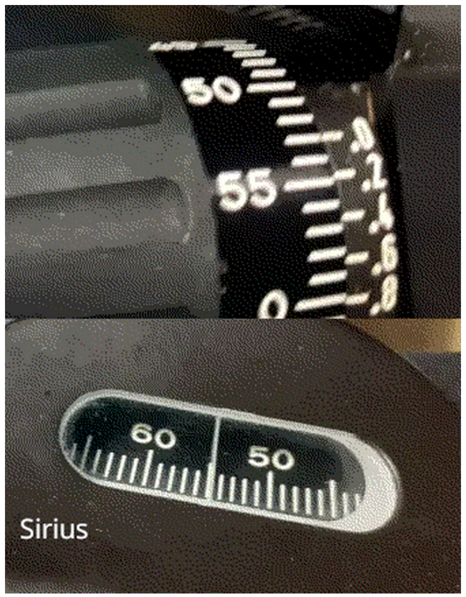
date	2023-04-06
timezone	GMT -6
watch error	0 "
index error	0 '
eye height	6 meter
temperature	28.1 °C
pressure	1024 mbar
fix at	19:44:27

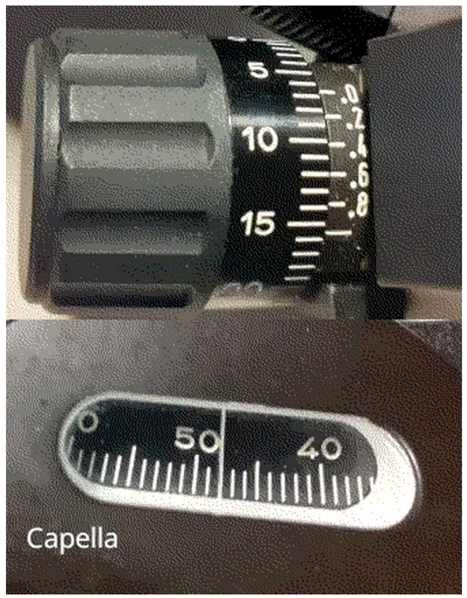
100 feet =	30.48 meter
(60 °F - 32) * 5 / 9 =	15.56 °C

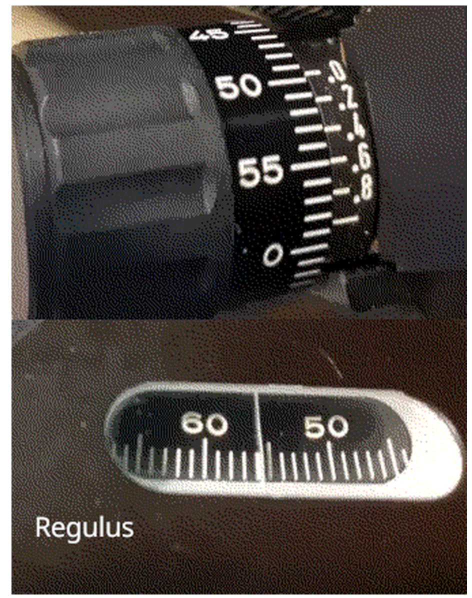
celestial body	local time	∡ altitude	
Sirius	19:14:02	54 °	53.00 '
Capella	19:20:43	48 °	6.60 '
Regulus	19:28:22	55 °	49.60 '

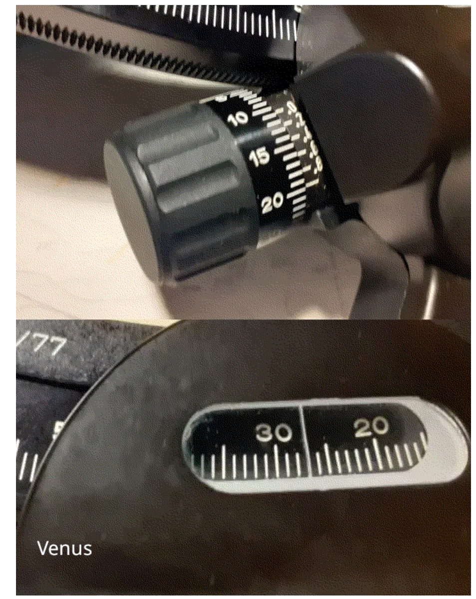
ded reckoning		
latitude	16 °	0.00 '
longitude	-99 °	0.00 '
course	290 °	24.00 '
speed	6.5	knot

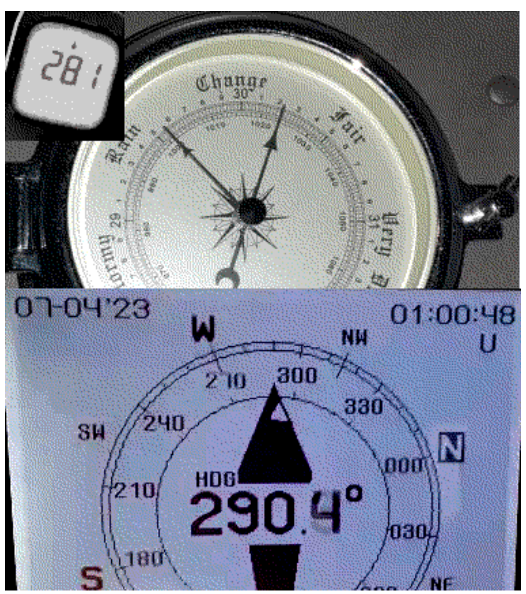
almanac data	Sirius		Capella		Regulus	
UTC	###		2023-04-07 01:20:43		2023-04-07 01:28:22	
GHA	210 °	3.11 '	210 °	3.11 '	210 °	3.11 '
SHA	258 °	27.59 '	280 °	24.3 '	207 °	35.79 '
DEC	-16 °	45.05 '	46 °	1.37 '	11 °	51.2 '
tabular v						
tabular d						
HP		'		'		'
SD		'		'		'
limb						

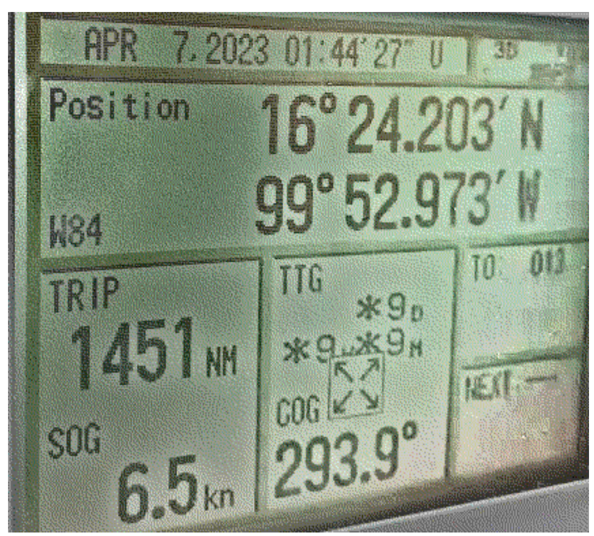


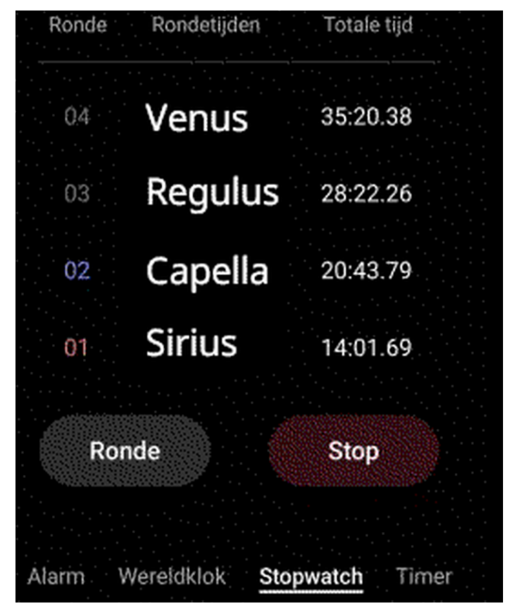












celestial body	date			time		
Sirius	year	month	day	hour	minute	second
∡ local date & time	2023	4	6	19	14	2
				GMT -6		0 WE
🖩 UTC date & time	2023	4	7	1	14	2
🖩 UTC time of fix	2023	4	7	1	44	27

🖩 Δt	1825 sec	t_{fix} - t_{UTC}	0 ·3600+	30 ·60+	25
∡ speed	6.5 knot	ded reckoning	lat (ϕ_0)	16°	0.000'
🖩 distance	3.295 nmi		lon (λ_0)	-99°	0.000'
🖩 x	-0.277045	∡	course	290°	24.000'
🖩 y	-0.215888	🖩	Zn	217°	55.650'

∡ : measurement
🖩 : calculation
📖 : almanac data

clock slow : +WE
clock fast : - WE
GMT-z : add z
GMT+z : subtract z
North, East : +
South, West : -
OFF the arc : +
ON the arc : -

$$atan2(y,x)=\begin{cases} atan\left(\frac{y}{x}\right) & x>0 \\ atan\left(\frac{y}{x}\right)+\pi & x<0 \quad y\geq 0 \\ atan\left(\frac{y}{x}\right)-\pi & x<0 \quad y<0 \\ \pi/2 & x=0 \quad y>0 \\ -\pi/2 & x=0 \quad y<0 \\ \text{undefined} & x=0 \quad y=0 \end{cases}$$

∡	T	28.1 °C	
∡	p	1024 mbar	
∡	h	6 meter	
🖩	f	0.952946	
🖩	R	0.701899'	
∡	limb	lower: + SD	upper: - SD
📖	HP	0'	
📖	HP_{Sun}	0.146569'	
📖	*tabular v*	0.000	
📖	*tabular d*	0.000	
🖩	t_{incr}	842 sec	

$$t_{incr}=minutes_{UTC}\cdot 60 \;+\; seconds_{UTC}$$

$$GHA_{incr}^{\substack{Sun,\\planets}}[']\;=\;900.00'\cdot\frac{t_{incr}}{3600}$$

$$GHA_{incr}^{Moon}[']\;=\;859.00'\cdot\frac{t_{incr}}{3600}$$

$$GHA_{incr}^{stars}[']\;=\;902.46'\cdot\frac{t_{incr}}{3600}$$

∡	Hs	=	54°	###'
∡	IE	±		0.000'
🖩	DIP	-		4.306'
🖩	Ha	=	54°	48.694'
🖩	f · R	-		0.669'
📖	SD	±		0.000'
🖩	P	+		0.000'
🖩	ΔHo	+		0.992'
🖩	Ho	=	54°	49.017'

📖	GHA_{hour}	=	210°	3.110'
🖩	GHA_{incr}	+	3°	31.075'
📖	SHA	+	258°	27.590'
🖩	v_{corr}	+		0.000'
🖩	GHA	=	112°	1.775'

📖	tabular DEC	=	-16°	45.050'
🖩	d_{corr}	+		0.000'
🖩	DEC	=	-16°	45.050'

$$\begin{aligned}
distance[nmi] &= \frac{S\cdot\Delta t}{3600} \\
Zn &= atan2(y,x) \\
DIP['] &= 1.758\cdot\sqrt{h} \\
Ha &= Hs\pm IE-DIP \\
f &= \frac{p}{1010}\cdot\frac{283.15}{273.15+T} \\
R['] &= \cot\left(Ha+\frac{7.31}{Ha+4.4}\right) \\
P['] &= HP\cdot\cos(Ha) \\
\Delta Ho['] &= distance\cdot\cos(Zn-C) \\
Ho &= Ha-f\cdot R\pm SD+P+\Delta Ho \\
v_{corr}['] &= v\cdot(minutes_{UTC}+0.5)\;/\;60 \\
GHA &= GHA_{hour}+GHA_{inc}+SHA+v_{corr} \\
d_{corr}['] &= d\cdot(minutes_{UTC}+0.5)\;/\;60 \\
DEC &= DEC_{tab}+d_{corr} \\
y &= \sin(-GHA-\lambda_0)\cdot\cos DEC \\
x &= \cos\phi_0\cdot\sin DEC-\sin\phi_0\cdot\cos DEC\cdot\cos(-GHA-\lambda_0)
\end{aligned}$$

	celestial body	year	month	day	hour	minute	second
	Capella	date			time		
∡	local date & time	2023	4	6	19	20	43
					GMT -6		0 WE
🖩	UTC date & time	2023	4	7	1	20	43
🖩	UTC time of fix	2023	4	7	1	44	27

🖩	Δt	1424	sec	$t_{fix} - t_{UTC}$	0 ·3600+	23 ·60+	44
∡	speed	6.5	knot	ded reckoning	lat (ϕ_0)	16°	0.000'
🖩	distance	2.571	nmi		lon (λ_0)	-99°	0.000'
🖩	x	0.691740		∡	course	290°	24.000'
🖩	y	-0.414489		🖩	Zn	329°	4.201'

∡ : measurement
🖩 : calculation
📖 : almanac data

clock slow : +WE
clock fast : - WE
GMT-z : add z
GMT+z : subtract z
North, East : +
South, West : -
OFF the arc : +
ON the arc : -

$$atan2(y,x)=\begin{cases} atan\left(\frac{y}{x}\right) & x>0 \\ atan\left(\frac{y}{x}\right)+\pi & x<0 \quad y\geq 0 \\ atan\left(\frac{y}{x}\right)-\pi & x<0 \quad y<0 \\ \pi/2 & x=0 \quad y>0 \\ -\pi/2 & x=0 \quad y<0 \\ \text{undefined} & x=0 \quad y=0 \end{cases}$$

∡	T	28.1	°C
∡	p	1024	mbar
∡	h	6	meter
🖩	f	0.952946	
🖩	R	0.894806'	
∡	limb	lower: + SD	upper: - SD
📖	HP	0'	
📖	HP_{Sun}	0.146569'	
📖	*tabular v*	0.000	
📖	*tabular d*	0.000	
🖩	t_{incr}	1243	sec

$$t_{incr} = minutes_{UTC}\cdot 60 \; + \; seconds_{UTC}$$

$$GHA_{incr}^{Sun,\,planets}['] = 900.00'\cdot\frac{t_{incr}}{3600}$$

$$GHA_{incr}^{Moon}['] = 859.00'\cdot\frac{t_{incr}}{3600}$$

$$GHA_{incr}^{stars}['] = 902.46'\cdot\frac{t_{incr}}{3600}$$

∡	Hs	=	48°	6.600'
∡	IE	±		0.000'
🖩	DIP	-		4.306'
🖩	Ha	=	48°	2.294'
🖩	f · R	-		0.853'
📖	SD	±		0.000'
🖩	P	+		0.000'
🖩	ΔHo	+		2.007'
🖩	Ho	=	48°	3.449'

📖	GHA_{hour}	=	210°	3.110'
🖩	GHA_{incr}	+	5°	11.599'
📖	SHA	+	280°	24.300'
🖩	v_{corr}	+		0.000'
🖩	GHA	=	135°	39.009'

📖	tabular DEC	=	46°	1.370'
🖩	d_{corr}	+		0.000'
🖩	DEC	=	46°	1.370'

$$\begin{aligned}
distance[nmi] &= \frac{S\cdot\Delta t}{3600} \\
Zn &= atan2(y,x) \\
DIP['] &= 1.758\cdot\sqrt{h} \\
Ha &= Hs \pm IE - DIP \\
f &= \frac{p}{1010}\cdot\frac{283.15}{273.15+T} \\
R['] &= \cot\left(Ha+\frac{7.31}{Ha+4.4}\right) \\
P['] &= HP\cdot\cos(Ha) \\
\Delta Ho['] &= distance\cdot\cos(Zn-C) \\
Ho &= Ha - f\cdot R \pm SD + P + \Delta Ho \\
v_{corr}['] &= v\cdot(minutes_{UTC}+0.5)\;/\;60 \\
GHA &= GHA_{hour}+GHA_{inc}+SHA+v_{corr} \\
d_{corr}['] &= d\cdot(minutes_{UTC}+0.5)\;/\;60 \\
DEC &= DEC_{tab}+d_{corr} \\
y &= \sin(-GHA-\lambda_0)\cdot\cos DEC \\
x &= \cos\phi_0\cdot\sin DEC - \sin\phi_0\cdot\cos DEC\cdot\cos(-GHA-\lambda_0)
\end{aligned}$$

	celestial body	date			time			
	Regulus	year	month	day	hour	minute	second	
∡	local date & time	2023	4	6	19	28	22	
					GMT -6		0	WE
🖩	UTC date & time	2023	4	7	1	28	22	
🖩	UTC time of fix	2023	4	7	1	44	27	

🖩	Δt	965	sec	t_{fix} - t_{UTC}	0 ·3600+	16 ·60+	5
∡	speed	6.5	knot	ded reckoning	lat (ϕ_0)	16 °	0.000 '
🖩	distance	1.742	nmi		lon (λ_0)	-99 °	0.000 '
🖩	x	0.197450		∡	course	290 °	24.000 '
🖩	y	0.550672		🖩	Zn	70 °	16.445 '

∡ : measurement
🖩 : calculation
📖 : almanac data

clock slow : +WE
clock fast : - WE
GMT-z : add z
GMT+z : subtract z
North, East : +
South, West : -
OFF the arc : +
ON the arc : -

$$atan2(y,x)=\begin{cases} atan\left(\frac{y}{x}\right) & x>0 \\ atan\left(\frac{y}{x}\right)+\pi & x<0 \quad y\geq 0 \\ atan\left(\frac{y}{x}\right)-\pi & x<0 \quad y<0 \\ \pi/2 & x=0 \quad y>0 \\ -\pi/2 & x=0 \quad y<0 \\ \text{undefined} & x=0 \quad y=0 \end{cases}$$

∡	T	28.1	°C
∡	p	1024	mbar
∡	h	6	meter
🖩	f	0.952946	
🖩	R	0.677651 '	
∡	limb	lower: + SD	upper: - SD
📖	HP	0 '	
📖	HP_{Sun}	0.146569 '	
📖	*tabular v*	0.000	
📖	*tabular d*	0.000	
🖩	t_{incr}	1702	sec

$$t_{incr}=minutes_{UTC}\cdot 60 + seconds_{UTC}$$

$$GHA_{incr}^{Sun, planets}['] = 900.00'\cdot\frac{t_{incr}}{3600}$$

$$GHA_{incr}^{Moon}['] = 859.00'\cdot\frac{t_{incr}}{3600}$$

$$GHA_{incr}^{stars}['] = 902.46'\cdot\frac{t_{incr}}{3600}$$

∡	Hs	=	55 °	49.600 '
∡	IE	±		0.000 '
🖩	DIP	-		4.306 '
🖩	Ha	=	55 °	45.294 '
🖩	f · R	-		0.646 '
📖	SD	±		0.000 '
🖩	P	+		0.000 '
🖩	ΔHo	+		-1.332 '
🖩	Ho	=	55 °	43.316 '

📖	GHA_{hour}	=	210 °	3.110 '
🖩	GHA_{incr}	+	7 °	6.663 '
📖	SHA	+	207 °	35.790 '
🖩	v_{corr}	+		0.000 '
🖩	GHA	=	64 °	45.563 '

📖	tabular DEC	=	11 °	51.200 '
🖩	d_{corr}	+		0.000 '
🖩	DEC	=	11 °	51.200 '

$$distance[nmi] = \frac{S\cdot\Delta t}{3600}$$
$$Zn = atan2(y,x)$$
$$DIP['] = 1.758\cdot\sqrt{h}$$
$$Ha = Hs\pm IE - DIP$$
$$f = \frac{p}{1010}\cdot\frac{283.15}{273.15+T}$$
$$R['] = \cot\left(Ha+\frac{7.31}{Ha+4.4}\right)$$
$$P['] = HP\cdot\cos(Ha)$$
$$\Delta Ho['] = distance\cdot\cos(Zn-C)$$
$$Ho = Ha - f\cdot R\pm SD + P + \Delta Ho$$
$$v_{corr}['] = v\cdot(minutes_{UTC}+0.5)\ /\ 60$$
$$GHA = GHA_{hour}+GHA_{inc}+SHA+v_{corr}$$
$$d_{corr}['] = d\cdot(minutes_{UTC}+0.5)\ /\ 60$$
$$DEC = DEC_{tab}+d_{corr}$$
$$y = \sin(-GHA-\lambda_0)\cdot\cos DEC$$
$$x = \cos\phi_0\cdot\sin DEC - \sin\phi_0\cdot\cos DEC\cdot\cos(-GHA-\lambda_0)$$

celestial body	latitude	longitude	altitude
	DEC	*- GHA*	*Ho*
Sirius	-16.750833	-112.029589	54.816955
Capella	46.022833	-135.650156	48.057475
Regulus	11.853333	-64.759384	55.721930

	x_n	y_n	z_n
	cos(lat)·cos(lon)	*cos(lat)·sin(lon)*	*sin(lat)*
$\overline{\text{GP 1}}$	-0.359169	-0.887655	-0.288210
$\overline{\text{GP 2}}$	-0.496535	-0.485392	0.719617
$\overline{\text{GP 3}}$	0.417328	-0.885237	0.205407
	x1+x2+x3	*y1+y2+y3*	*z1+z2+z3*
$\overline{\text{SUM}}$	-0.438376	-2.258285	0.636814

$\overline{P} = B\cdot\overline{GP1} - A\cdot\overline{GP2}$	Px	Py	Pz
	B·x1 - A·x2	*B·y1 - A·y2*	*B·z1 - A·z2*
	0.138670	-0.263534	-0.802529

$\overline{Q} = C\cdot\overline{GP2} - B\cdot\overline{GP3}$	Qx	Qy	Qz
	C·x2 - B·x3	*C·y2 - B·y3*	*C·z2 - B·z3*
	-0.720709	0.257367	0.441844

$\overline{V} = \overline{P} \times \overline{Q}$	x	y	z
	Py·Qz - Pz·Qy	*Pz·Qx - Px·Qz*	*Px·Qy - Py·Qx*
	0.090104	0.517119	-0.154242
$\overline{\text{SUM}}\cdot\overline{V}$	*x·(x1+x2+x3)*	*y·(y1+y2+y3)*	*z·(z1+z2+z3)*
	-0.039499	-1.167802	-0.098223

fix:	latitude	longitude
	16° 22.51' N	99° 53.05' W

Three star fix
At least 6 decimal place precision is required.

	sin(Ho)
A =	0.817315
B =	0.743816
C =	0.826314

$$\operatorname{atan2}(y,x) = \begin{cases} \operatorname{atan}\left(\frac{y}{x}\right) & x>0 \\ \operatorname{atan}\left(\frac{y}{x}\right)+\pi & x<0 \quad y\geq 0 \\ \operatorname{atan}\left(\frac{y}{x}\right)-\pi & x<0 \quad y<0 \\ \pi/2 & x=0 \quad y>0 \\ -\pi/2 & x=0 \quad y<0 \\ \text{undefined} & x=0 \quad y=0 \end{cases}$$

$s = sign(\overline{SUM}\cdot\overline{V}) =$	-1
$d = \sqrt{x^2 + y^2} =$	0.524910
$lat = atan2(s\cdot z, d)$	
$lon = atan2(s\cdot y, s\cdot x)$	

crooked hat	
latitude	longitude
16° 21.55' N	99° 57.64' W
16° 27.02' N	99° 50.17' W
16° 19.03' N	99° 50.6' W

celestial body	latitude	longitude	altitude
	DEC	*- GHA*	*Ho*
Sirius	-16.750833	-112.029589	54.816955
Capella	46.022833	-135.650156	48.057475

	x_n	y_n	z_n
	cos(lat)·cos(lon)	*cos(lat)·sin(lon)*	*sin(lat)*
$\overline{\textbf{GP 1}}$	-0.359169	-0.887655	-0.288210
$\overline{\textbf{GP 2}}$	-0.496535	-0.485392	0.719617

	Cx	Cy	Cz
$\overline{\textbf{cross}} = \overline{\textbf{GP1}} \times \overline{\textbf{GP2}}$	*y1·z2 - y2·z1*	*z1·x2 - z2·x1*	*x1·y2 - x2·y1*
	-0.778666	0.401571	-0.266414

	CPx	CPy	CPz
$\overline{\textbf{center}} = \textbf{k1}\cdot\overline{\textbf{GP1}}+\textbf{k2}\cdot\overline{\textbf{GP2}}$	*k1·x1 + k2·x2*	*k1·y1 + k2·y2*	*k1·z1 + k2·z2*
	-0.392481	-0.661846	0.149519

	vx	vy	vz
$\overline{\textbf{V1}} = \overline{\textbf{center}} + \textbf{sc}\cdot\overline{\textbf{cross}}$	*CPx + sc·Cx*	*CPy + sc·Cy*	*CPz + sc·Cz*
	-0.645787	-0.531212	0.062853
$\overline{\textbf{V2}} = \overline{\textbf{center}} - \textbf{sc}\cdot\overline{\textbf{cross}}$	*CPx - sc·Cx*	*CPy - sc·Cy*	*CPz - sc·Cz*
	-0.139175	-0.792480	0.236186

	latitude	longitude
fix:	*atan2(z, d)*	*atan2(y, x)*
	16° 21.55' N	99° 57.64' W

Two star fix

Intersection points between two small circles.

Use the GP and altitude of the 1st and 2nd stars.

At least 6 decimal place precision is required.

A =	*sin Ho1*	0.817315
B =	*sin Ho2*	0.743816
cos α =	*x1·x2 + y1·y2 + z1·z2*	0.401800
k1 =	*A - B·cos α*	0.518450
k2 =	*B - A·cos α*	0.415418
sc =	$\sqrt{1 - \cos^2\alpha - A\cdot k1 - B\cdot k2}$	0.325307

		intersection 1	intersection 2
d =	$\sqrt{vx^2 + vy^2}$	0.836198	0.804608
lat =	*atan2(vz, d)*	4.298565	16.359135
lon =	*atan2(vy, vx)*	-140.559923	-99.960720

Pick the closest point to ded reckoning. The haversine distance can help if the decision is not obvious.

		intersection 1	intersection 2
a =	$\sin((lat - \phi_0) / 2)$	-0.101937	0.003134
b =	$\sin((lon - \lambda_0) / 2)$	-0.354780	-0.008384
c =	$a^2 + b^2\cdot\cos(lat)\cdot\cos(\phi_0)$	131.04E-03	74.65E-06
R =	*6378.137 km*		
L =	$2\cdot R\cdot\mathrm{asin}(\sqrt{c})$	4725.077 km	110.217 km

celestial body	latitude	longitude	altitude
	DEC	*- GHA*	*Ho*
Capella	46.022833	-135.650156	48.057475
Regulus	11.853333	-64.759384	55.721930

	x_n	y_n	z_n
	cos(lat)·cos(lon)	*cos(lat)·sin(lon)*	*sin(lat)*
$\overline{\text{GP 1}}$	-0.496535	-0.485392	0.719617
$\overline{\text{GP 2}}$	0.417328	-0.885237	0.205407

	Cx	Cy	Cz
$\overline{\text{cross}}$ = $\overline{\text{GP1}} \times \overline{\text{GP2}}$	*y1·z2 - y2·z1*	*z1·x2 - z2·x1*	*x1·y2 - x2·y1*
	0.537329	0.402308	0.642119

	CPx	CPy	CPz
$\overline{\text{center}}$ = k1·$\overline{\text{GP1}}$+k2·$\overline{\text{GP2}}$	*k1·x1 + k2·x2*	*k1·y1 + k2·y2*	*k1·z1 + k2·z2*
	0.012497	-0.700196	0.428238

	vx	vy	vz
$\overline{\text{V1}}$ = $\overline{\text{center}}$ + sc·$\overline{\text{cross}}$	*CPx + sc·Cx*	*CPy + sc·Cy*	*CPz + sc·Cz*
	0.166369	-0.584989	0.612118
$\overline{\text{V2}}$ = $\overline{\text{center}}$ - sc·$\overline{\text{cross}}$	*CPx - sc·Cx*	*CPy - sc·Cy*	*CPz - sc·Cz*
	-0.141375	-0.815403	0.244357

	latitude	longitude
fix:	*atan2(z, d)*	*atan2(y, x)*
	16° 27.02' N	99° 50.17' W

Two star fix

Intersection points between two small circles.

Use the GP and altitude of the 2nd and 3rd stars.

At least 6 decimal place precision is required.

A =	*sin Ho1*	0.743816
B =	*sin Ho2*	0.826314
cos α =	*x1·x2 + y1·y2 + z1·z2*	0.370284
k1 =	*A - B·cos α*	0.437845
k2 =	*B - A·cos α*	0.550891
sc =	$\sqrt{1 - \cos^2\alpha - A\cdot k1 - B\cdot k2}$	0.286365

		intersection 1	intersection 2
d =	$\sqrt{vx^2 + vy^2}$	0.608186	0.827568
lat =	*atan2(vz, d)*	45.184589	16.450366
lon =	*atan2(vy, vx)*	-74.124409	-99.836205

Pick the closest point to ded reckoning. The haversine distance can help if the decision is not obvious.

		intersection 1	intersection 2
a =	$\sin((lat - \phi_0) / 2)$	0.251939	0.003930
b =	$\sin((lon - \lambda_0) / 2)$	0.215380	-0.007297
c =	$a^2 + b^2\cdot\cos(lat)\cdot\cos(\phi_0)$	94.90E-03	64.54E-06
R =	*6378.137 km*		
L =	$2\cdot R\cdot \mathrm{asin}(\sqrt{c})$	3994.700 km	102.479 km

celestial body	latitude	longitude	altitude
	DEC	*- GHA*	*Ho*
Regulus	11.853333	-64.759384	55.721930
Sirius	-16.750833	-112.029589	54.816955

	x_n	y_n	z_n
	cos(lat)·cos(lon)	*cos(lat)·sin(lon)*	*sin(lat)*
$\overline{\text{GP 1}}$	0.417328	-0.885237	0.205407
$\overline{\text{GP 2}}$	-0.359169	-0.887655	-0.288210

	Cx	Cy	Cz
$\overline{\text{cross}} = \overline{\text{GP1}} \times \overline{\text{GP2}}$	*y1·z2 - y2·z1*	*z1·x2 - z2·x1*	*x1·y2 - x2·y1*
	0.437465	0.046502	-0.688394

	CPx	CPy	CPz
$\overline{\text{center}} = \text{k1}\cdot\overline{\text{GP1}}+\text{k2}\cdot\overline{\text{GP2}}$	*k1·x1 + k2·x2*	*k1·y1 + k2·y2*	*k1·z1 + k2·z2*
	0.025741	-0.616735	-0.025304

	vx	vy	vz
$\overline{\text{V1}} = \overline{\text{center}} + \text{sc}\cdot\overline{\text{cross}}$	*CPx + sc·Cx*	*CPy + sc·Cy*	*CPz + sc·Cz*
	0.160984	-0.602359	-0.238123
$\overline{\text{V2}} = \overline{\text{center}} - \text{sc}\cdot\overline{\text{cross}}$	*CPx - sc·Cx*	*CPy - sc·Cy*	*CPz - sc·Cz*
	-0.109503	-0.631112	0.187516

fix:	latitude	longitude
	atan2(z, d)	*atan2(y, x)*
	16° 19.03' N	99° 50.6' W

Two star fix

Intersection points between two small circles.

Use the GP and altitude of the 3rd and 1st stars.

At least 6 decimal place precision is required.

A =	*sin Ho1*	0.826314
B =	*sin Ho2*	0.817315
cos α =	*x1·x2 + y1·y2 + z1·z2*	0.576694
k1 =	*A - B·cos α*	0.354973
k2 =	*B - A·cos α*	0.340785
sc =	$\sqrt{1 - \cos^2 \alpha - A\cdot k1 - B\cdot k2}$	0.309154

		intersection 1	intersection 2
d =	$\sqrt{vx^2 + vy^2}$	0.623500	0.640541
lat =	*atan2(vz, d)*	-20.902535	16.317160
lon =	*atan2(vy, vx)*	-75.037026	-99.843325

Pick the closest point to ded reckoning. The haversine distance can help if the decision is not obvious.

a =	$\sin((lat - \phi_0) / 2)$	-0.316498	0.002768
b =	$\sin((lon - \lambda_0) / 2)$	0.207596	-0.007359
c =	$a^2 + b^2\cdot\cos(lat)\cdot\cos(\phi_0)$	138.87E-03	57.63E-06
R =	*6378.137 km*		
L =	$2\cdot R\cdot \text{asin}(\sqrt{c})$	4871.207 km	96.835 km