Homework 4: Supervised learning

Rostislav Andreev 97040
João Aparício 97155

(a) $$\hat{L}_N(\pi) = -\frac{1}{N}\sum_{n=1}^{N} \log \pi(a_n|x_n)$$

$$= -\frac{1}{N}\sum_{n=1}^{N} \log \frac{\exp(z_{a_n})}{\sum_{a'\in A} \exp(z_{a'})}$$

$$= -\frac{1}{N}\sum_{n=1}^{N} \left[\log \exp(z_{a_n}) - \log \sum_{a'\in A} \exp(z_{a'})\right]$$

$$= -\frac{1}{N}\sum_{n=1}^{N} \left[z_{a_n} - \log \sum_{a'\in A} \exp(z_{a'})\right]$$

$$= -\frac{1}{N}\sum_{n=1}^{N} \left[w_{a_n}^T \phi(x_n) - \log \sum_{a'\in A} \exp(w_{a'}^T \phi(x_n))\right] \text{ cqd}$$

(b) $$\frac{\partial \hat{L}_N(\pi)}{\partial w_{k,a}} = \frac{\partial}{\partial w_{k,a}} -\frac{1}{N}\sum_{n=1}^{N} \left[w_{a_n}^T \phi(x_n) - \log \sum_{a'\in A} \exp(w_{a'}^T \phi(x_n))\right]$$

$$= -\frac{1}{N}\sum_{n=1}^{N} \left[\frac{\partial}{\partial w_{k,a}} w_{a_n}^T \phi(x_n) - \frac{\partial}{\partial w_{k,a}} \log \sum_{a'\in A} \exp(w_{a'}^T \phi(x_n))\right]$$

<u>Cálculos auxiliares:</u>

$$\frac{\partial}{\partial w_{k,a}} w_{a_n}^T \phi(x_n) = \frac{\partial}{\partial w_{k,a}}\left[w_{0,a_n} + \sum_{k'=1}^{K} w_{k',a_n} \phi_{k'}(x_n)\right]$$

$$= \frac{\partial}{\partial w_{k,a}} w_{0,a_n} + \frac{\partial}{\partial w_{k,a}} \sum_{k'=1}^{K} w_{k',a_n} \phi_{k'}(x_n)$$

$$= 0 + \phi_k(x_n)\mathbb{I}(a = a_n)$$

$$\frac{\partial}{\partial w_{k,a}} \log \sum_{a'\in A} \exp(w_{a'}^T \phi(x_n)) = \frac{\frac{\partial}{\partial w_{k,a}} \sum_{a'\in A} \exp(w_{a'}^T \phi(x_n))}{\sum_{a'\in A} \exp(w_{a'}^T \phi(x_n))} = \frac{\frac{\partial}{\partial w_{k,a}}\left[w_a^T \phi(x_n)\right] \exp(w_a^T \phi(x_n))}{\sum_{a'\in A} \exp(w_{a'}^T \phi(x_n))}$$

$$= \phi_k(x_n) \cdot \frac{\exp(w_a^T \phi(x_n))}{\sum_{a'\in A} \exp(w_{a'}^T \phi(x_n))} = \phi_k(x_n) \cdot \pi(a|x_n)$$

$$\frac{\partial \hat{L}_N(\pi)}{\partial w_{k,a}} = -\frac{1}{N}\sum_{n=1}^{N}\left[\frac{\partial}{\partial w_{k,a}} w_{a_n}^T \phi(x_n) - \frac{\partial}{\partial w_{k,a}} \log \sum_{a' \in A} \exp(w_{a'}^T \phi(x_n))\right]$$

$$= -\frac{1}{N}\sum_{n=1}^{N}\left[\phi_k(x_n)\mathbb{I}(a_n=a) - \phi_k(x_n)\cdot\pi(a|x_n)\right]$$

$$= -\frac{1}{N}\sum_{n=1}^{N}\phi_k(x_n)\left(\mathbb{I}(a_n=a) - \pi(a|x_n)\right) \quad \text{cqd}$$

(c) $z_a = 1 + \phi_2(x)$

$z_b = 1 + \phi_1(x)$

$z_c = 1 + \phi_1(x) + \phi_2(x)$

$z_a > z_b \iff 1+\phi_2(x) > 1+\phi_1(x)$

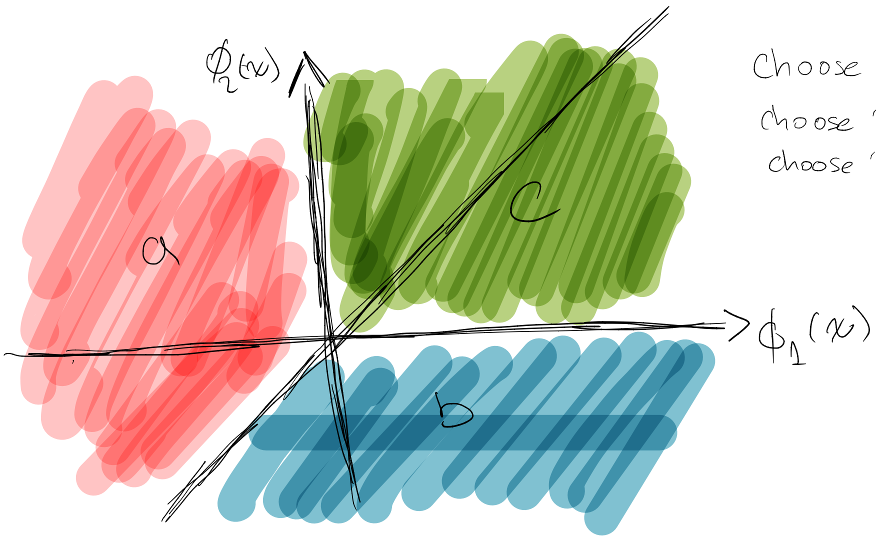
$\iff \phi_2(x) > \phi_1(x)$

$z_a > z_c \iff 1+\phi_2(x) > 1+\phi_1(x)+\phi_2(x)$

$\iff \phi_1(x) < 0$

$z_b > z_c \iff 1+\phi_1(x) > 1+\phi_1(x)+\phi_2(x)$

$\iff \phi_2(x) < 0$



Choose 'a' when $z_a > z_b$ and $z_a > z_c$

choose 'b' when $z_b > z_a$ and $z_b > z_c$

choose 'c' when $z_c > z_a$ and $z_c > z_b$