1}{4}$$
$$\lambda = \frac{1}{2} \vee \lambda = -\frac{1}{2}$$

Now we are calculating both x_1 *and* x_2 according to lambda:

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$x = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

Now to establish the min condition:

$$\min_{x}\{x_1 + x_2\}$$

$$for x = \begin{bmatrix}1\\1\end{bmatrix} : x_1 + x_2 = 2$$
$$for x = \begin{bmatrix}-1\\-1\end{bmatrix} : x_1 + x_2 = -2$$

Hence, the solution is: $x = \begin{bmatrix}-1\\-1\end{bmatrix}$

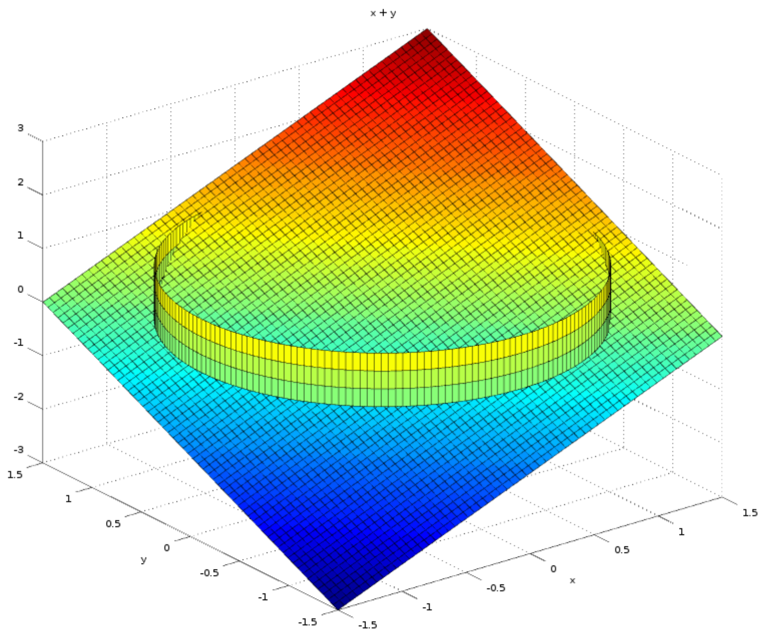

Figure 1.1 The graphical interpretation of given problem (cylinder of radius $\sqrt{2}$) and the plane x+y ($x_1 + x_2$). The solution is in point [-1,-1] - the lowest point in Z-axis that is intersection of the mentioned two surfaces.

The following code was used to illustrate the problem:

```
pkg load symbolic

syms x y;

ezsurf (@(x, y) x + y, [-1.5 1.5 -1.5 1.5])
hold
[X, Y, Z] = cylinder([sqrt(2) sqrt(2) sqrt(2) sqrt(2)],200);
surf(X,Y,Z);
```

Chapter 2

Listings of algorithms

2.1 Coded selected algorithms

Algorithm 1 -

Algorithm 2 -

Bibliography

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[4] Nonlinear Programming course webpage, North Carolina State University, http://www4.ncsu.edu/ kksivara/ma706/