# Homework 1. Markov Chains

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## Exercise 1.

(a) $\mathcal{X} = \{A, B, C\}$ $\qquad P = \begin{bmatrix} 0 & 1 & 0 \\ 0{,}5 & 0 & 0{,}5 \\ 1 & 0 & 0 \end{bmatrix}$

$$\begin{tikzcd}
A \arrow[r, shift left, "1"] & B \arrow[l, shift left, "0{,}5"] \arrow[dl, "0{,}5"] \\
C \arrow[u, "1"] &
\end{tikzcd}$$

(b) $$T_{AA} = T_{AB} + 0{,}5\,T_{BA} + 0{,}5\left(T_{BC} + T_{CA}\right)$$

$$= 1 + 0{,}5 \cdot 1 + 0{,}5(1 + 1)$$

$$= 2{,}5$$

$$T_{BB} = 0,5(T_{BA} + T_{AB}) + 0,5(T_{BC} + T_{CA} + T_{AC})$$

$$= 0,5(1+1) + 0,5(1+1+1)$$

$$= 2,5$$

$$T_{AC} = T_{AB} + 0,5\,T_{BC} + 0,5(T_{BA} + T_{AC})$$

$$\Leftrightarrow T_{AC} = T_{AB} + 0,5\,T_{BC} + 0,5\,T_{BA} + 0,5\,T_{AC}$$

$$\Leftrightarrow 0,5\,T_{AC} = T_{AB} + 0,5\,T_{BC} + 0,5\,T_{BA}$$

$$\Leftrightarrow T_{AC} = 2(T_{AB} + 0,5\,T_{BC} + 0,5\,T_{BA})$$

$$\Leftrightarrow T_{AC} = 2(1 + 0,5 + 0,5)$$

$$\Leftrightarrow T_{AC} = 4$$

$$T_{CC} = T_{CA} + T_{AC}$$

$$= 1 + 4$$

$$= 5$$

(c) $$\mu_A = \frac{1}{T_{AA}} = \frac{1}{2{,}5} = 0{,}4$$

$$\mu_B = \frac{1}{T_{BB}} = \frac{1}{2{,}5} = 0{,}4$$

$$\mu_C = \frac{1}{T_{CC}} = \frac{1}{5} = 0{,}2$$

Let $\mu = [\mu_A \ \mu_B \ \mu_C] = [0{,}4 \ \ 0{,}4 \ \ 0{,}2]$

$$\mu P = [0{,}4 \quad 0{,}4 \quad 0{,}2] \begin{bmatrix} 0 & 1 & 0 \\ 0{,}5 & 0 & 0{,}5 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= [0{,}4 \quad 0{,}4 \quad 0{,}2]$$

$$= \mu$$

Given that $\mu = \mu P$ we can conclude that the distribution is invariant for the chain