

## Tables de vérité

**Questions :** Etablir la table de vérité des expressions booléennes suivantes en faisant apparaître des variables intermédiaires de calcul.

**Réponses :** Au moins 4 variables intermédiaires ( $s, t, u, v$ ) seront introduites pour « suivre » plus facilement les différentes étapes du calcul de la table de vérité de la fonction  $z = f(a, b, c)$ . Le code PYTHON ci-dessous permet de vérifier la table de vérité obtenue.

```
print('a','b','c','|','s','t','u','v','|','z')
print('-----|---')
for a in [0,1] :
    for b in [0,1] :
        for c in [0,1] :
            s = ...
            t = ...
            u = ...
            v = ...
            z = ...
            print(a,b,c,'|',int(s),int(t),int(u),int(v),'|',int(z))
```

Dans ce qui suit, l'implication  $p \Rightarrow q$  est systématiquement remplacée par son expression équivalente  $\bar{p} + q$  (`not p or q` en PYTHON), l'identité  $\bar{\bar{p}}$  par  $p$  et le « ou exclusif »  $p \oplus q$  par  $p \neq q$  en PYTHON.

$$1. z = ((a \Rightarrow b) + \overline{(b \Rightarrow c)}) \Rightarrow (\bar{c} + \bar{a})$$

```
s = not a or b
t = not (not b or c)
u = s or t
v = not c or not a
z = not u or v
```

a	b	c		s	t	u	v		z
----- ---									
0	0	0		1	0	1	1		1
0	0	1		1	0	1	1		1
0	1	0		1	1	1	1		1
0	1	1		1	0	1	1		1
1	0	0		0	0	0	1		1
1	0	1		0	0	0	0		1
1	1	0		1	1	1	1		1
1	1	1		1	0	1	0		0

$$2. z = ((a \Rightarrow b) \cdot (b \Rightarrow \bar{c})) \Rightarrow (\bar{c} \oplus \bar{a})$$

```
s = not a or b
t = not b or not c
u = s and t
v = (not c) != (not a)
z = not u or v
```

a	b	c		s	t	u	v		z
----- ---									
0	0	0		1	1	1	0		0
0	0	1		1	1	1	1		1
0	1	0		1	1	1	0		0
0	1	1		1	0	0	1		1
1	0	0		0	1	0	1		1
1	0	1		0	1	0	0		1
1	1	0		1	1	1	1		1
1	1	1		1	0	0	0		1

3.  $z = \overline{((a \oplus \bar{b}) \Rightarrow (b \cdot c))} \Rightarrow (\bar{c} \oplus \bar{a})$

s = not (a != (not b))  
t = b and c  
u = not s or t  
v = (not c) != (not a)  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	0	0	1	0	0
0	0	1	0	0	1	1	1
0	1	0	1	0	0	0	1
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	1
1	0	1	1	0	0	0	1
1	1	0	0	0	1	1	1
1	1	1	0	1	1	0	0

4.  $z = ((a \Rightarrow \bar{b}) \cdot (b + c)) \Rightarrow \overline{(\bar{c} \Rightarrow \bar{a})}$

s = not a or not b  
t = b or c  
u = s and t  
v = not (c or not a)  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	0	0	0	1
0	0	1	1	1	1	0	0
0	1	0	1	1	1	0	0
0	1	1	1	1	1	0	0
1	0	0	1	0	0	1	1
1	0	1	1	1	1	0	0
1	1	0	0	1	0	1	1
1	1	1	0	1	0	0	1

5.  $z = ((\bar{a} \Rightarrow \bar{b}) \cdot (b \Rightarrow c)) \Rightarrow (c \Rightarrow a)$

s = a or not b  
t = not b or c  
u = s and t  
v = not c or a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	1	1	1	1
0	0	1	1	1	1	0	0
0	1	0	0	0	0	1	1
0	1	1	0	1	0	0	1
1	0	0	1	1	1	1	1
1	0	1	1	1	1	1	1
1	1	0	1	0	0	1	1
1	1	1	1	1	1	1	1

6.  $z = ((a \Rightarrow b) + \overline{(b \Rightarrow \bar{c})}) \Rightarrow (\bar{c} \oplus \bar{a})$

s = not a or b  
t = not (not b or not c)  
u = s or t  
v = (not c) != (not a)  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	0	1	0	0
0	0	1	1	0	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	0	0	1	1
1	0	1	0	0	0	0	1
1	1	0	1	0	1	1	1
1	1	1	1	1	1	0	0

7.  $z = ((a \cdot b) \oplus (b \cdot c)) \Rightarrow (c + \bar{a})$

s = a and b  
t = b and c  
u = s != t  
v = c or not a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	0	0	0	1	1
0	0	1	0	0	0	1	1
0	1	0	0	0	0	1	1
0	1	1	0	1	1	1	1
1	0	0	0	0	0	0	1
1	0	1	0	0	0	1	1
1	1	0	1	0	1	0	0
1	1	1	1	1	0	1	1

8.  $z = ((a \Rightarrow b) \cdot (b \Rightarrow \bar{c})) \Rightarrow (\bar{c} \Rightarrow \bar{a})$

s = not a or b  
t = not b or not c  
u = s and t  
v = c or not a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	1	0	0	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	1	1	0	0
1	1	1	1	0	0	1	1

9.  $z = ((a \Rightarrow b) \cdot \overline{(b \Rightarrow c)}) \oplus (c \Rightarrow a)$

s = not a or b  
t = not (not b or c)  
u = s and t  
v = not c or a  
z = u != v

a	b	c	s	t	u	v	z
0	0	0	1	0	0	1	1
0	0	1	1	0	0	0	0
0	1	0	1	1	1	1	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	1	1
1	0	1	0	0	0	1	1
1	1	0	1	1	1	1	0
1	1	1	1	0	0	1	1

10.  $z = ((a \cdot b) \Rightarrow (b \cdot c)) \Rightarrow (c + \bar{a})$

s = a and b  
t = b and c  
u = not s or t  
v = c or not a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	0	0	1	1	1
0	0	1	0	0	1	1	1
0	1	0	0	0	1	1	1
0	1	1	0	1	1	1	1
1	0	0	0	0	1	0	0
1	0	1	0	0	1	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

11.  $z = \overline{((a \oplus \bar{b}) \Rightarrow (b \cdot c))} \Rightarrow (\bar{c} \oplus \bar{a})$

s = not (a != (not b))  
t = b and c  
u = not s or t  
v = (not c) != (not a)  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	0	0	1	0	0
0	0	1	0	0	1	1	1
0	1	0	1	0	0	0	1
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	1
1	0	1	1	0	0	0	1
1	1	0	0	0	1	1	1
1	1	1	0	1	1	0	0

12.  $z = ((a + b) \oplus \overline{(b \cdot c)}) \Rightarrow (c + \bar{a})$

s = a or b  
t = not (b and c)  
u = s != (not t)  
v = c or not a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	0	1	0	1	1
0	0	1	0	1	0	1	1
0	1	0	1	1	1	1	1
0	1	1	1	0	0	1	1
1	0	0	1	1	1	0	0
1	0	1	1	1	1	1	1
1	1	0	1	1	1	0	0
1	1	1	1	0	0	1	1

13.  $z = ((a \Rightarrow b) + (b \oplus c)) \Rightarrow (\bar{c} \Rightarrow \bar{a})$

s = not a or b  
t = b != c  
u = s or t  
v = c or not a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	0	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	1	0	1	1	1
1	0	0	0	0	0	0	1
1	0	1	0	1	1	1	1
1	1	0	1	1	1	0	0
1	1	1	1	0	1	1	1

14.  $z = ((\bar{a} \Rightarrow \bar{b}) \cdot (b \Rightarrow c)) \oplus (c \Rightarrow a)$

s = a or not b  
t = not b or c  
u = s and t  
v = not c or a  
z = u != v

a	b	c	s	t	u	v	z
0	0	0	1	1	1	1	0
0	0	1	1	1	1	0	1
0	1	0	0	0	0	1	1
0	1	1	0	1	0	0	0
1	0	0	1	1	1	1	0
1	0	1	1	1	1	1	0
1	1	0	1	0	0	1	1
1	1	1	1	1	1	1	0

15.  $z = ((a \Rightarrow b) + (b \Rightarrow c)) \Rightarrow (\bar{c} \Rightarrow \bar{a})$

s = not a or b  
t = not b or c  
u = s or t  
v = c or not a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	1	0	0
1	0	1	0	1	1	1	1
1	1	0	1	0	1	0	0
1	1	1	1	1	1	1	1

16.  $z = ((a \Rightarrow b) + (b \Rightarrow c)) \Rightarrow (\bar{c} + \bar{a})$

s = not a or b  
t = not b or c  
u = s or t  
v = not c or not a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	0	0
1	1	0	1	0	1	1	1
1	1	1	1	1	1	0	0

17.  $z = ((a \Rightarrow \bar{b}) \cdot (b \oplus c)) \Rightarrow \overline{(\bar{c} \Rightarrow \bar{a})}$

s = not a or not b  
t = b != c  
u = s and t  
v = not (c or not a)  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	0	0	0	1
0	0	1	1	1	0	0	0
0	1	0	1	1	0	0	0
0	1	1	1	0	0	0	1
1	0	0	1	0	0	1	1
1	0	1	1	1	0	0	0
1	1	0	0	1	0	1	1
1	1	1	0	0	0	0	1

18.  $z = ((a \Rightarrow b) + \overline{(b \Rightarrow c)}) \oplus (c \Rightarrow a)$

s = not a or b  
t = not (not b or c)  
u = s or t  
v = not c or a  
z = u != v

a	b	c	s	t	u	v	z
0	0	0	1	0	1	1	0
0	0	1	1	0	1	0	1
0	1	0	1	1	1	1	0
0	1	1	1	0	1	0	1
1	0	0	0	0	1	1	1
1	0	1	0	0	0	1	1
1	1	0	1	1	1	1	0
1	1	1	1	0	1	1	0

19.  $z = ((a \oplus b) \Rightarrow (b + c)) \Rightarrow (\bar{c} \oplus \bar{a})$

s = a != b  
t = b or c  
u = not s or t  
v = (not c) != (not a)  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	0	0	1	0	0
0	0	1	0	1	1	1	1
0	1	0	1	1	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	1
1	0	1	1	1	1	0	0
1	1	0	0	1	1	1	1
1	1	1	0	1	1	0	0

20.  $z = ((a \Rightarrow \bar{b}) \cdot \overline{(b \Rightarrow c)}) \Rightarrow (c + a)$

s = not a or not b  
t = not (not b or c)  
u = s and t  
v = c or a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	1
0	1	0	1	1	1	0	0
0	1	1	1	0	0	1	1
1	0	0	1	0	0	1	1
1	0	1	1	0	0	1	1
1	1	0	0	1	0	1	1
1	1	1	0	0	0	1	1

21.  $z = ((a + b) + \overline{(b \cdot c)}) \Rightarrow (c + \bar{a})$

s = a or b  
t = not (b and c)  
u = s or t  
v = c or not a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	1	0	1	1	1
1	0	0	1	1	1	0	0
1	0	1	1	1	1	1	1
1	1	0	1	1	1	0	0
1	1	1	1	0	1	1	1

22.  $z = ((a \Rightarrow \bar{b}) \cdot \overline{(b \Rightarrow c)}) \Rightarrow (c \cdot a)$

s = not a or not b  
t = not (not b or c)  
u = s and t  
v = c and a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	0	0	0	1
0	0	1	1	0	0	0	1
0	1	0	1	1	1	0	0
0	1	1	1	0	0	0	1
1	0	0	1	0	0	0	1
1	0	1	1	0	0	1	1
1	1	0	0	1	0	0	1
1	1	1	0	0	0	1	1

23.  $z = ((a + b) \Rightarrow (b + c)) \Rightarrow (\bar{c} \Rightarrow \bar{a})$

s = a or b  
t = b or c  
u = not s or t  
v = c or not a  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	0	0	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	0	0	0	1
1	0	1	1	1	1	1	1
1	1	0	1	1	1	0	0
1	1	1	1	1	1	1	1

24.  $z = ((a \Rightarrow b) \cdot \overline{(b \Rightarrow \bar{c})}) \Rightarrow (\bar{c} \oplus \bar{a})$

s = not a or b  
t = not (not b or not c)  
u = s and t  
v = (not c) != (not a)  
z = not u or v

a	b	c	s	t	u	v	z
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	1
0	1	0	1	0	0	0	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	1	1
1	0	1	0	0	0	0	1
1	1	0	1	0	0	1	1
1	1	1	1	1	1	0	0