celestial body		date			time			
		year	month	day	hour	minute	second	
∡	local date & time							
					GMT			WE
🖩	UTC date & time							
🖩	UTC time of fix							

🖩	Δt	sec	$t_{fix} - t_{UTC}$	·3600+	·60+	
∡	speed	knot	ded reckoning	lat (ϕ_0)	°	'
🖩	distance	nmi		lon (λ_0)	°	'
🖩	x		∡	course	°	'
🖩	y		🖩	Zn	°	'

∡ : measurement
🖩 : calculation
📖 : almanac data

clock slow : +WE
clock fast : - WE
GMT-z : add z
GMT+z : subtract z
North, East : +
South, West : -
OFF the arc : +
ON the arc : -

$$atan2(y,x) = \begin{cases} atan\left(\frac{y}{x}\right) & x>0 \\ atan\left(\frac{y}{x}\right)+\pi & x<0 \quad y\geq 0 \\ atan\left(\frac{y}{x}\right)-\pi & x<0 \quad y<0 \\ \pi/2 & x=0 \quad y>0 \\ -\pi/2 & x=0 \quad y<0 \\ \text{undefined} & x=0 \quad y=0 \end{cases}$$

∡	T		°C
∡	p		mbar
∡	h		meter
🖩	f		
🖩	R	'	
∡	limb	lower: + SD	upper: - SD
📖	HP	'	
📖	HP_{Sun}	0.146569 '	
📖	*tabular v*		
📖	*tabular d*		
🖩	t_{incr}		sec

$$t_{incr} = minutes_{UTC}\cdot 60 \ + \ seconds_{UTC}$$

$$GHA_{incr}^{Sun,\,planets}['] = 900.00'\cdot\frac{t_{incr}}{3600}$$

$$GHA_{incr}^{Moon}['] = 859.00'\cdot\frac{t_{incr}}{3600}$$

$$GHA_{incr}^{stars}['] = 902.46'\cdot\frac{t_{incr}}{3600}$$

∡	Hs	=	°	'
∡	IE	±		'
🖩	DIP	-		'
🖩	Ha	=	°	'
🖩	f · R	-		'
📖	SD	±		'
🖩	P	+		'
🖩	ΔHo	+		'
🖩	Ho	=	°	'

📖	GHA_{hour}	=	°	'
🖩	GHA_{incr}	+	°	'
📖	SHA	+	°	'
🖩	v_{corr}	+		'
🖩	GHA	=	°	'

📖	tabular DEC	=	°	'
🖩	d_{corr}	+		'
🖩	DEC	=	°	'

$$\begin{aligned}
distance[nmi] &= \frac{S\cdot\Delta t}{3600} \\
Zn &= atan2(y,x) \\
DIP['] &= 1.758\cdot\sqrt{h} \\
Ha &= Hs \pm IE - DIP \\
f &= \frac{p}{1010}\cdot\frac{283.15}{273.15+T} \\
R['] &= \cot\left(Ha+\frac{7.31}{Ha+4.4}\right) \\
P['] &= HP\cdot\cos(Ha) \\
\Delta Ho['] &= distance\cdot\cos(Zn-C) \\
Ho &= Ha - f\cdot R \pm SD + P + \Delta Ho \\
v_{corr}['] &= v\cdot(minutes_{UTC}+0.5)\ /\ 60 \\
GHA &= GHA_{hour}+GHA_{inc}+SHA+v_{corr} \\
d_{corr}['] &= d\cdot(minutes_{UTC}+0.5)\ /\ 60 \\
DEC &= DEC_{tab}+d_{corr} \\
y &= \sin(-GHA-\lambda_0)\cdot\cos DEC \\
x &= \cos\phi_0\cdot\sin DEC - \sin\phi_0\cdot\cos DEC\cdot\cos(-GHA-\lambda_0) \\
1\ \ \text{feet} &= 0.3048\ \text{meter} \\
(50°F-32)\cdot 5/9 &= 10.00°C
\end{aligned}$$

	celestial body	date			time		
		year	month	day	hour	minute	second
∡	local date & time						
					GMT		WE
🖩	UTC date & time						
🖩	UTC time of fix						

🖩	Δt	sec	t_{fix} - t_{UTC}		·3600+	·60+
∡	speed	knot	ded reckoning	lat (ϕ_0)	°	'
🖩	distance	nmi		lon (λ_0)	°	'
🖩	x		∡	course	°	'
🖩	y		🖩	Zn	°	'

∡ : measurement
🖩 : calculation
📖 : almanac data

clock slow : +WE
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∡	T	°C	
∡	p	mbar	
∡	h	meter	
🖩	f		
🖩	R	'	
∡	limb	lower: + SD	upper: - SD
📖	HP	'	
📖	HP_{Sun}	0.146569 '	
📖	*tabular v*		
📖	*tabular d*		
🖩	t_{incr}	sec	

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$$GHA_{incr}^{stars}['] = 902.46'\cdot\frac{t_{incr}}{3600}$$

∡	Hs	=	°	'
∡	IE	±		'
🖩	DIP	-		'
🖩	Ha	=	°	'
🖩	f · R	-		'
📖	SD	±		'
🖩	P	+		'
🖩	ΔHo	+		'
🖩	Ho	=	°	'

📖	GHA_{hour}	=	°	'
🖩	GHA_{incr}	+	°	'
📖	SHA	+	°	'
🖩	v_{corr}	+		'
🖩	GHA	=	°	'

📖	tabular DEC	=	°	'
🖩	d_{corr}	+		'
🖩	DEC	=	°	'

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$$P['] = HP\cdot\cos(Ha)$$
$$\Delta Ho['] = distance\cdot\cos(Zn-C)$$
$$Ho = Ha-f\cdot R\pm SD+P+\Delta Ho$$
$$v_{corr}['] = v\cdot(minutes_{UTC}+0.5)\;/\;60$$
$$GHA = GHA_{hour}+GHA_{inc}+SHA+v_{corr}$$
$$d_{corr}['] = d\cdot(minutes_{UTC}+0.5)\;/\;60$$
$$DEC = DEC_{tab}+d_{corr}$$
$$y = \sin(-GHA-\lambda_0)\cdot\cos DEC$$
$$x = \cos\phi_0\cdot\sin DEC-\sin\phi_0\cdot\cos DEC\cdot\cos(-GHA-\lambda_0)$$
$$1\ \text{feet} = 0.3048\ \text{meter}$$
$$(50°F-32)\cdot 5/9 = 10.00°C$$

	celestial body	date			time			
		year	month	day	hour		minute	second
∡	local date & time							
					GMT			WE
🖩	UTC date & time							
🖩	UTC time of fix							

🖩	Δt	sec	$t_{fix} - t_{UTC}$	·3600+	·60+	
∡	speed	knot	ded reckoning	lat (ϕ_0)	°	'
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∡	T	°C	
∡	p	mbar	
∡	h	meter	
🖩	f		
🖩	R	'	
∡	limb	lower: + SD	upper: - SD
📖	HP	'	
📖	HP_{Sun}	0.146569 '	
📖	***tabular v***		
📖	***tabular d***		
🖩	t_{incr}	sec	

$$t_{incr}=minutes_{UTC}\cdot 60 \;+\; seconds_{UTC}$$

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∡	Hs	=	°	'
∡	IE	±		'
🖩	DIP	-		'
🖩	Ha	=	°	'
🖩	f · R	-		'
📖	SD	±		'
🖩	P	+		'
🖩	ΔHo	+		'
🖩	Ho	=	°	'

📖	GHA_{hour}	=	°	'
🖩	GHA_{incr}	+	°	'
📖	SHA	+	°	'
🖩	v_{corr}	+		'
🖩	GHA	=	°	'

📖	tabular DEC	=	°	'
🖩	d_{corr}	+		'
🖩	DEC	=	°	'

$$\begin{aligned}
distance[nmi] &= \frac{S\cdot\Delta t}{3600} \\
Zn &= atan2(y,x) \\
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P['] &= HP\cdot\cos(Ha) \\
\Delta Ho['] &= distance\cdot\cos(Zn-C) \\
Ho &= Ha-f\cdot R\pm SD+P+\Delta Ho \\
v_{corr}['] &= v\cdot(minutes_{UTC}+0.5)\;/\;60 \\
GHA &= GHA_{hour}+GHA_{inc}+SHA+v_{corr} \\
d_{corr}['] &= d\cdot(minutes_{UTC}+0.5)\;/\;60 \\
DEC &= DEC_{tab}+d_{corr} \\
y &= \sin(-GHA-\lambda_0)\cdot\cos DEC \\
x &= \cos\phi_0\cdot\sin DEC-\sin\phi_0\cdot\cos DEC\cdot\cos(-GHA-\lambda_0) \\
1\ \text{feet} &= 0.3048\ \text{meter} \\
(50°F-32)\cdot 5/9 &= 10.00°C
\end{aligned}$$

celestial body	latitude	longitude	altitude
	DEC	*- GHA*	*Ho*

	x_n	y_n	z_n
	$\cos(lat)\cdot\cos(lon)$	$\cos(lat)\cdot\sin(lon)$	$\sin(lat)$
$\overline{\text{GP 1}}$			
$\overline{\text{GP 2}}$			
$\overline{\text{GP 3}}$			
	$x1+x2+x3$	$y1+y2+y3$	$z1+z2+z3$
$\overline{\text{SUM}}$			

$\overline{P}$ = $B\cdot\overline{GP1}-A\cdot\overline{GP2}$	**Px**	**Py**	**Pz**
	$B\cdot x1 - A\cdot x2$	$B\cdot y1 - A\cdot y2$	$B\cdot z1 - A\cdot z2$

$\overline{Q}$ = $C\cdot\overline{GP2}-B\cdot\overline{GP3}$	**Qx**	**Qy**	**Qz**
	$C\cdot x2 - B\cdot x3$	$C\cdot y2 - B\cdot y3$	$C\cdot z2 - B\cdot z3$

$\overline{V}$ = $\overline{P}\times\overline{Q}$	**x**	**y**	**z**
	$Py\cdot Qz - Pz\cdot Qy$	$Pz\cdot Qx - Px\cdot Qz$	$Px\cdot Qy - Py\cdot Qx$
$\overline{SUM}\cdot\overline{V}$	$x\cdot(x1+x2+x3)$	$y\cdot(y1+y2+y3)$	$z\cdot(z1+z2+z3)$

fix:	**latitude**	**longitude**

Three star fix
At least 6 decimal place precision is required.

	sin(Ho)
A =	
B =	
C =	

$$atan2(y,x) = \begin{cases} atan\left(\frac{y}{x}\right) & x>0 \\ atan\left(\frac{y}{x}\right)+\pi & x<0 \quad y\geq 0 \\ atan\left(\frac{y}{x}\right)-\pi & x<0 \quad y<0 \\ \pi/2 & x=0 \quad y>0 \\ -\pi/2 & x=0 \quad y<0 \\ \text{undefined} & x=0 \quad y=0 \end{cases}$$

$s = sign(\overline{SUM}\cdot\overline{V}) =$
$d = \sqrt{x^2+y^2} =$
$lat = atan2(s\cdot z, d)$
$lon = atan2(s\cdot y, s\cdot x)$

celestial body	latitude	longitude	altitude
	DEC	*- GHA*	*Ho*

	x_n	y_n	z_n
	cos(lat)·cos(lon)	*cos(lat)·sin(lon)*	*sin(lat)*
$\overline{\text{GP 1}}$			
$\overline{\text{GP 2}}$			

	Cx	**Cy**	**Cz**
$\overline{\text{cross}}$ = $\overline{\text{GP1}}$ × $\overline{\text{GP2}}$	*y1·z2 - y2·z1*	*z1·x2 - z2·x1*	*x1·y2 - x2·y1*

	CPx	**CPy**	**CPz**
$\overline{\text{center}}$ = k1·$\overline{\text{GP1}}$+k2·$\overline{\text{GP2}}$	*k1·x1 + k2·x2*	*k1·y1 + k2·y2*	*k1·z1 + k2·z2*

	vx	**vy**	**vz**
$\overline{\text{V1}}$ = $\overline{\text{center}}$ + sc·$\overline{\text{cross}}$	*CPx + sc·Cx*	*CPy + sc·Cy*	*CPz + sc·Cz*
$\overline{\text{V2}}$ = $\overline{\text{center}}$ - sc·$\overline{\text{cross}}$	*CPx - sc·Cx*	*CPy - sc·Cy*	*CPz - sc·Cz*

	latitude	**longitude**
fix:	*atan2(z, d)*	*atan2(y, x)*

Two star fix

Intersection points between two small circles.

Use the GP and altitude of the 1st and 2nd stars.

At least 6 decimal place precision is required.

A =	$\sin Ho1$	
B =	$\sin Ho2$	
cos α =	$x1\cdot x2 + y1\cdot y2 + z1\cdot z2$	
k1 =	$A - B\cdot\cos\alpha$	
k2 =	$B - A\cdot\cos\alpha$	
sc =	$\sqrt{1 - \cos^2\alpha - A\cdot k1 - B\cdot k2}$	

		intersection 1	intersection 2
d =	$\sqrt{vx^2 + vy^2}$		
lat =	*atan2(vz, d)*		
lon =	*atan2(vy, vx)*		

Pick the closest point to ded reckoning. The haversine distance can help if the decision is not obvious.

a =	$\sin((lat - \phi_0) / 2)$		
b =	$\sin((lon - \lambda_0) / 2)$		
c =	$a^2 + b^2\cdot\cos(lat)\cdot\cos(\phi_0)$		
R =	*6378.137 km*		
L =	$2\cdot R\cdot \mathrm{asin}(\sqrt{c})$		

celestial body	latitude	longitude	altitude
	DEC	*- GHA*	*Ho*

	x_n	y_n	z_n
	$\cos(lat)\cdot\cos(lon)$	$\cos(lat)\cdot\sin(lon)$	$\sin(lat)$
$\overline{\text{GP 1}}$			
$\overline{\text{GP 2}}$			

$\overline{\text{cross}} = \overline{\text{GP1}} \times \overline{\text{GP2}}$	Cx	Cy	Cz
	$y1\cdot z2 - y2\cdot z1$	$z1\cdot x2 - z2\cdot x1$	$x1\cdot y2 - x2\cdot y1$

$\overline{\text{center}} = k1\cdot\overline{\text{GP1}}+k2\cdot\overline{\text{GP2}}$	CPx	CPy	CPz
	$k1\cdot x1 + k2\cdot x2$	$k1\cdot y1 + k2\cdot y2$	$k1\cdot z1 + k2\cdot z2$

	vx	vy	vz
$\overline{\text{V1}} = \overline{\text{center}} + sc\cdot\overline{\text{cross}}$	$CPx + sc\cdot Cx$	$CPy + sc\cdot Cy$	$CPz + sc\cdot Cz$
$\overline{\text{V2}} = \overline{\text{center}} - sc\cdot\overline{\text{cross}}$	$CPx - sc\cdot Cx$	$CPy - sc\cdot Cy$	$CPz - sc\cdot Cz$

fix:	latitude	longitude
	$\operatorname{atan2}(z, d)$	$\operatorname{atan2}(y, x)$

Two star fix

Intersection points between two small circles.

Use the GP and altitude of the 2nd and 3rd stars.

At least 6 decimal place precision is required.

A =	$\sin Ho1$		
B =	$\sin Ho2$		
cos α =	$x1\cdot x2 + y1\cdot y2 + z1\cdot z2$		
k1 =	$A - B\cdot\cos\alpha$		
k2 =	$B - A\cdot\cos\alpha$		
sc =	$\sqrt{1 - \cos^2\alpha - A\cdot k1 - B\cdot k2}$		
		intersection 1	intersection 2
d =	$\sqrt{vx^2 + vy^2}$		
lat =	$\operatorname{atan2}(vz, d)$		
lon =	$\operatorname{atan2}(vy, vx)$		

Pick the closest point to ded reckoning. The haversine distance can help if the decision is not obvious.

a =	$\sin((lat - \phi_0) / 2)$		
b =	$\sin((lon - \lambda_0) / 2)$		
c =	$a^2 + b^2\cdot\cos(lat)\cdot\cos(\phi_0)$		
R =	6378.137 km		
L =	$2\cdot R\cdot\operatorname{asin}(\sqrt{c})$		

celestial body	latitude	longitude	altitude
	DEC	*- GHA*	*Ho*

	x_n	y_n	z_n
	cos(lat)·cos(lon)	*cos(lat)·sin(lon)*	*sin(lat)*
$\overline{\text{GP}}$ **1**			
$\overline{\text{GP}}$ **2**			

$\overline{\text{cross}}$ = $\overline{\text{GP1}} \times \overline{\text{GP2}}$	**Cx**	**Cy**	**Cz**
	y1·z2 - y2·z1	*z1·x2 - z2·x1*	*x1·y2 - x2·y1*

$\overline{\text{center}}$ = k1·$\overline{\text{GP1}}$+k2·$\overline{\text{GP2}}$	**CPx**	**CPy**	**CPz**
	k1·x1 + k2·x2	*k1·y1 + k2·y2*	*k1·z1 + k2·z2*

	vx	**vy**	**vz**
$\overline{\text{V1}}$ = $\overline{\text{center}}$ + sc·$\overline{\text{cross}}$	*CPx + sc·Cx*	*CPy + sc·Cy*	*CPz + sc·Cz*
$\overline{\text{V2}}$ = $\overline{\text{center}}$ - sc·$\overline{\text{cross}}$	*CPx - sc·Cx*	*CPy - sc·Cy*	*CPz - sc·Cz*

fix:	latitude	longitude
	atan2(z, d)	*atan2(y, x)*

Two star fix

Intersection points between two small circles.

Use the GP and altitude of the 3rd and 1st stars.

At least 6 decimal place precision is required.

A =	$\sin Ho1$	
B =	$\sin Ho2$	
cos α =	$x1 \cdot x2 + y1 \cdot y2 + z1 \cdot z2$	
k1 =	$A - B \cdot \cos\alpha$	
k2 =	$B - A \cdot \cos\alpha$	
sc =	$\sqrt{1 - \cos^2\alpha - A \cdot k1 - B \cdot k2}$	

		intersection 1	intersection 2
d =	$\sqrt{vx^2 + vy^2}$		
lat =	*atan2(vz, d)*		
lon =	*atan2(vy, vx)*		

Pick the closest point to ded reckoning. The haversine distance can help if the decision is not obvious.

a =	$\sin((lat - \phi_0) / 2)$		
b =	$\sin((lon - \lambda_0) / 2)$		
c =	$a^2 + b^2 \cdot \cos(lat) \cdot \cos(\phi_0)$		
R =	*6378.137 km*		
L =	$2 \cdot R \cdot \operatorname{asin}(\sqrt{c})$		