

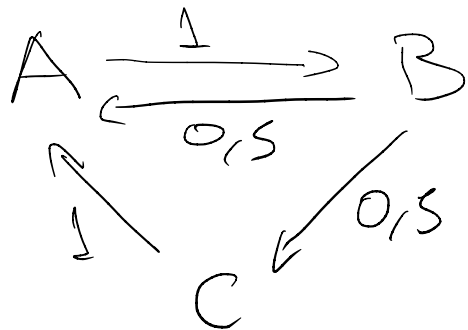
Homework 1. Markov Chains

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Exercise 1.

(a) $X = \{A, B, C\}$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0,5 & 0 & 0,5 \\ 1 & 0 & 0 \end{bmatrix}$$



$$\begin{aligned} (b) \quad T_{AA} &= T_{AB} + 0,5T_{BA} + 0,5(T_{BC} + T_{CA}) \\ &= 1 + 0,5 \cdot 1 + 0,5(1 + 1) \\ &= 2,5 \end{aligned}$$

$$\begin{aligned}T_{BB} &= 0,5(T_{BA} + T_{AB}) + 0,5(T_{BC} + T_{CA} + T_{AC}) \\&= 0,5(1 + 1) + 0,5(1 + 1 + 1) \\&= 2,5\end{aligned}$$

$$T_{AC} = T_{AB} + 0,5T_{BC} + 0,5(T_{BA} + T_{AC})$$

$$\Leftrightarrow T_{AC} = T_{AB} + 0,5T_{BC} + 0,5T_{BA} + 0,5T_{AC}$$

$$\Leftrightarrow 0,5T_{AC} = T_{AB} + 0,5T_{BC} + 0,5T_{BA}$$

$$\Leftrightarrow T_{AC} = 2(T_{AB} + 0,5T_{BC} + 0,5T_{BA})$$

$$\Leftrightarrow T_{AC} = 2(1 + 0,5 + 0,5)$$

$$\Leftrightarrow T_{AC} = 4$$

$$\begin{aligned}T_{CC} &= T_{CA} + T_{AC} \\&= 1 + 4 \\&= 5\end{aligned}$$

$$(c) \quad \mu_A = \frac{1}{T_{AA}} = \frac{1}{2,5} = 0,4$$

$$\mu_B = \frac{1}{T_{BB}} = \frac{1}{2,5} = 0,4$$

$$\mu_C = \frac{1}{T_{CC}} = \frac{1}{5} = 0,2$$

$$\text{Let } \mu = [\mu_A \ \mu_B \ \mu_C] = [0,4 \ 0,4 \ 0,2]$$

$$\mu P = [0,4 \ 0,4 \ 0,2] \begin{bmatrix} 0 & 1 & 0 \\ 0,5 & 0 & 0,5 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= [0,4 \ 0,4 \ 0,2]$$

$$= \mu$$

Given that $\mu = \mu P$ we can conclude that the distribution is invariant for the chain