

# UAL/ETEAPOT Results (Augmented) for Proton EDM Benchmark Lattices

J. D. Talman and R. M. Talman

October 29, 2012

## **Abstract**

The three benchmark lattices for an all-electric proton EDM ring are investigated using UAL/ETEAPOT. They have field indices  $m = -1$ ,  $m \approx 0$ , and  $m = 1$ . Values of horizontal and vertical tunes, and beta functions at the origin (midway between adjacent bends) are tabulated. Results are compared to values obtained from a linearized transfer matrix formalism. In all cases the design vertical tunes have been adjusted to  $Q_y=0.2$ .

# 1 Introduction

All-electric lattices for a storage ring to measure the electric dipole moment (EDM) of the proton are described in the BNL proposal by the Storage Ring EDM Collaboration, *A Proposal to Measure the Proton Electric Dipole Moment with  $10^{-29}$  e-cm Sensitivity*[1]. The present report provides lattices stripped down to their bare essentials in order to simplify benchmark comparisons of various simulation codes.

An electric field with index  $m$  power law dependence on radius  $r$  for  $y=0$  is

$$\mathbf{E}(r, 0) = -E_0 \frac{r_0^{1+m}}{r^{1+m}} \hat{\mathbf{r}}, \quad (1)$$

and the electric potential  $V(r)$ , adjusted to vanish at  $r = r_0$ , is

$$V(r) = -\frac{E_0 r_0}{m} \left( \frac{r_0^m}{r^m} - 1 \right). \quad (2)$$

The “cleanest” case theoretically has  $m=1$ , in which case it is known as the Kepler or the Coulomb electric field. Our problem is a bit more general than this terminology suggests, since our treatment is necessarily relativistic. For  $m=1$  the Kepler problem can be solved with the same generality in the relativistic as in the nonrelativistic case; the orbits are no longer exactly elliptical, nor exactly closed however[3].

The full ring is circular and consists of 40 identical cells. A single cell is shown in Figure 1. With curved planar plates, which is the easiest configuration to build, the field boundaries are concentric cylinders and the field index is  $m=0$ . The optimal (i.e. longest) spin coherence time (SCT) is expected to be close to this value. For  $m=1$  the field boundaries are concentric spheres.

For  $m=0$  the fields are independent of vertical displacement  $y$  so there is no vertical focusing. Vertical stability requires focusing quadrupoles, labeled  $q$  in Figure 1. Even if not required for vertical stability the  $q$  quadrupoles are necessary for adjusting the vertical tune  $Q_y$  of the storage ring. The vertical tunes have been adjusted to  $Q_y = 0.2$  for all three of the benchmark lattices described in the next section.

# 2 Parameters of Benchmark Lattices

Analytic, linearized transfer matrix MAPLE programs with names `E_BM_P1.0.mw`, `E_BM_Z.mw`, and `E_BM_M1.0.mw` are in `/home/talman/EDM/EDM-lattice/maple/benchmark`. The corresponding SXF lattice files are produced in `/home/oxygen/XML2ADXF` and `/home/oxygen/ADXF2SXF/`. The filenames are column headings in the following table.

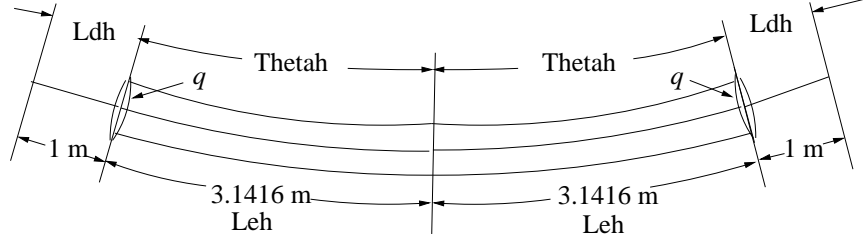


Figure 1: Single cell geometry. The full lattice consists of 40 identical cells.

Table 1: Parameters of benchmark all-electric EDM lattices, and Twiss parameters calculated from linearized transfer matrices. Note that the "half bend", and "half drift", elements are treated as individual elements in the sxf file. This gives 80 bends. The software treats these as 160 after slicing.

file name	variable name	unit	E_BM_M1.0.sxf	E_BM_Z.sxf	E_BM_P1.0.sxf
cells/arc	NCellPerArc		20	20	20
bend radius	r0	m	40.0	40.0	40.0
half drift length	Ldh	m	1.0	1.0	1.0
half bend per cell	Thetah	r	0.078539816	0.078539816	0.078539816
half bend length	Leh	m	3.141592	3.141592	3.141592
circumference	circum	m	331.327	331.327	331.327
inverse focal length	q	1/m	-0.002019	-0.00005960	0.0019075
field index	m		-1.0	1.0e-10	1.0
horizontal beta	betax	m	35.9237	36.1018	36.1910
vertical beta	betay	m	264.182	263.620	262.237
horizontal tune	Qx		1.4605	1.4578	1.4588
vertical tune	Qy		0.20024	0.20004	0.20047

### 3 Twiss Parameter Extraction

A once around transfer matrix at any point in the ring has the standard Twiss parameterization,

$$\mathbf{M} = \begin{pmatrix} \cos \mu + \alpha \sin \mu & \beta \sin \mu \\ -\frac{1+\alpha^2}{\beta} \sin \mu & \cos \mu - \alpha \sin \mu \end{pmatrix}. \quad (3)$$

At a mirror symmetry point in the lattice,  $\alpha$  vanishes, though not in general. The phase advance  $\mu = 2\pi Q$  can be obtained from

$$\cos \mu = \frac{M_{11} + M_{22}}{2}, \quad \text{and} \quad M_{12} = \beta \sin \mu. \quad (4)$$

$\beta$  is, by definition, positive. These equations determine the signs of both  $\cos \mu$  and  $\sin \mu$ . The four phase space quadrants, I/II/III/IV, are characterized by

cosine, sine sign pairs  $++/-+/- -/+--$ . We then obtain the fractional parts of the phase advance from

$$\mu = \begin{cases} \cos^{-1}(\frac{M_{11}+M_{22}}{2}), & \text{if quadrant is I or II} \\ 2\pi - \cos^{-1}(\frac{M_{11}+M_{22}}{2}), & \text{if quadrant is III or IV} \end{cases} \quad (5)$$

This still only partially resolves the aliasing ambiguity. Resolving the integer tune ambiguity requires keeping track of phase space quadrants while tracking through the lattice. This is described in captions to graphs below. We then obtain  $\beta$  from Eq. (4), and  $\alpha$  from

$$\alpha = \frac{M_{11} - M_{22}}{2 \sin \mu}. \quad (6)$$

## 4 UAL/ETEAPOT Comparisons

### 4.1 Dependence on Numerical Differentiation Parameter `tiny`

For each of the three benchmark lattices, bunches of 21 standard particles are tracked for one turn. Each of the 80 bend elements is sliced in half for this analysis. The graphs show horizontal and vertical positions for two of the standard particles at the exits from each of the bends. The approximate tunes are calculated in the figure captions.

Once-around transfer matrices are obtained from the 21 standard particles using numerical differences. This differencing is based on the smallness (typically 1 micron spatial displacement, 1 microradian angular displacement) of the initial offsets. Sensitivity to these choices has not yet been investigated. Twiss parameters,  $\beta_x, \beta_y$ , and  $\mu_x, \mu_y$  are calculated using the procedure described above. The integer tunes (which this procedure does not provide) are obtained from the graphs. Values are given in the tables following the graphs.

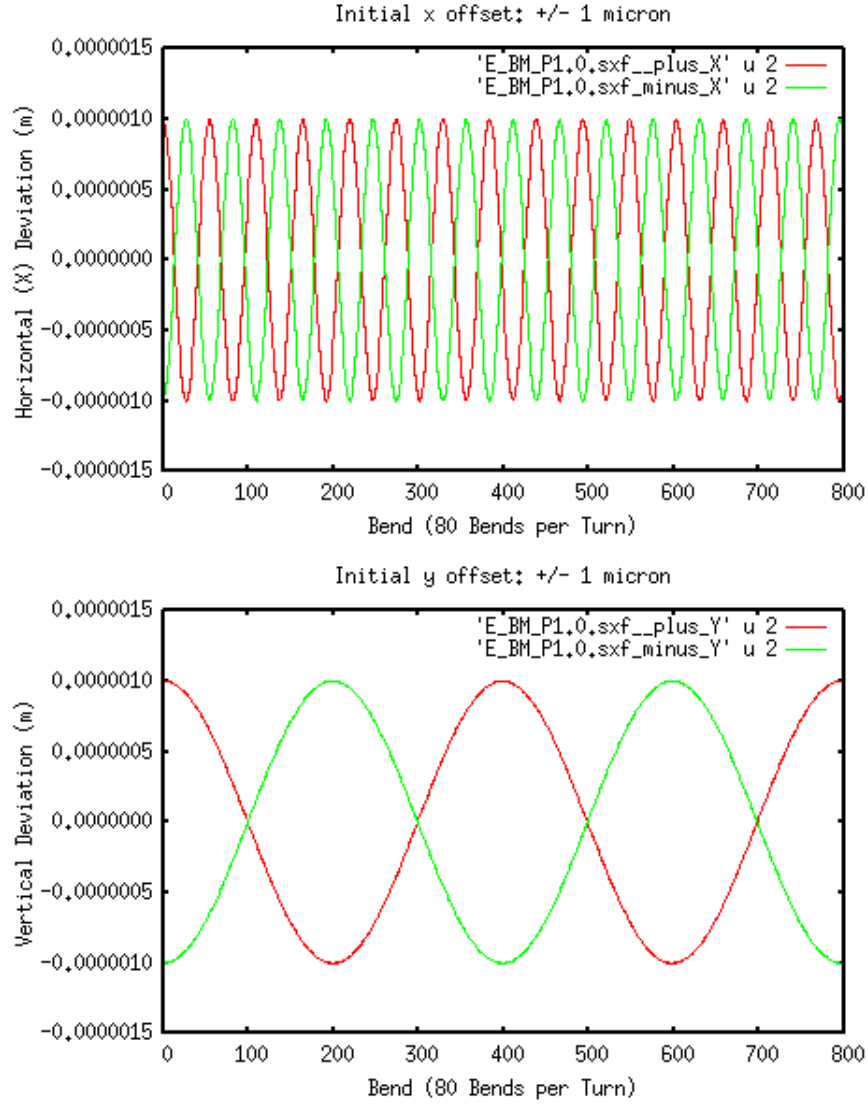


Figure 2: E\_BM\_P1.0.sxf: Horizontal Displacement: 10 turns, 1 sample per bend.  $Q_x \approx 14.5 \text{ osc}/800 \text{ bends} \cdot 80 \text{ bends/turn} = 1.45 \text{ osc/turn}$ . The integer part of the tune is  $[Q_x] = 1$ . Vertical Displacement: 10 turns, 1 sample per bend.  $Q_y \approx 2.0 \text{ osc}/800 \text{ bends} \cdot 80 \text{ bends/turn} = 0.20 \text{ osc/turn}$ . The integer part of the tune is  $[Q_y] = 0$ .

Table 2: E-TEAPOT comparisons for `./tracker ./data/E_BM_P1.0.sxf 30 1`  
`> & ! JTout`. All `typ` numerical differentiation displacements are proportional  
to `tiny`.

file name	variable name	unit	linearized	2 slices/bend	$\Delta$ 's
cells/arc	NCellPerArc		20		
bend radius	r0	m	40.0		
half drift length	Ldh	m	1.0		
half bend per cell	Thetah	r	0.078539816		
half bend length	Leh	m	3.141592		
circumference	circum	m	331.327		
inverse focal length	q	1/m	0.0019075		
field index	m		1.0		
<b>tiny=1.0e-6</b>					
horizontal beta	betax	m	36.1910	36.1910	0.0000
vertical beta	betay	m	262.2370	262.2370	0.0000
horizontal tune	Qx		1.4588	1.4588	0.0000
vertical tune	Qy		0.2005	0.2005	0.0000
<b>tiny=1.0e-5</b>					
horizontal beta	betax	m	36.1910	36.1910	0.0000
vertical beta	betay	m	262.2370	262.2370	0.0000
horizontal tune	Qx		1.4588	1.4588	0.0000
vertical tune	Qy		0.2005	0.2005	0.0000
<b>tiny=1.0e-4</b>					
horizontal beta	betax	m	36.1910	36.1925	0.0015
vertical beta	betay	m	262.2370	262.2336	-0.0034
horizontal tune	Qx		1.4588	1.4588	0.0000
vertical tune	Qy		0.2005	0.2005	0.0000
<b>tiny=1.0e-3</b>					
horizontal beta	betax	m	36.1910	36.3349	0.1439
vertical beta	betay	m	262.2370	261.8966	-0.3404
horizontal tune	Qx		1.4588	1.4589	0.0001
vertical tune	Qy		0.2005	0.2006	0.0001

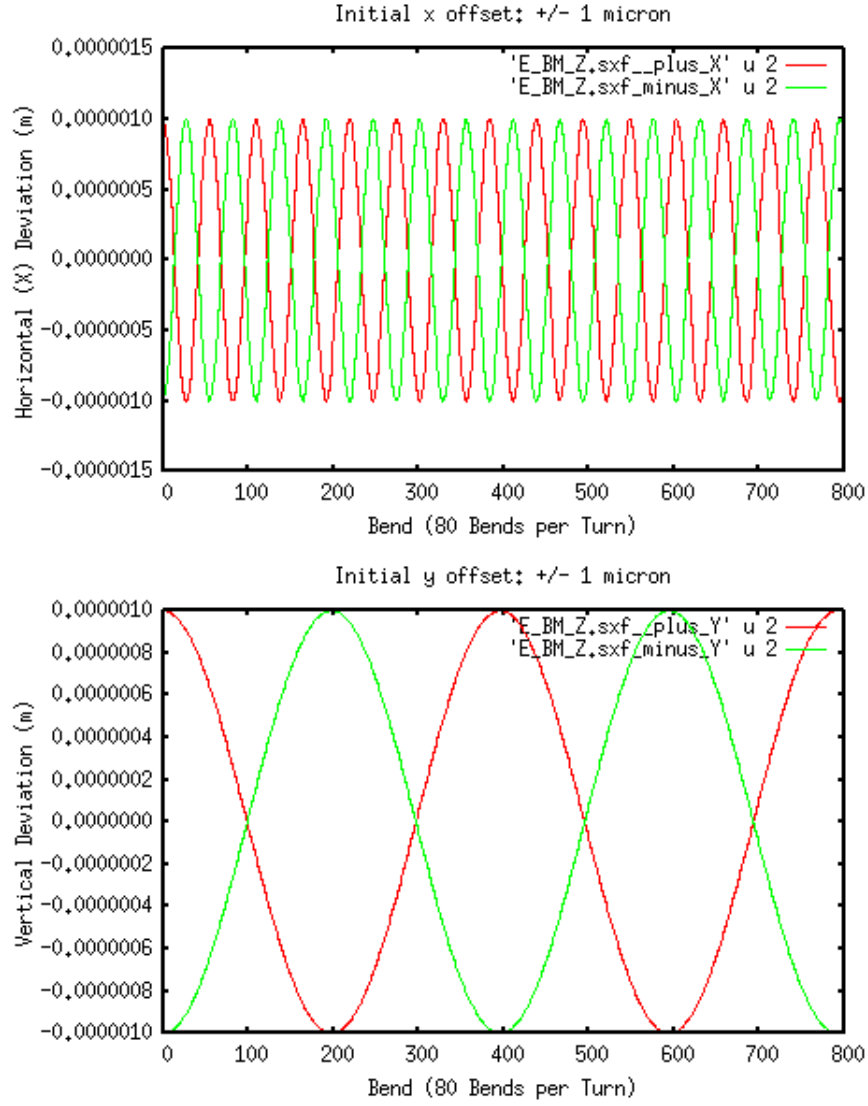


Figure 3: E\_BM\_Z.sxf: Horizontal Displacement: 10 turns, 1 sample per bend.  $Q_x \approx 14.5 \text{ osc}/800 \text{ bends} \cdot 80 \text{ bends/turn} = 1.45 \text{ osc/turn}$ . The integer part of the tune is  $[Q_x] = 1$ . Vertical Displacement: 10 turns, 1 sample per bend.  $Q_y \approx 2.0 \text{ osc}/800 \text{ bends} \cdot 80 \text{ bends/turn} = 0.20 \text{ osc/turn}$ . The integer part of the tune is  $[Q_y] = 0$ .

Table 3: E-TEAPOTComparisons for `./tracker ./data/E_BM_Z.sxf 30 0 >`  
`& ! JTout`. All `typ` numerical differentiation displacements are proportional  
to `tiny`.

file name	variable name	unit	linearized	2 slices/bend	$\Delta$ 's
cells/arc	NCellPerArc		20		
bend radius	r0	m	40.0		
half drift length	Ldh	m	1.0		
half bend per cell	Thetah	r	0.078539816		
half bend length	Leh	m	3.141592		
circumference	circum	m	331.327		
inverse focal length	q	1/m	-0.00005960		
field index	m		1.0e-10		
<b>tiny=1.0e-6</b>					
horizontal beta	betax	m	36.1018	36.0795	-0.0223
vertical beta	betay	m	263.6200	261.4688	-2.1512
horizontal tune	Qx		1.4578	1.4581	0.0003
vertical tune	Qy		0.2000	0.2018	0.0018
<b>tiny=1.0e-5</b>					
horizontal beta	betax	m	36.1018	36.0795	-0.0223
vertical beta	betay	m	263.6200	261.4688	-2.1512
horizontal tune	Qx		1.4578	1.4581	0.0003
vertical tune	Qy		0.2000	0.2018	0.0018
<b>tiny=1.0e-4</b>					
horizontal beta	betax	m	36.1018	36.0809	-0.0209
vertical beta	betay	m	263.6200	261.4654	-2.1546
horizontal tune	Qx		1.4578	1.4581	0.0003
vertical tune	Qy		0.2000	0.2018	0.0018
<b>tiny=1.0e-3</b>					
horizontal beta	betax	m	36.1018	36.2190	0.1172
vertical beta	betay	m	263.6200	261.1318	-2.4882
horizontal tune	Qx		1.4578	1.4582	0.0004
vertical tune	Qy		0.2000	0.2019	0.0019



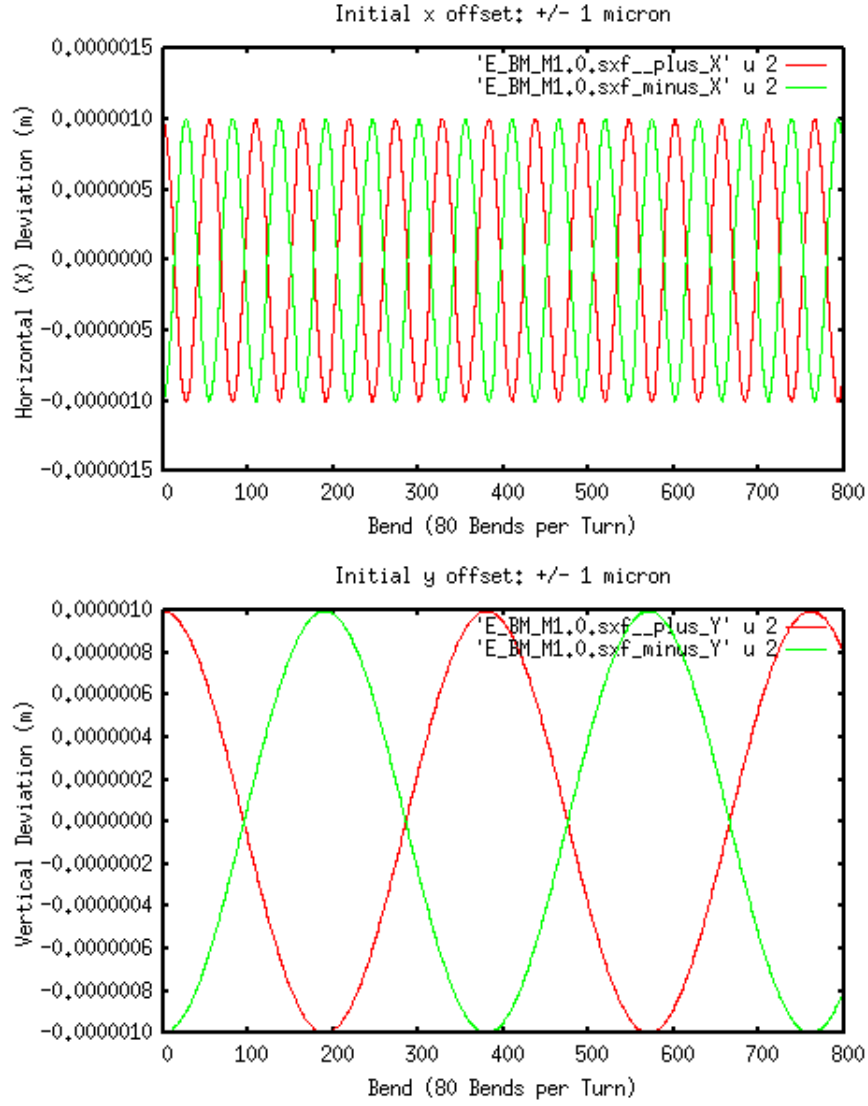


Figure 4: E\_BM\_M1.0.sxf: Horizontal Displacement: 10 turns, 1 sample per bend.  $Q_x \approx 14.5 \text{ osc}/800 \text{ bends} \cdot 80 \text{ bends/turn} = 1.45 \text{ osc/turn}$ . The integer part of the tune is  $[Q_x] = 1$ . Vertical Displacement: 10 turns, 1 sample per bend.  $Q_y \approx 2.1 \text{ osc}/800 \text{ bends} \cdot 80 \text{ bends/turn} = 0.21 \text{ osc/turn}$ . The integer part of the tune is  $[Q_y] = 0$ .

Table 4: E-TEAPOT comparisons for `./tracker ./data/E_BM_M1.0.sxf 30 -1 > & ! JTout`. All `typ` numerical differentiation displacements are proportional to `tiny`.

file name	variable name	unit	linearized	2 slices/bend	$\Delta$ 's
cells/arc	NCellPerArc		20		
bend radius	r0	m	40.0		
half drift length	Ldh	m	1.0		
half bend per cell	Thetah	r	0.078539816		
half bend length	Leh	m	3.141592		
circumference	circum	m	331.327		
inverse focal length	q	1/m	-0.002019		
field index	m		-1.0		
<b>tiny=1.0e-6</b>					
horizontal beta	betax	m	35.9237	35.8566	-0.0671
vertical beta	betay	m	264.1820	251.8522	-12.3298
horizontal tune	Qx		1.4605	1.4620	0.0015
vertical tune	Qy		0.2002	0.2102	0.0100
<b>tiny=1.0e-5</b>					
horizontal beta	betax	m	35.9237	35.8566	-0.0671
vertical beta	betay	m	264.1820	251.8522	-12.3298
horizontal tune	Qx		1.4605	1.4620	0.0015
vertical tune	Qy		0.2002	0.2102	0.0100
<b>tiny=1.0e-4</b>					
horizontal beta	betax	m	35.9237	35.8582	-0.0655
vertical beta	betay	m	264.1820	251.8492	-12.3328
horizontal tune	Qx		1.4605	1.4620	0.0015
vertical tune	Qy		0.2002	0.2102	0.0100
<b>tiny=1.0e-3</b>					
horizontal beta	betax	m	35.9237	36.0209	0.0972
vertical beta	betay	m	264.1820	251.5553	-12.6267
horizontal tune	Qx		1.4605	1.4621	0.0016
vertical tune	Qy		0.2002	0.2103	0.0101

## 4.2 Dependence on Slicing Granularity

Table 5: E\_TEAPOT comparisons for `./tracker ./data/E_BM_P1.0.sxf 30 1 > & ! JTout. tiny=1.0e-6.`

file name	variable name	unit	linearized	2 sl./bend	$\Delta$ 's	4 sl./bend	$\Delta$ 's
cells/arc	NCellPerArc		20				
bend radius	r0	m	40.0				
half drift length	Ldh	m	1.0				
half bend per cell	Thetah	r	0.078539816				
half bend length	Leh	m	3.141592				
circumference	circum	m	331.327				
inverse focal length	q	1/m	0.0019075				
field index	m		1.0				
horizontal beta	betax	m	36.191030	36.19103		36.19103	
vertical beta	betay	m	262.236988	262.23701		262.23701	
horizontal tune	Qx		1.458782	1.45878		1.45878	
vertical tune	Qy		0.200472	0.20047		0.20047	

Table 6: E\_TEAPOT comparisons for `./tracker ./data/E_BM_Z.sxf 30 0 > & ! JTout. tiny=1.0e-6.`

file name	variable name	unit	linearized	2 sl./bend	$\Delta$ 's	4 sl./bend	$\Delta$ 's
cells/arc	NCellPerArc		20				
bend radius	r0	m	40.0				
half drift length	Ldh	m	1.0				
half bend per cell	Thetah	r	0.078539816				
half bend length	Leh	m	3.141592				
circumference	circum	m	331.327				
inverse focal length	q	1/m	-0.00005960				
field index	m		1.0e-10				
horizontal beta	betax	m	36.101819	36.07949		36.09624	
vertical beta	betay	m	263.620065	261.46880		263.07673	
horizontal tune	Qx		1.457801	1.45813		1.45788	
vertical tune	Qy		0.200044	0.20177		0.20048	

Table 7: E<sub>TEAPOT</sub> comparisons for ./tracker ./data/E<sub>BM</sub>M1.0.sxf 30  
-1 > & ! JTout. tiny=1.0e-6.

file name	variable name	unit	linearized	2 sl./bend	$\Delta$ 's	4 sl./bend	$\Delta$ 's
cells/arc	NCellPerArc		20				
bend radius	r0	m	40.0				
half drift length	Ldh	m	1.0				
half bend per cell	Thetah	r	0.078539816				
half bend length	Leh	m	3.141592				
circumference	circum	m	331.327				
inverse focal length	q	1/m	-0.002019				
field index	m		-1.0				
horizontal beta	betax	m	35.923719	35.85659		35.90695	
vertical beta	betay	m	264.182337	251.85222		260.92409	
horizontal tune	Qx		1.460454	1.46204		1.46085	
vertical tune	Qy		0.200242	0.21021		0.20278	

## 5 Comments and Conclusions

For the  $m = 1$  case, there is exact agreement between the linearized analytic formulae and the tracking formalism. This is "as should be", but is far from automatic. The equations of the linearized transfer matrix formalism and the arbitrary-amplitude Munoz-Pavic seem utterly different. So the exact agreement confirms *both* approaches.

The agreement for  $m \neq 1$  is excellent, but clearly not as good as  $m = 1$ . This strongly suggests that the kick compensation is not quite perfect. It is also not surprising that the deviation is proportional to  $(m - 1)^2$ ; for  $m = 1$ , the kick correction strength vanishes.

For most practical purposes, the code performance in the range  $-1 \leq m \leq 1$  has been shown to be satisfactory. Nevertheless, minor further study seems to be justified. Two investigations seem natural. One is to vary the  $(x_{\text{typ}}, x'_{\text{typ}}, \dots)$  parameters bigger and smaller by factors of, say, 10, to investigate this sensitivity. The other is to study the same 3 benchmark lattices with 4 slices per bend. (The code does not support 1 slice per bend).

## References

- [1] Storage Ring EDM Collaboration, *A Proposal to Measure the Proton Electric Dipole Moment with  $10^{-29}$  e-cm Sensitivity*, especially Appendix 1. October, 2011
- [2] C. Møller, *The Theory of Relativity*, Clarendon Press, Oxford, 1952,
- [3] G. Muñoz and I. Pavic, *A Hamilton-like vector for the special-relativistic Coulomb problem*, Eur. J. Phys. **27**, 1007-1018, 2006
- [4] R. Talman, *Geometric Mechanics*, John Wiley and Sons, 2000
- [5] J. Aguirregabiria et al., Archiv:physics/0407049v1 [physics.ed-ph] 2004,
- [6] U. Torkelsson, Eur. J. Phys., **19**, 459, 1998,
- [7] T. Boyer, Am. J. Phys. **72** (8) 992, 2004