Reconstruction: Rational Approximation of the Complex Error Function and the Electric Field of a Two-Dimensional Gaussian Charge Distribution

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Abstract

Rationale is provided for the resurection and archiving of an unpublished original Cornell Labortory of Nuclear Studies report by Yuko Okamoto and Richard Talman, "Rational Approximation of the Complex Error Function and the Electric Field of a Two-Dimensional Gaussian Charge Distribution" CBN 80-13, dating from September 1980.

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Rational Approximation of the Complex Error Function and the Electric Field of a Two-Dimensional Gaussian Charge Distribution

by

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ABSTRACT

To simulate the beam-beam interaction one needs efficient formulae for the evaluation of the electric field of a two-dimensional Gaussian charge distribution which can be expressed in terms of the complex error function w(z). This paper shows how to approximate w(z) by a set of rational functions. The percent error of the approximation is extremely small ($\sim 10^{-4}$% except near the real axis). Computer programs to evaluate w(z) and the electric field are also provided.

1. Introduction

For the simulation of the beam-beam interaction one needs to evaluate the electric field of a two-dimensional Gaussian charge distribution. The electric field at the position (x,y) has been found by M. Bassetti and G.A. Erskine[1] to have the following form:[2]

$$E_x = \frac{Q}{2\varepsilon_0\sqrt{2\pi(s_x^2-s_y^2)}}\,\mathrm{Im}\left[w\left(\frac{x+iy}{\sqrt{2(s_x^2-s_y^2)}}\right) - e^{-\left(\frac{x^2}{2s_x^2}+\frac{y^2}{2s_y^2}\right)}\,w\left(\frac{x\frac{s_y}{s_x}+iy\frac{s_x}{s_y}}{\sqrt{2(s_x^2-s_y^2)}}\right)\right], \quad (1.1)$$

$$E_y = \frac{Q}{2\varepsilon_0\sqrt{2\pi(s_x^2-s_y^2)}}\,\mathrm{Re}\left[w\left(\frac{x+iy}{\sqrt{2(s_x^2-s_y^2)}}\right) - e^{-\left(\frac{x^2}{2s_x^2}+\frac{y^2}{2s_y^2}\right)}\,w\left(\frac{x\frac{s_y}{s_x}+iy\frac{s_x}{s_y}}{\sqrt{2(s_x^2-s_y^2)}}\right)\right], \quad (1.2)$$

where Q is a constant with a dimension of electric charge, ε_0 is the electric permittivity of free space, s_x and s_y ($s_x > s_y$ assumed) are the standard deviations of the charge distribution in the x and y directions, respectively, and w(z) is the complex error function[3] defined by

$$w(z) = e^{-z^2}\left[1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{u^2}\,du\right] \quad (1.3)$$

We shall approximate w(z) by rational functions so that a computer can <u>quickly</u> handle the evaluation of w(z) and thus the electric field of a two-dimensional Gaussian charge distribution. Though we were originally interested in an approximation good within 1 % percent error, the result turned out to be a much better approximation. We note that after the approximation of w(z) the only transcendental function in (1.1) and (1.2) which spend a longer computing time than rational functions are the exponential factors. We also note that by the symmetry properties of w(z)[3] it suffices to approximate w(z) only in the first quadrant of the complex plane.

2. Padé Approximation

We shall briefly discuss how the Pade approximation is done first, then apply the approximation to the function w(z).

Suppose we have a complex-valued function f(z) which is analytic at a point z_0, and suppose we want to approximate it around z_0 by a rational function of the form

$$f_{Pade}(z) = \frac{\sum_{k=0}^{M} c_k(z-z_0)^k}{1 + \sum_{k=1}^{N} d_k(z-z_0)^k} \tag{2.1}$$

where b_k, $c_k \in \mathbb{C}$ are unknown (possibly complex) coefficients to be determined. Note: We must have $d_0 \neq 0$ because f(z) is well-behaved at z_0. We may set $d_0 = 1$. For, otherwise, we can always divide both the numerator and the denominator by d_0.

Here we choose M and N according to how much accuracy we need. In order to determine the coefficients c_k and d_k we impose a condition on f_{Pade}:

$$f - f_{Pade} = A_1(z-z_0)^{M+N+1} + A_2(z-z_0)^{M+N+2} + \cdots\cdots \tag{2.2}$$

where $A_1, A_2, \cdots \in \mathbb{C}$ are some constants. That is, the error introduced by the approximation at z with $|z-z_0|<1$ is of the order of $|z-z_0|^{M+N+1}$ and very small if M and N are large. Since f is analytic at z_0, we have a Taylor series at z_0:

$$f(z) = \sum_{j=0}^{\infty} a_j(z-z_0)^j \quad ; \quad a_j \in \mathbb{C} \tag{2.3}$$

Then using (2.3) for f in (2.2), multiplying both sides of (2.2) by the denominator of f_{Pade}, and equating the coefficients of the powers of $(z-z_0)$ in both sides of the equation, we have the following relationships among a_k, c_k, and d_k:

Powers	Relation among Coefficients
$(z-z_0)^0$	$c_0 = a_0$
$(z-z_0)^1$	$c_1 - a_0 d_1 = a_1$
$(z-z_0)^2$	$c_2 - a_1 d_1 - a_0 d_2 = a_2$
$(z-z_0)^3$	$c_3 - a_2 d_1 - a_1 d_2 - a_0 d_3 = a_3$
....	....
$(z-z_0)^k$	$c_k - a_{k-1} d_1 - a_{k-2} d_2 - \cdots - a_0 d_k = a_k$

(2.4)

where $c_k=0$ for $k > M$ and $d_k=0$ for $k > N$.

In a matrix language we have

$$\begin{pmatrix}
1 & & & & -a_0 & & & \\
& 1 & & 0 & -a_1 & -a_0 & & 0 \\
& & 1 & & -a_2 & -a_1 & -a_0 & \\
& & & \ddots & \vdots & \vdots & & \ddots \\
& & & 1 & -a_{M-1} & -a_{M-2} & \cdots & -a_{M-N} \\
& & & & -a_M & -a_{M-1} & \cdots & -a_{M-N+1} \\
& 0 & & & -a_{M+1} & -a_M & \cdots & -a_{M-N+2} \\
& & & & \vdots & \vdots & & \vdots \\
& & & & -a_{M+N-1} & -a_{M+N-2} & \cdots & -a_M
\end{pmatrix}
\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_M \\ d_1 \\ d_2 \\ \vdots \\ d_N \end{pmatrix}
=
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_M \\ a_{M+1} \\ a_{M+2} \\ \vdots \\ a_{M+N} \end{pmatrix} \tag{2.5}$$

where $a_k=0$ for $k < 0$. By inverting this matrix, we can determine the coefficients c_j and d_k ($j=1,\cdots,M$ and $k=1,\cdots,N$).

Note: The inversion of this kind of matrices is easily done by computer. (Cf. IBM 360 Scientific Subroutine Package (SSP))

(PADE 1)

The Taylor series of w(z) around the origin is [3)]

$$w(z) = \sum_{j=0}^{\infty} a_j z^j = \sum_{j=0}^{\infty} \frac{(iz)^j}{\Gamma(j/2 + 1)} \tag{2.6}$$

Let

$$u = iz = -ZI + iZR \,, \quad \text{where } z = ZR + iZI \tag{2.7}$$

Then

$$w(z) \overset{\text{Def.}}{=} G(u) = \sum_{j=0}^{\infty} \frac{u^j}{\Gamma(j/2 + 1)} \tag{2.8}$$

We shall apply a Pade approximation to G(u). Considering the behavior of w(z)

$$w(z) \to 0 \quad \text{as} \quad |z| \to \infty$$

for those z such that $|ZR| > |ZI|$, we take

$$M = 6 \quad \text{and} \quad N = 7$$

By inverting the matrix (2.5), we obtain, up to nine significant figures,

$$\begin{array}{ll} c_0 = 1 \quad (\text{Cf. } (2.4)\ ) & d_1 = -2.38485635 \\ c_1 = -1.25647718 & d_2 = 2.51608137 \\ c_2 = 8.25059158 \times 10^{-1} & d_3 = -1.52579040 \\ c_3 = -3.19300157 \times 10^{-1} & d_4 = 5.75922693 \times 10^{-1} \\ c_4 = 7.63191605 \times 10^{-2} & d_5 = -1.35740709 \times 10^{-1} \\ c_5 = -1.04697938 \times 10^{-2} & d_6 = 1.85678083 \times 10^{-2} \\ c_6 = 6.44878652 \times 10^{-4} & d_7 = -1.14243694 \times 10^{-3} \end{array} \tag{2.9}$$

Hence, the approximation of w(z) near the origin is, by (2.1),

$$w(z) = G(u) \simeq \frac{1 + c_1 u + c_2 u^2 + c_3 u^3 + c_4 u^4 + c_5 u^5 + c_6 u^6}{1 + d_1 u + d_2 u^2 + d_3 u^3 + d_4 u^4 + d_5 u^5 + d_6 u^6 + d_7 u^7} \tag{2.10}$$

where $u = -ZI + iZR$, and the coefficients c_k and d_k are given by (2.9).

(PADE 2)

Since the approximation PADE 1 behaves rather poorly along the real axis right around z = 3 (Cf. Table 2), we need a Pade approximation around z = 3. The Taylor series of w(z) at z = 3 is

$$w(z) = \sum_{j=0}^{\infty} a_j (z-3)^j \tag{2.11}$$

where

$$a_j = \frac{w^{(j)}(3)}{j!} \tag{2.12}$$

The derivatives $w^{(j)}(3)$ can be expressed in terms of w(3) by use of the relation[3]

$$\begin{aligned} &w^{(j+2)}(z) + 2zw^{(j+1)}(z) + 2(j+1)w^{(j)}(z) = 0 \ , \quad (j=0,1,2,\cdots) \\ &w^{(0)}(z) = w(z), \ w'(z) = -2zw(z) + \frac{2i}{\sqrt{\pi}} \end{aligned} \tag{2.13}$$

On the other hand, the value of w(3) is, by (1.3),

$$w(3) = e^{-9} + \frac{2i}{\sqrt{\pi}} e^{-9} \int_0^3 e^{u^2} du \tag{2.14}$$

By using Table 2 in Rosser[4] for the value of the second term, we have w(3) up to nine significant figures:

$$w(3) = 1.23409804 \times 10^{-4} + i2.01157318 \times 10^{-1} \tag{2.15}$$

This time we choose

$$M = 3 \quad \text{and} \quad N = 4$$

By inverting the matrix (2.5), we obtain, up to nine significant figures,

$$
\begin{aligned}
c_0 &= 1.23409804 \times 10^{-4} + i2.01157318 \times 10^{-1} \quad \text{(Cf. (2.4))} \\
c_1 &= 2.33746715 \times 10^{-1} + i1.61133338 \times 10^{-1} \\
c_2 &= 1.25689814 \times 10^{-1} - i4.04227250 \times 10^{-2} \\
c_3 &= 8.92089179 \times 10^{-3} - i1.81293213 \times 10^{-2} \\
d_1 &= 1.19230984 - i1.16495901 \\
d_2 &= 8.94015450 \times 10^{-2} - i1.07372867 \\
d_3 &= -1.68547429 \times 10^{-1} - i2.70096451 \times 10^{-1} \\
d_4 &= -3.20997564 \times 10^{-2} - i1.58578639 \times 10^{-2}
\end{aligned}
\tag{2.16}
$$

Hence, the approximation of $w(z)$ near $z = 3$ is, by (2.1),

$$
w(z) \simeq \frac{c_0 + c_1 z + c_2 z^2 + c_3 z^3}{1 + d_1 z + d_2 z^2 + d_3 z^3 + d_4 z^4} \tag{2.17}
$$

where the coefficients c_k and d_k are given by (2.16).

3. Asymptotic Expression

Away from the origin and z = 3 we can use the asymptotic expression of w(z) given in Faddeyeva and Terent'ev (Eqn. (10))[5]. The formula is

$$w(z) \sim \sum_{k=1}^{n} \frac{i\lambda_k^{(n)}}{\pi(z-x_k^{(n)})}$$

$$= \sum_{k=1}^{n} \frac{ia_k^{(n)}}{z - x_k^{(n)}} \quad , \quad a_k^{(n)} = \frac{\lambda_k^{(n)}}{\pi} \tag{3.1}$$

where $x_k^{(n)}$ are the roots of Hermite polynomials and $\lambda_k^{(n)}$ are the corresponding coefficients (and n is an integer related to the accuracy of the approximation). The values of $x_k^{(n)}$ and $\lambda_k^{(n)}$ can be found in Greenwood and Miller[6]. By choosing n = 10, we have an asymptotic expression of w(z) as

$$w(z) \simeq \frac{ia_1}{z-x_1} + \frac{ia_1}{z+x_1} + \frac{ia_2}{z-x_2} + \frac{ia_2}{z+x_2} + \frac{ia_3}{z-x_3} + \frac{ia_3}{z+x_3} + \frac{ia_4}{z-x_4} + \frac{ia_4}{z+x_4} +$$

$$\frac{ia_5}{z-x_5} + \frac{ia_5}{z+x_5} \tag{3.2}$$

where the constants are, up to nine or ten significant figures,

$$\begin{aligned} a_1 &= 1.94443615 \times 10^{-1} & x_1 &= 3.42901327 \times 10^{-1} \\ a_2 &= 7.64384940 \times 10^{-2} & x_2 &= 1.036610830 \\ a_3 &= 1.07825546 \times 10^{-2} & x_3 &= 1.756683649 \\ a_4 &= 4.27695730 \times 10^{-4} & x_4 &= 2.532731674 \\ a_5 &= 2.43202531 \times 10^{-6} & x_5 &= 3.436159119 \end{aligned} \tag{3.3}$$

4. Regions of Validity of the Three Approximations

The regions of validity of the three approximations are illustrated in Figures 1 and 2, which will be explained below in detail.

In order to check our approximations we used the tables of w(z) by Faddeyeva and Terent'ev[5)]. The tables give six-place values of w(z) for the square $0 \le ZR \le 3$, $0 \le ZI \le 3$ with tabular step of 0.02 for each of the variables and six-place values of w(z) for the range $3 \le ZR \le 5$, $0 \le ZI \le 3$ and $0 \le ZR \le 5$, $3 \le ZI \le 5$ with tabular step of 0.1 for each of the variables. We also used, as a reference in computing, the formulae(Cf. Abramowitz and Stegun, Eqn. 7.1.26 and 7.1.29)

$$\mathrm{erf}(ZR) \simeq 1 - (a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5) e^{-ZR^2} \tag{4.1}$$

where $t = \dfrac{1}{1 + pZR}$ and p, a_1, a_2, a_3, a_4, and a_5 are real constants and

$$\mathrm{erf}(ZR+iZI) \simeq \mathrm{erf}(ZR) + \frac{e^{-ZR^2}}{2\pi ZR}\{(1 - \cos 2ZRZI) + i\sin 2ZRZI\} +$$

$$\frac{2e^{-ZR^2}}{\pi}\sum_{n=1}^{\infty}\frac{e^{-n^2/4}}{n^2+4ZR^2}\{f_n(ZR,ZI) + ig_n(ZR,ZI)\} \tag{4.2}$$

where

$$f_n(ZR,ZI) = 2ZR - 2ZR\cosh(nZI)\cos 2ZRZI + n\sinh(nZI)\sin 2ZRZI \quad \text{and}$$

$$g_n(ZR,ZI) = 2ZR\cosh(nZI)\sin 2ZRZI + n\sinh(nZI)\cos 2ZRZI$$

These formulae allow us to calculate the percent error of the approximations, i.e. 100 x (Approximation - Exact Value) / Exact Value, by computer (Cf. Program 6). Unfortunately, as we can tell from Table 1, Program 6 which evaluates w(z) through (4.1) and (4.2) does not give quite accurate values, especially for those regions where ZR is small and ZI is large simultaneously.

Thence, the percent errors given in Table 2 through Table 6 are not very reliable in those "bad" regions. In other words our rational approximations are normally more accurate than the reference formula and hence the listed errors are over-estimated.

(PADE 1)

(The region of validity of PADE 1 is illustrated in Figure 1.)

We computed PADE 1, i.e. Eqn. (2.10) (Cf. Program 3), up to nine significant places in the range $0 \leq ZR \leq 5$, $0 \leq ZI \leq 5$ with step of 0.1 and checked the results against the tables by Faddeyeva and Terent'ev. The agreement was excellent except along the real axis with ZR large; even at ZR=ZI=5, the real part of PADE 1 agreed with the table up to six places, maximum accuracy of the table, and the imaginary part of PADE 1 agreed with that of the table up to five places. On the real axis, we found percent errors of $\sim$1.3% at ZR=2.9 and $\sim$2.9% at ZR=3.0 for the <u>real</u> part of w(z), and even larger error for larger ZR (Cf. Table 2). But we note that PADE 1 is very accurate for ZI=0.1 (even with ZR=5). The breakdown does not occur unless ZI is very small ($\sim$0.01 or smaller). We also note that the imaginary part of PADE 1 is very accurate even in this area.

(PADE 2)

(The region of validity of PADE 2 is illustrated in Figures 1 and 2.)

We computed PADE 2, i.e. Eqn. (2.17) (Cf. Program 4), up to nine significant places in exactly the same region as in PADE 1 and checked the results against the tables by Faddeyeva and Terent'ev. The agreement was good (not as good as in PADE 1) even away from the point $z=3$. The percent errors were much less than 1% in most of the region except for the points along the real axis with ZR large and the points near the imaginary axis (e.g. at the origin, ∿2% error and at ZR=4.0, ZI=0, ∿15% error) (Cf. Table 3; also see Figure 1 for the errors near the real axis). We note that the breakdown near the real axis is abrupt just as for PADE 1, i.e. the approximation is good until ZI gets very small (∿0.01 or smaller). Again the imaginary part of PADE 2 is very accurate even on the real axis.

(ASYMP)

(The region of validity of ASYMP, i.e. the asymptotic formula (3.2) (Cf. Program 5), is illustrated in Figures 1 and 2.)

Exactly the same procedures as for PADE 1 and PADE 2 were followed. The approximation is excellent for ZI large enough ($\gtrsim 1.0$) or ZR large. But again the real part is a poor approximation on the real axis (Cf. Table 4). In fact, Eqn. (3.2) implies that the real part of w(z) is zero on the real axis, which is a 100% error. Hence, even though ASYMP becomes a better approximation as ZR gets larger, the valid region of the real part of ASYMP never reaches the real axis (e.g. Figure 2 implies that ASYMP is good for ZI ∿0.002 at ZR∿4.2). Again the imaginary part of ASYMP is very accurate even in this region.
To overcome the difficulty we expanded w(z) in powers of ZI and kept only the first power in ZI as follows. For ZI $\ll$ 1 and ZRZI $\ll$ 1 we have, keeping only the first power of ZI in (1.3),

$$w(z) \simeq e^{-ZR^2}(1-i2ZRZI)(1 + \frac{2i}{\sqrt{\pi}}\int_0^{ZR} e^{u^2}du - \frac{2}{\sqrt{\pi}}e^{ZR^2}ZI)$$

Thus, the real part is, for $ZRZI \ll 1$ and $ZI \ll 1$,

$$Rew(z) \simeq e^{-ZR^2} + 2\{ZRImw(ZR+i0) - \frac{1}{\sqrt{\pi}}\}ZI \tag{4.3}$$

Note: The formula (4.3) is plausible because the imaginary part of ASYMP is very accurate for ZR large enough.

The condition for (4.3) to be valid within 1% error is

$$ZRZI \lesssim 0.01 \tag{4.4}$$

We shall discuss this region of validity more in detail in the next section.

5. Boundaries of the Valid Regions of the Three Approximations

(The reader is again referred to Figures 1 and 2 for illustrations.)

Having examined the regions of validity of the three approximations, our next task is to determine where we should set the boundaries of the three approximations so that we have minimum possible errors. Given any two of the three approximations, the idea is to find ZI (or ZR) for fixed ZR (or ZI) where we have the least (or minimum) discontinuity between the two approximations. The points of least discontinuity are plotted in Figures 1 and 2. The boundaries were set so that they go through as many points of least discontinuity as possible.

From the discussions in the previous section we recall that there are bad points for the real part of w(z) on the real axis inside the PADE 2 region and the ASYMP region. Since the power expansion formula (4.3) is a good approximation near the real axis (exact on the real axis), we use it there. In Figure 2 we plot the points of least discontinuity both between PADE 2 and the power expansion and between ASYMP and the power expansion. The boundary between PADE 2 and the power expansion is fitted by a straight line

$$\text{ZRZI} = 0.0625(\text{ZR}-3.5) \tag{5.1}$$

The boundary between ASYMP and the power expansion is fitted by

$$\text{ZRZI} = \frac{a}{\text{ZR}-b} + c \quad , \quad (a,b,c \cdots \text{constants}) \tag{5.2}$$

Using the three points of least discontinuity, (ZR,ZRZI) = (3.8,0.044), (3.9,0.0312) and (4.0,0.022), we find

$$a = 0.04 \ , \quad b = 3.29 \quad \text{and} \quad c = -0.034 \tag{5.3}$$

For ZR > 4.2 we use the boundary

$$\text{ZRZI} = 0.01 \tag{5.4}$$

To sum up:

ASYMP is modified so that it calculates the power expansion formula (4.3)

if $ZR < 4.2$ and $ZRZI < \dfrac{0.04}{ZR-3.29} - 0.034$ or

if $ZR \geq 4.2$ and $ZRZI < 0.01$

After this modification,

for $3.5 \leq ZR < 4.1$

use ASYMP if $ZRZI < 0.0625(ZR-3.5)$

use PADE 2 if $ZRZI \geq 0.0625(ZR-3.5)$

for $ZR \geq 4.1$

use ASYMP.

6. Electric Field

Once we have the function $w(z)$, we can find the electric field by simply using the formulae (1.1) and (1.2). We set, for simplicity,

$$\frac{Q}{2\varepsilon_0\sqrt{\pi}} = 1 \tag{6.1}$$

in those formulae.

Unfortunately, there is one problem: By symmetry $E_y = 0$ for $y = 0$. But we know $\mathrm{Re}\,w(z)$ is not approximated well near the real axis, so the two terms in (1.2) might not cancel out each other to give exactly zero at $y = 0$. This might cause the percent error for E_y to be rather large for $y = 0$ and y small. To overcome this difficulty we first set $E_y = 0$ if $y = 0$ and linearly interpolate the values of E_y for y very small. That is, for

$$\frac{y}{\sqrt{2(s_x^2-s_y^2)}} < 0.002 \quad ,$$

we set

$$E_y(x,y) = \frac{\dfrac{y}{\sqrt{2(s_x^2-s_y^2)}}}{0.002}\, E_y(x, 0.002\sqrt{2(s_x^2-s_y^2)}\,) \tag{6.2}$$

(Cf. Program 1 and Table 5) This also serves to guarantee that $E_y(x,y)$ will be continuous between the first and fourth quadrants.

7. Concluding Remarks

The program FNCTNW calculates w(z) quite accurately. The percent error in most of the region is $\sim 10^{-4}\%$ except for the real part of w(z) near the real axis for certain values of ZR (near ZR = 2.2, 3.5 and 4.2) where the percent error could be at most 0.1%.

The program GAFELD likewise calculates the electric field with the percent error $\sim 10^{-4}\%$ except for E_y near the real axis where the percent error is at most of the order of 0.1%.

Even though we have rather large percent errors ($\sim 0.1\%$) for Rew(z) and E_y near the real axis, the absolute errors are small because Rew(z) and E_y take on small absolute values there.

We have discussed the accurate evaluation over the entire first quadrant. If used in a computer simulation of beam-beam effects, PADE 1 would be called by far the most as its region of validity more or less corresponds to where the particles reside. One may be justified, for the sake of simplicity, in regarding PADE 1 as an adequate replacement for the true field, but further investigation would be necessary to confirm this.

Acknowledgements:

We would like to thank Professor W. Fuchs in Mathematics Department of Cornell University for various useful discussions.

References

1) M. Bassetti and G.A. Erskine, Closed expression for the electrical field of a two-dimensional Gaussian chrge, CERN-ISR-TH/80-06 (unpublished).

2) A typographical error in the formula of Bassetti and Erskine has been corrected. (The sign of the second term in the exponential factor has been reversed.)

3) M. Abramowitz and I.A. Stegun, Handbook of Mathematical Functions (National Bureau of Standards, Washington, 1966), Chapter 7.

4) J.B. Rosser, Theory and Application of

$$\int_0^z e^{-x^2}dx \text{ and } \int_0^z e^{-p^2y^2}dy\int_0^y e^{-x^2}dx$$

(Mapleton House, New York, 1948), p.190.

5) V.N. Faddeyeva and N.M. Terent'ev, Tables of Values of the Function

$$w(z) = e^{-z^2}(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt)$$

for Complex Argument (Pergamon Press, London, 1961).

6) R.E. Greenwood and J.J. Miller, Zeros of the Hermite polynomials and weights for Gauss' mechanical quadrature formula, Amer. Math. Soc. Bull., 54, 765 (1948).

Figure Captions

1. Points of least discontinuity among the three approximations and the boundaries of separating regions of the three approximations.

   The numbers, 2,3,etc., represent the numbers of decimal places of disagreement out of nine significant figures, i.e. 2 means the first seven significant places of agreement and 3 means the first six signigicant places of agreement. Those numbers are taken to be the larger one of the two discontinuities at a point corresponding to the real part and the imaginary part. The real part and the imaginary part have similar degrees of discontinuity at each point in most of the region except for those points near the real axis where the discontinuity of the real part tends to be much bigger than that of the imaginary part.

2. Points of least discontinuity between ASYMP (without the power expansion modification) and the power expansion formula (4.3) and between PADE 2 and the power expansion formula.

   6, etc. represent the number of decimal places of disagreement between ASYMP and the power expansion formula.

   5, etc. represent the number of decimal places of disagreement between PADE 2 and the power expansion formula.

Programs

1. GAFELD.FORTRAN
2. FNCTNW.FORTRAN
3. WPADE1.FORTRAN
4. WPADE2.FORTRAN
5. WASYMP.FORTRAN
6. WEXCT.FORTRAN

To run the computer program for the electric field from the PDP 10 terminal, we just type

```
.EXE GAFELD,FNCTNW,WPADE1,WPADE2,WASYMP,WEXCT
```

Tables

1. WEXCT

The hand-written numbers under some entries represent the exact values taken from the tables by Faddeyeva and Terent'ev. Entries without any hand-written numbers under them represent those values which agree with the tables completely (up to six places).

2. PADE 1
3. PADE 2
4. ASYMP
5. Function w(z) (FNCTNW.FORTRAN)
6. Electric Field (GAFELD.FORTRAN)

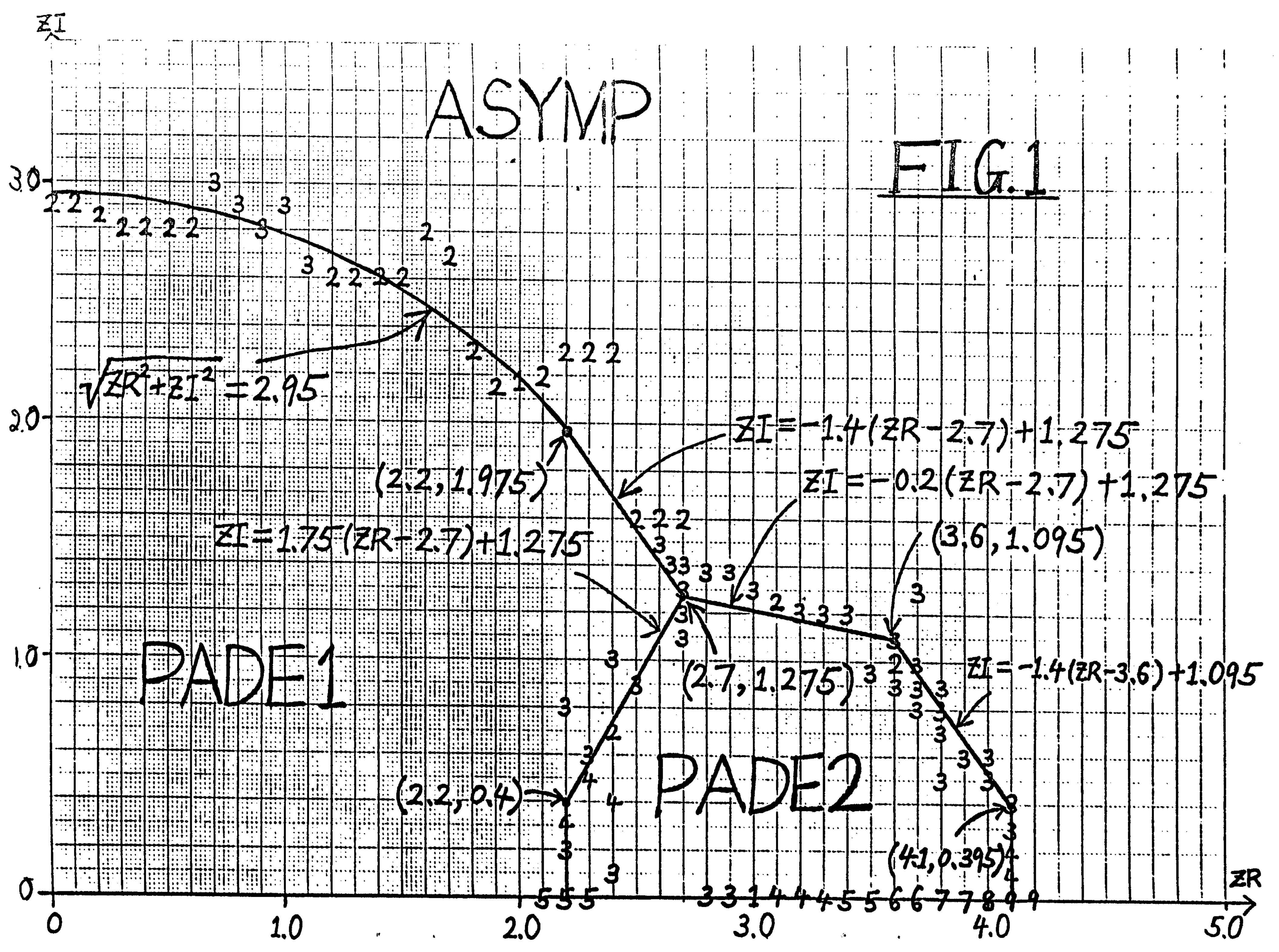
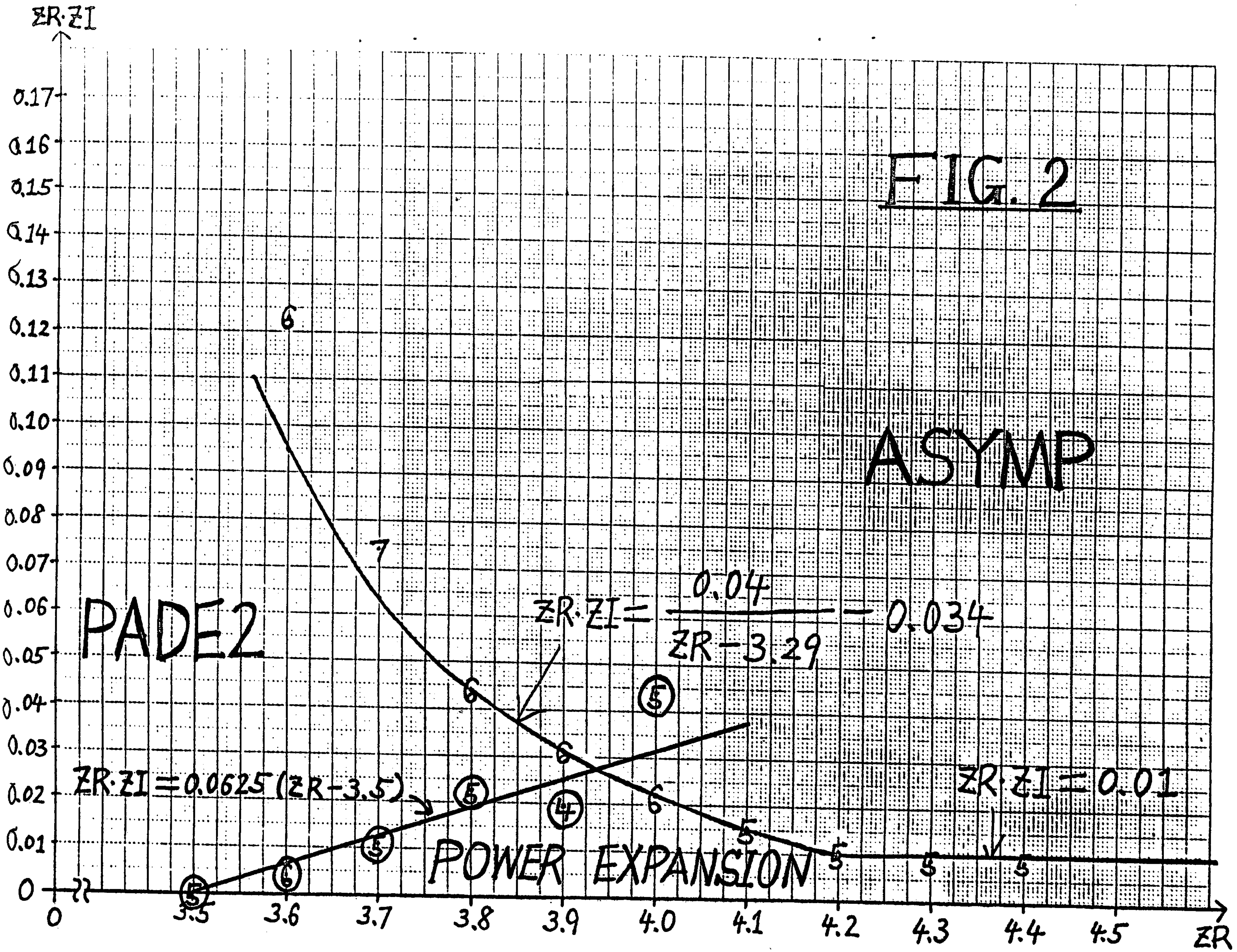


FIG.1
ASYMP
PADE1
PADE2
$\sqrt{ZR^2+ZI^2}=2.95$
(2.2, 1.975)
$ZI=-1.4(ZR-2.7)+1.275$
$ZI=-0.2(ZR-2.7)+1.275$
$ZI=1.75(ZR-2.7)+1.275$
(3.6, 1.095)
(2.7, 1.275)
$ZI=-1.4(ZR-3.6)+1.095$
(2.2, 0.4)
(4.1, 0.395)
ZI
ZR
0
1.0
2.0
3.0
4.0
5.0

FIG. 2


ZR·ZI
ZR
PADE2
ASYMP
POWER EXPANSION
ZR·ZI = 0.04/(ZR−3.29) − 0.034
ZR·ZI = 0.0625 (ZR−3.5)
ZR·ZI = 0.01
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
3.5
3.6
3.7
3.8
3.9
4.0
4.1
4.2
4.3
4.4
4.5

PROGRAM 1

```
C
C        THIS PROGRAM EVALUATES THR ELECTRIC FIELD OF A
C       TWO-DIMENSIONAL GAUSSIAN CHARGE DISTRIBUTION.
C
        DATA SX/1.E0/
        DATA DELTA/.002E0/,CHECK/0.E0/
        DATA ICTR/4/
        DO 10 I=1,3
          SY=.2E0*I
          AB=2.E0*(SX*SX-SY*SY)
          C=SQRT(AB)
         DO 20 J=1,7
           ICTR=ICTR+1
           IF(ICTR.LT.4)GO TO 1
           ICTR=0
           WRITE(30,999)
 999       FORMAT(/X/T35,'ELECTRIC FIELD'/X/X/X/
     &           X,T6,'SX',T12,'SY',T18,'X',T24,'Y',T35,'EX',
     &           T48,'% ERROR',T64,'EY',T77,'% ERROR')
 1         CONTINUE
        DO 30 K=1,7
          X=.5E0*(J-1)
          Y=.5E0*(K-1)
          XS=X/SX
 100      YS=Y/SY
          E=EXP(-XS*XS/2.E0-YS*YS/2.E0)
          ZR=X/C
          ZI=Y/C
          CALL FCNW(WR,WI,ZR,ZI)
          W1R=WR
          W1I=WI
          IF(1.E0-CHECK)200,200,150
 150      CALL WEXCT(WER,WEI,ZR,ZI)
          WE1R=WER
          WE1I=WEI
          ZI1=ZI
 200      ZR=XS*SY/C
          ZI=YS*SX/C
          CALL FCNW(WR,WI,ZR,ZI)
          W2R=WR
          W2I=WI
          IF(1.E0-CHECK)450,450,250
 250      CALL WEXCT(WER,WEI,ZR,ZI)
          WE2R=WER
          WE2I=WEI
          EXEXCT=(WE1I-E*WE2I)/C
          EYEXCT=(WE1R-E*WE2R)/C
          EX=(W1I-E*W2I)/C
          IF(Y)350,300,350
 300      EY=0.E0
          GO TO 550
 350      IF(ZI1-DELTA)400,450,450
 400      CHECK=1.E0
          Y1=Y
          Y=Y*DELTA/ZI1
          GO TO 100
 450      EY=(W1R-E*W2R)/C
          IF(1.E0-CHECK)500,500,550
 500      EY=EY*ZI1/DELTA
          CHECK=0.E0
          Y=Y1
 550      PCNTX=100.E0*(EX-EXEXCT)/EXEXCT
```

(CONTINUED)

GAFELD No.2

```
      PCNTY=100.E0*(EY-EYEXCT)/EYEXCT
      WRITE(30,1000)SX,SY,X,Y,EX,PCNTX,EY,PCNTY
1000  FORMAT(/1X,4F6.1,F17.9,F12.5,F17.9,F12.5)
30    CONTINUE
20    CONTINUE
10    CONTINUE
      STOP
      END
```

PROGRAM 1

PROGRAM 2

```
C                  FNCTNW.FORTRAN
C
C          THIS PROGRAM EVALUATES THE FUNCTION W(Z)
C       (WHERE Z = ZR + i ZI) IN THE FIRST QUADRANT OF
C       THE COMPLEX PLANE (I.E. ZR ≥ 0 AND ZI ≥ 0).
C       THREE DIFFERENT EXPRESSIONS, PADE1, PADE2, AND
C       ASYMP, ARE USED, DEPENDING ON WHERE Z LIES IN
C       THE QUADRANT.
C
        SUBROUTINE FCNW(WR,WI,ZR,ZI)
        DATA X1/4.1E0/,X2/3.6E0/,X3/3.5E0/,X4/2.7E0/,X5/2.2E0/
        DATA Y1/1.275E0/,Y2/1.095E0/
        DATA R2/8.7025E0/
100      IF(ZR-X1)200,30,30
200      EPS1=.0625E0*(ZR-X3)
         IF(ZR-X2)300,210,210
210        YC=-1.4E0*(ZR-X2)+Y2
           IF(ZI-YC)220,30,30
220        IF(ZR*ZI.LT.EPS1)30,20
300      IF(ZR-X4)400,310,310
310        YC=-.2E0*(ZR-X4)+Y1
           IF(ZI-YC)320,30,30
320        IF(ZR.GE.X3.AND.ZR*ZI.LT.EPS1)30,20
400      IF(ZR-X5)500,410,410
410        YC1=-1.4E0*(ZR-X4)+Y1
           YC2=1.75E0*(ZR-X4)+Y1
           IF(ZI-YC1)420,30,30
420        IF(ZI-YC2)20,10,10
500      IF(ZR*ZR+ZI*ZI-R2)10,30,30
10      CALL PADE1(WR,WI,ZR,ZI)
        RETURN
20      CALL PADE2(WR,WI,ZR,ZI)
        RETURN
30      CALL ASYMP(WR,WI,ZR,ZI)
        RETURN
        END
```

PROGRAM 3

```
C                 WPADE1.FORTRAN
C
C        THIS PROGRAM CALCULATES A PADE APPROXIMATION OF W(Z)
C     AROUND THE ORIGIN.
C
      SUBROUTINE PADE1(WR,WI,ZR,ZI)
      DATA C1/-1.25647718E0/,C2/8.25059158E-1/,
     & C3/-3.19300157E-1/,C4/7.63191605E-2/,
     & C5/-1.04697938E-2/,C6/6.44878652E-4/
      DATA D1/-2.38485635E0/,D2/2.51608137E0/,
     & D3/-1.52579040E0/,D4/5.75922693E-1/,
     & D5/-1.35740709E-1/,D6/1.85678083E-2/,
     & D7/-1.14243694E-3/
       U2R=ZI*ZI-ZR*ZR
       U2I=-2.E0*ZR*ZI
       U3R=-U2R*ZI-U2I*ZR
       U3I=U2R*ZR-U2I*ZI
       U4R=-U3R*ZI-U3I*ZR
       U4I=U3R*ZR-U3I*ZI
       U5R=-U4R*ZI-U4I*ZR
       U5I=U4R*ZR-U4I*ZI
       U6R=-U5R*ZI-U5I*ZR
       U6I=U5R*ZR+-U5I*ZI
       U7R=-U6R*ZI-U6I*ZR
       U7I=U6R*ZR-U6I*ZI
       FR=1.E0-C1*ZI+C2*U2R+C3*U3R+C4*U4R+C5*U5R+C6*U6R
       FI=C1*ZR+C2*U2I+C3*U3I+C4*U4I+C5*U5I+C6*U6I
       DR=1.E0-D1*ZI+D2*U2R+D3*U3R+D4*U4R+D5*U5R+D6*U6R+D7*U7R
       DI=D1*ZR+D2*U2I+D3*U3I+D4*U4I+D5*U5I+D6*U6I+D7*U7I
       DE=DR*DR+DI*DI
       WR=(FR*DR+FI*DI)/DE
       WI=(FI*DR-FR*DI)/DE
      RETURN
      END
```

PROGRAM 4

```
C                       WPADE2.FORTRAN
C
C          THIS PROGRAM CALCULATES A PADE APPROXIMATION OF W(Z)
C       AROUND THE POINT Z = 3.
C
      SUBROUTINE PADE2(WR,WI,ZR,ZI)
      DATA C0R/1.23409804E-4/,C0I/2.01157318E-1/,
     & C1R/2.33746715E-1/,C1I/1.61133338E-1/,
     & C2R/1.25689814E-1/,C2I/-4.0422725E-2/,
     & C3R/8.92089179E-3/,C3I/-1.81293213E-2/
      DATA D1R/1.19230984E0/,D1I/-1.16495901E0/,
     & D2R/8.9401545E-2/,D2I/-1.07372867E0/,
     & D3R/-1.68547429E-1/,D3I/-2.70096451E-1/,
     & D4R/-3.20997564E-2/,D4I/-1.58578639E-2/
       ZR=ZR-3.E0
       Z2R=ZR*ZR-ZI*ZI
       Z2I=2.E0*ZR*ZI
       Z3R=Z2R*ZR-Z2I*ZI
       Z3I=Z2R*ZI+Z2I*ZR
       Z4R=Z3R*ZR-Z3I*ZI
       Z4I=Z3R*ZI+Z3I*ZR
       FR=C0R+C1R*ZR-C1I*ZI+C2R*Z2R-C2I*Z2I+C3R*Z3R-C3I*Z3I
       FI=C0I+C1R*ZI+C1I*ZR+C2R*Z2I+C2I*Z2R+C3R*Z3I+C3I*Z3R
       DR=1.E0+D1R*ZR-D1I*ZI+D2R*Z2R-D2I*Z2I+D3R*Z3R-D3I*Z3I+
     &    D4R*Z4R-D4I*Z4I
       DI=D1R*ZI+D1I*ZR+D2R*Z2I+D2I*Z2R+D3R*Z3I+D3I*Z3R+D4R*Z4I+
     &    D4I*Z4R
       DE=DR*DR+DI*DI
       WR=(FR*DR+FI*DI)/DE
       WI=(FI*DR-FR*DI)/DE
       ZR=ZR+3.E0
      RETURN
      END
```

PROGRAM 5

```
C                  WASYMP.FORTRAN
C
C          THIS PROGRAM CALCULATES AN ASYMPTOTIC EXPRESSION OF
C        W(Z) VALID AWAY FROM THE ORIGIN.
C
         SUBROUTINE ASYMP(WR,WI,ZR,ZI)
         DATA A1P/1.94443615E-1/,A2P/7.64384940E-2/,
      &   A3P/1.07825546E-2/,A4P/4.27695730E-4/,A5P/2.43202531E-6/
         DATA B1/3.42901327E-1/,B2/1.036610830E0/,B3/1.756683649E0/,
      &   B4/2.532731674E0/,B5/3.436159119E0/
         DATA PI2/1.12837917E0/
         DATA X1/3.5E0/,X2/4.2E0/
         DATA EPS/.01E0/,CHECK/0.E0/
 10       DR1=ZR+B1
          D1R=ZR-B1
          DR2=ZR+B2
          D2R=ZR-B2
          DR3=ZR+B3
          D3R=ZR-B3
          DR4=ZR+B4
          D4R=ZR-B4
          DR5=ZR+B5
          D5R=ZR-B5
          DE1=DR1*DR1+ZI*ZI
          D1E=D1R*D1R+ZI*ZI
          DE2=DR2*DR2+ZI*ZI
          D2E=D2R*D2R+ZI*ZI
          DE3=DR3*DR3+ZI*ZI
          D3E=D3R*D3R+ZI*ZI
          DE4=DR4*DR4+ZI*ZI
          D4E=D4R*D4R+ZI*ZI
          DE5=DR5*DR5+ZI*ZI
          D5E=D5R*D5R+ZI*ZI
          IF(1.E0-CHECK)70,70,20
 20         IF(ZR.GE.X1)30,60
 30           EPS1=.04E0/(ZR-3.29E0)-.034E0
              IF(ZR*ZI.LT.EPS1)50,40
 40           IF(ZR.GE.X2.AND.ZR*ZI.LT.EPS)50,60
 50         CHECK=1.E0
            WI0=A1P*(DR1/DE1+D1R/D1E)+A2P*(DR2/DE2+D2R/D2E)+
      &        A3P*(DR3/DE3+D3R/D3E)+A4P*(DR4/DE4+D4R/D4E)+
      &        A5P*(DR5/DE5+D5R/D5E)
            ZI0=ZI
            ZI=0.E0
            GO TO 10
 60       WR=(A1P*(1.E0/DE1+1.E0/D1E)+A2P*(1.E0/DE2+1.E0/D2E)+
      &      A3P*(1.E0/DE3+1.E0/D3E)+A4P*(1.E0/DE4+1.E0/D4E)+
      &      A5P*(1.E0/DE5+1.E0/D5E))*ZI
 70       WI=A1P*(DR1/DE1+D1R/D1E)+A2P*(DR2/DE2+D2R/D2E)+
      &      A3P*(DR3/DE3+D3R/D3E)+A4P*(DR4/DE4+D4R/D4E)+
      &      A5P*(DR5/DE5+D5R/D5E)
          IF(1.E0-CHECK)80,80,90
 80       WR=EXP(-ZR*ZR)+2.E0*WI*ZR*ZI0-PI2*ZI0
          WI=WI0
          ZI=ZI0
          CHECK=0.E0
 90       RETURN
         END
```

PROGRAM6

```
C                 WEXCT.FORTRAN
C
C         THIS PROGRAM GIVES AN APPROXIMATE VALUE FOR AN
C       INFINITE SERIES EXPRESSION OF W(Z).
C
      SUBROUTINE WEXCT(WER,WEI,ZR,ZI)
      DATA P/3.275911E-1/,A1/2.54829592E-1/,A2/-2.84496736E-1/,
     & A3/1.421413741E0/,A4/-1.453152027E0/,A5/1.061405429E0/
      DATA NMAX/35/
      DATA PI/3.14159265E0/
      ZR=-ZR
      EX=EXP(-ZR*ZR)
      T=1.E0/(1.E0+P*ZI)
      TWXY=2.E0*ZR*ZI
      C2XY=COS(TWXY)
      S2XY=SIN(TWXY)
      DEN=2.E0*PI*ZI
      ER=((((A5*T+A4)*T+A3)*T+A2)*T+A1)*T
      IF(ZI)20,10,20
10    ACR=ER
      ACI=-ZR/PI
      GO TO 30
20    ACR=ER+(C2XY-1.E0)/DEN
      ACI=-S2XY/DEN
30    DO 100 N=1,NMAX
       XN=N*ZR
       N2=N*N
       ARG=.25E0*N2
       ARG1=2.E0/PI/(4.E0*ZI*ZI+N2)
       EARG=0.E0
       EXPAR=0.E0
       EXMAR=0.E0
       IF(ARG-XN-80.E0)40,40,50
40     EARG=EXP(-ARG)
       EXPAR=EXP(XN-ARG)
       EXMAR=EXP(-XN-ARG)
50     CH2=EXPAR+EXMAR
       SH2=EXPAR-EXMAR
       TERMR=-ARG1*(2.E0*ZI*EARG-ZI*CH2*C2XY+N/2.E0*SH2*S2XY)
       TERMI=-ARG1*(ZI*CH2*S2XY+N/2.E0*SH2*C2XY)
       ACR=ACR+TERMR
       ACI=ACI+TERMI
100   CONTINUE
      WER=EX*(C2XY*ACR-S2XY*ACI)
      WEI=EX*(C2XY*ACI+S2XY*ACR)
      ZR=-ZR
      RETURN
      END
```

WEXCT

TABLE 1

ZR	ZI	REAL PART	IMAGINARY PART
0.00	0.00	1.0000000000000	0.0000000000000
0.00	0.50	0.6156901790000	0.0000000000000
0.00	1.00	0.4275838580000	0.0000000000000
0.00	1.50	0.3215842770000 5	0.0000000000000
0.00	2.00	0.2554025240000 396	0.0000000000000
0.00	2.50	0.2108495910000 06	0.0000000000000
0.00	3.00	0.1791198760000 001	0.0000000000000
0.50	0.00	0.7788007780000	0.4789251690000
0.50	0.50	0.5331565960000	0.2304883000000
0.50	1.00	0.3912341370000	0.1272022220000
0.50	1.50	0.3033550380000	0.0778517555000 1
0.50	2.00	0.2452737730000 6	0.0515166279000 21
0.50	2.50	0.2046958470000 723	0.0361758047000 96
0.50	3.00	0.1750136710000 105	0.0266231191000 36
1.00	0.00	0.3678794350000	0.6071577000000
1.00	0.50	0.3549002820000	0.3428717550000
1.00	1.00	0.3047441650000	0.2082188390000
1.00	1.50	0.2571283430000	0.1352423320000
1.00	2.00	0.2184909600000 3	0.0929997098000 8
1.00	2.50	0.1881438330000 39	0.0670397030000 24
1.00	3.00	0.1643030720000 261	0.0502093383000 197
1.50	0.00	0.1053992240000	0.4832273280000
1.50	0.50	0.1966360170000	0.3377203230000
1.50	1.00	0.2118365490000	0.2331709690000
1.50	1.50	0.2011151350000	0.1643484610000 9
1.50	2.00	0.1833354550000	0.1192984350000
1.50	2.50	0.1651373900000 6	0.0892175231000 22
1.50	3.00	0.1486067920000 18	0.0625801087000 5

TABLE 1

WEXCT

ZR	ZI	REAL PART	IMAGINARY PART
2.00	0.00	0.0183156391000	0.3400262260000
2.00	0.50	0.1033588290000	0.2847858970000
2.00	1.00	0.1402395760000	0.2222134380000
2.00	1.50	0.1504154240000	0.1703711390000
2.00	2.00	0.1479527390000	0.1311795900000
2.00	2.50	0.1402194690000	0.1023294110000
2.00	3.00	0.1307592970000	0.0811138125000
2.50	0.00	0.0019304541400	0.2517230210000
2.50	0.50	0.0584374736000	0.2324204350000
2.50	1.00	0.0937507432000	0.1983071180000
2.50	1.50	0.1112334560000	0.1632367450000
2.50	2.00	0.1172385700000	0.1327199040000
2.50	2.50	0.1167372050000	0.1079085900000
2.50	3.00	0.1128778080000	0.0882829148000
3.00	0.00	0.0001234098050	0.2011571970000
3.00	0.50	0.0371263563000	0.1929836350000
3.00	1.00	0.0653177612000	0.1739182010000
3.00	1.50	0.0832095193000	0.1508796890000
3.00	2.00	0.0927107269000	0.1283168420000
3.00	2.50	0.0963932602000	0.1082492820000
3.00	3.00	0.0964024877000	0.0912362561000

PADE1

TABLE 2

ZR	ZI	REAL PART	% ERROR	IMAGINARY PART	% ERROR
0.00	0.00	1.000000000000	0.00000	0.000000000000	**************
0.00	0.50	0.615690358000	0.00003	0.000000000000	**************
0.00	1.00	0.427583579000	-0.00007	0.000000000000	**************
0.00	1.50	0.321585421000	0.00036	0.000000000000	**************
0.00	2.00	0.255395673000	-0.00268	0.000000000000	**************
0.00	2.50	0.210806368000	-0.02050	0.000000000000	**************
0.00	3.00	0.179001164000	-0.06628	0.000000000000	**************
0.50	0.00	0.778800778000	0.00000	0.478925183000	0.00000
0.50	0.50	0.533156708000	0.00002	0.230488228000	-0.00003
0.50	1.00	0.391234022000	-0.00003	0.127202412000	0.00015
0.50	1.50	0.303355116000	0.00003	0.077850871700	-0.00114
0.50	2.00	0.245275991000	0.00090	0.051521476800	0.00941
0.50	2.50	0.204722822000	0.01318	0.036195945900	0.05568
0.50	3.00	0.175105222000	0.05231	0.026636157200	0.04897
1.00	0.00	0.367879450000	0.00000	0.607157707000	0.00000
1.00	0.50	0.354900342000	0.00002	0.342871722000	-0.00001
1.00	1.00	0.304744210000	0.00001	0.208218942000	0.00005
1.00	1.50	0.257127944000	-0.00016	0.135242280000	-0.00004
1.00	2.00	0.218492612000	0.00076	0.092997808900	-0.00204
1.00	2.50	0.188139319000	-0.00240	0.067024451700	-0.02275
1.00	3.00	0.164261133000	-0.02553	0.050197116100	-0.02434
1.50	0.00	0.105399224000	0.00000	0.483227320000	-0.00000
1.50	0.50	0.196636034000	0.00001	0.337720331000	0.00000
1.50	1.00	0.211836586000	0.00002	0.233170971000	0.00000
1.50	1.50	0.201115120000	-0.00001	0.164348582000	0.00007
1.50	2.00	0.183334759000	-0.00038	0.119298241000	-0.00016
1.50	2.50	0.165135801000	-0.00096	0.089221801600	0.00480
1.50	3.00	0.148618160000	0.00765	0.068585246800	0.00749

PADE1

TABLE 2

ZR	ZI	REAL PART	% ERROR	IMAGINARY PART	% ERROR
2.00	0.00	0.018315787400	0.00081	0.340026181000	-0.00001
2.00	0.50	0.103358807000	-0.00002	0.284785919000	0.00001
2.00	1.00	0.140239574000	-0.00000	0.222213427000	-0.00001
2.00	1.50	0.150415460000	0.00002	0.170371143000	0.00000
2.00	2.00	0.147952765000	0.00002	0.131179744000	0.00012
2.00	2.50	0.140220096000	0.00045	0.102329001000	-0.00040
2.00	3.00	0.130757406000	-0.00145	0.081112649300	-0.00143
2.50	0.00	0.001930428570	-0.00132	0.251724064000	0.00041
2.50	0.50	0.058437210500	-0.00045	0.232420432000	-0.00000
2.50	1.00	0.093750747900	0.00000	0.198306996000	-0.00006
2.50	1.50	0.111233548000	0.00008	0.163236737000	-0.00000
2.50	2.00	0.117238612000	0.00004	0.132719971000	0.00005
2.50	2.50	0.116737078000	-0.00011	0.107908675000	0.00008
2.50	3.00	0.112877866000	0.00005	0.088283104800	0.00022
3.00	0.00	0.000119885849	-2.85549	0.201157194000	-0.00000
3.00	0.50	0.037125889200	-0.00126	0.192982849000	-0.00041
3.00	1.00	0.065318054500	0.00045	0.173917951000	-0.00014
3.00	1.50	0.083209819200	0.00036	0.150879838000	0.00010
3.00	2.00	0.092710864700	0.00015	0.128317172000	0.00026
3.00	2.50	0.096393215500	-0.00005	0.108249595000	0.00029
3.00	3.00	0.096402295900	-0.00020	0.091236437700	0.00020

PADE2

ZR	ZI	REAL PART	% ERROR	IMAGINARY PART	% ERROR
0.00	0.00	1.022405980000	2.24060	0.042636918400	*************
0.00	0.50	0.622718848000	1.14159	0.001742482960	*************
0.00	1.00	0.429081336000	0.35022	-0.001014851590	*************
0.00	1.50	0.321704991000	0.03754	-0.000659049867	*************
0.00	2.00	0.255225577000	-0.06928	-0.000284117326	*************
0.00	2.50	0.210621439000	-0.10821	-0.000078917138	*************
0.00	3.00	0.178861629000	-0.14418	0.000018885423	*************
0.50	0.00	0.763372660000	-1.98101	0.477050964000	-0.39134
0.50	0.50	0.532502919000	-0.12261	0.232789729000	0.99850
0.50	1.00	0.391791366000	0.14243	0.127532197000	0.25941
0.50	1.50	0.303575892000	0.07280	0.077695237500	-0.20105
0.50	2.00	0.245274648000	0.00036	0.051369078000	-0.28641
0.50	2.50	0.204647340000	-0.02370	0.036123098300	-0.14569
0.50	3.00	0.175023861000	0.00582	0.026626379900	0.01225
1.00	0.00	0.369620483000	0.47327	0.605242446000	-0.31545
1.00	0.50	0.354463331000	-0.12312	0.342784230000	-0.02553
1.00	1.00	0.304756124000	0.00392	0.208369095000	0.07216
1.00	1.50	0.257208850000	0.03131	0.135240307000	-0.00150
1.00	2.00	0.218512351000	0.00979	0.092943469100	-0.06047
1.00	2.50	0.188112568000	-0.01662	0.066984078900	-0.08297
1.00	3.00	0.164218407000	-0.05153	0.050185957500	-0.04657
1.50	0.00	0.105454601000	0.05254	0.483398523000	0.03543
1.50	0.50	0.196629060000	-0.00354	0.337678753000	-0.01231
1.50	1.00	0.211825313000	-0.00530	0.233190071000	0.00819
1.50	1.50	0.201133007000	0.00889	0.164352974000	0.00275
1.50	2.00	0.183342803000	0.00401	0.119281172000	-0.01447
1.50	2.50	0.165124679000	-0.00770	0.089204394300	-0.01472
1.50	3.00	0.148595996000	-0.00726	0.068579793500	-0.00046

PADE2

ZR	ZI	REAL PART	% ERROR	IMAGINARY PART	% ERROR
2.00	0.00	0.018312219500	-0.01867	0.340026416000	0.00006
2.00	0.50	0.103359606000	0.00075	0.284784224000	-0.00059
2.00	1.00	0.140238488000	-0.00078	0.222215220000	0.00080
2.00	1.50	0.150418915000	0.00232	0.170371166000	0.00002
2.00	2.00	0.147953784000	0.00071	0.131173976000	-0.00428
2.00	2.50	0.140213843000	-0.00401	0.102322839000	-0.00642
2.00	3.00	0.130745517000	-0.01054	0.081111950800	-0.00230
2.50	0.00	0.001930453870	-0.00001	0.251723051000	0.00001
2.50	0.50	0.058437486700	0.00002	0.232420389000	-0.00002
2.50	1.00	0.093750804700	0.00007	0.198307235000	0.00006
2.50	1.50	0.111233878000	0.00038	0.163236188000	-0.00034
2.50	2.00	0.117237615000	-0.00081	0.132718099000	-0.00136
2.50	2.50	0.116733180000	-0.00345	0.107907374000	-0.00113
2.50	3.00	0.112871493060	-0.00559	0.088284975900	0.00233
3.00	0.00	0.000123409804	-0.00000	0.201157318000	0.00006
3.00	0.50	0.037126366100	0.00003	0.192983750000	0.00006
3.00	1.00	0.065317763000	0.00000	0.173918294000	0.00005
3.00	1.50	0.083209325600	-0.00023	0.150879636000	-0.00003
3.00	2.00	0.092709777900	-0.00102	0.128316864000	0.00002
3.00	2.50	0.096391078100	-0.00226	0.108250235000	0.00088
3.00	3.00	0.096399325900	-0.00328	0.091239312700	0.00335

TABLE4

ZR	ZI	REAL PART	% ERROR	IMAGINARY PART	% ERROR
0.00	0.00	0.000000000000	-100.00000	0.000000000000	************
0.00	0.50	0.589986347000	-4.17480	0.000000000000	************
0.00	1.00	0.427057043000	-0.12321	0.000000000000	************
0.00	1.50	0.321569581000	-0.00457	0.000000000000	************
0.00	2.00	0.255395003000	-0.00294	0.000000000000	************
0.00	2.50	0.210806325000	-0.02052	0.000000000000	************
0.00	3.00	0.179001147000	-0.06628	0.000000000000	************
0.50	0.00	0.000000000000	-100.00000	1.371826560000	186.43860
0.50	0.50	0.544878312000	2.19855	0.247668594000	7.45387
0.50	1.00	0.391604431000	0.09465	0.127425754000	0.17573
0.50	1.50	0.303368252000	0.00436	0.077853184200	0.00184
0.50	2.00	0.245276563000	0.00114	0.051521393500	0.00925
0.50	2.50	0.204722852000	0.01319	0.036195935700	0.05565
0.50	3.00	0.175105218000	0.05231	0.026636167400	0.04901
1.00	0.00	0.000000000000	-100.00000	-1.620124360000	-366.83749
1.00	0.50	0.358040016000	0.88468	0.332568303000	-3.00505
1.00	1.00	0.304621380000	-0.04029	0.208012585000	-0.09906
1.00	1.50	0.257120330000	-0.00312	0.135239914000	-0.00179
1.00	2.00	0.218492266000	0.00060	0.092997931900	-0.00191
1.00	2.50	0.188139310000	-0.00240	0.067024474000	-0.02272
1.00	3.00	0.164261138000	-0.02552	0.050197136600	-0.02430
1.50	0.00	0.000000000000	-100.00000	0.429637887000	-11.08990
1.50	0.50	0.193549179000	-1.56982	0.339345198000	0.48113
1.50	1.00	0.211850043000	0.00637	0.233260548000	0.03842
1.50	1.50	0.201118335000	0.00159	0.164349698000	0.00075
1.50	2.00	0.183334906000	-0.00030	0.119298131000	-0.00025
1.50	2.50	0.165135816000	-0.00095	0.089221787600	0.00478
1.50	3.00	0.148618188000	0.00767	0.068585262600	0.00752

TABLE 4

ASYMP

ZR	ZI	REAL PART	% ERROR	IMAGINARY PART	% ERROR
2.00	0.00	0.000000000000	-100.00000	0.351323549000	3.32249
2.00	0.50	0.104089985000	0.70740	0.284910675000	0.04381
2.00	1.00	0.140241457000	0.00134	0.222190067000	-0.01052
2.00	1.50	0.150414409000	-0.00067	0.170370910000	-0.00013
2.00	2.00	0.147952728000	-0.00001	0.131179771000	0.00014
2.00	2.50	0.140220126000	0.00047	0.102328976000	-0.00043
2.00	3.00	0.130757472000	-0.00140	0.081112650200	-0.00143
2.50	0.00	0.000000000000	-100.00000	0.236439999000	-6.07136
2.50	0.50	0.058346787000	-0.15519	0.232365089000	-0.02381
2.50	1.00	0.093750361400	-0.00041	0.198311338000	0.00213
2.50	1.50	0.111233702000	0.00022	0.163236728000	-0.00001
2.50	2.00	0.117238574000	0.00000	0.132719873000	-0.00002
2.50	2.50	0.116737123000	-0.00007	0.107908586000	-0.00000
2.50	3.00	0.112877983000	0.00016	0.088283065700	0.00017
3.00	0.00	0.000000000000	-100.00000	0.201139914000	-0.00859
3.00	0.50	0.037133972200	0.02051	0.192989158000	0.00286
3.00	1.00	0.065317676400	-0.00013	0.173917778000	-0.00024
3.00	1.50	0.083209501600	-0.00002	0.150879815000	0.00008
3.00	2.00	0.092710769700	0.00005	0.128316965000	0.00010
3.00	2.50	0.096393304900	0.00005	0.108249388000	0.00010
3.00	3.00	0.096402505400	0.00002	0.091236325000	0.00008

FUNCTION W(Z)

ZR	ZI	REAL PART	% ERROR	IMAGINARY PART	% ERROR
0.00	0.00	1.000000000000	0.00000	0.000000000000	************
0.00	0.50	0.615690358000	0.00003	0.000000000000	************
0.00	1.00	0.427583579000	-0.00007	0.000000000000	************
0.00	1.50	0.321585421000	0.00036	0.000000000000	************
0.00	2.00	0.255395673000	-0.00268	0.000000000000	************
0.00	2.50	0.210806368000	-0.02050	0.000000000000	************
0.00	3.00	0.179001147000	-0.06628	0.000000000000	************
0.50	0.00	0.778800778000	0.00000	0.478925183000	0.00000
0.50	0.50	0.533156708000	0.00002	0.230488228000	-0.00003
0.50	1.00	0.391234022000	-0.00003	0.127202412000	0.00015
0.50	1.50	0.303355116000	0.00003	0.077850871700	-0.00114
0.50	2.00	0.245275991000	0.00090	0.051521476800	0.00941
0.50	2.50	0.204722822000	0.01318	0.036195945900	0.05568
0.50	3.00	0.175105218000	0.05231	0.026636167400	0.04901
1.00	0.00	0.367879450000	0.00000	0.607157707000	0.00000
1.00	0.50	0.354900342000	0.00002	0.342871722000	-0.00001
1.00	1.00	0.304744210000	0.00001	0.208218942000	0.00005
1.00	1.50	0.257127944000	-0.00016	0.135242280000	-0.00004
1.00	2.00	0.218492612000	0.00076	0.092997808900	-0.00204
1.00	2.50	0.188139319000	-0.00240	0.067024451700	-0.02275
1.00	3.00	0.164261138000	-0.02552	0.050197136600	-0.02430
1.50	0.00	0.105399224000	0.00000	0.483227320000	-0.00000
1.50	0.50	0.196636034000	0.00001	0.337720331000	0.00000
1.50	1.00	0.211836586000	0.00002	0.233170971000	0.00000
1.50	1.50	0.201115120000	-0.00001	0.164348582000	0.00007
1.50	2.00	0.183334759000	-0.00038	0.119298241000	-0.00016
1.50	2.50	0.165135801000	-0.00096	0.089221801600	0.00480
1.50	3.00	0.148618188000	0.00767	0.068585262600	0.00752

FUNCTION W(Z)

ZR	ZI	REAL PART	% ERROR	IMAGINARY PART	% ERROR
2.00	0.00	0.018315787400	0.00081	0.340026181000	-0.00001
2.00	0.50	0.103358807000	-0.00002	0.284785919000	0.00001
2.00	1.00	0.140239574000	-0.00000	0.222213427000	-0.00001
2.00	1.50	0.150415460000	0.00002	0.170371143000	0.00000
2.00	2.00	0.147952765000	0.00002	0.131179744000	0.00012
2.00	2.50	0.140220126000	0.00047	0.102328976000	-0.00043
2.00	3.00	0.130757472000	-0.00140	0.081112650200	-0.00143
2.50	0.00	0.001930453870	-0.00001	0.251723051000	0.00001
2.50	0.50	0.058437486700	0.00002	0.232420389000	-0.00002
2.50	1.00	0.093750747900	0.00000	0.198306996000	-0.00006
2.50	1.50	0.111233548000	0.00008	0.163236737000	-0.00000
2.50	2.00	0.117238574000	0.00000	0.132719873000	-0.00002
2.50	2.50	0.116737123000	-0.00007	0.107908586000	-0.00000
2.50	3.00	0.112877983000	0.00016	0.088283065700	0.00017
3.00	0.00	0.000123409804	-0.00000	0.201157318000	0.00006
3.00	0.50	0.037126366100	0.00003	0.192983750000	0.00006
3.00	1.00	0.065317763000	0.00000	0.173918294000	0.00005
3.00	1.50	0.083209501600	-0.00002	0.150879815000	0.00008
3.00	2.00	0.092710769700	0.00005	0.128316965000	0.00010
3.00	2.50	0.096393304900	0.00005	0.108249388000	0.00010
3.00	3.00	0.096402505400	0.00002	0.091236325000	0.00008

TABLE 6

ELECTRIC FIELD

SX	SY	X	Y	EX	% ERROR	EY	% ERROR
1.0	0.2	0.0	0.0	0.000000000	**********	0.000000000	**********
1.0	0.2	0.0	0.5	0.000000000	**********	0.492497943	0.00001
1.0	0.2	0.0	1.0	0.000000000	**********	0.373504713	-0.00001
1.0	0.2	0.0	1.5	0.000000000	**********	0.293036692	-0.00003
1.0	0.2	0.0	2.0	0.000000000	**********	0.238951376	0.00033
1.0	0.2	0.0	2.5	0.000000000	**********	0.200651741	-0.00046
1.0	0.2	0.0	3.0	0.000000000	**********	0.172368452	-0.00626
1.0	0.2	0.5	0.0	0.217937123	0.00000	0.000000000	-100.00000
1.0	0.2	0.5	0.5	0.153103616	-0.00001	0.451645646	0.00001
1.0	0.2	0.5	1.0	0.094795180	0.00002	0.351584937	-0.00001
1.0	0.2	0.5	1.5	0.062670697	0.00010	0.280612830	-0.00002
1.0	0.2	0.5	2.0	0.043741168	-0.00134	0.231423989	0.00015
1.0	0.2	0.5	2.5	0.031899321	0.00247	0.195830308	-0.00011
1.0	0.2	0.5	3.0	0.024111172	0.03932	0.169133326	-0.00004
1.0	0.2	1.0	0.0	0.350181088	0.00000	0.000000000	-100.00000
1.0	0.2	1.0	0.5	0.255508177	-0.00000	0.350579359	0.00001
1.0	0.2	1.0	1.0	0.165700084	0.00001	0.294965837	-0.00000
1.0	0.2	1.0	1.5	0.113182262	0.00005	0.247453943	0.00000
1.0	0.2	1.0	2.0	0.080851011	-0.00049	0.210839519	-0.00010
1.0	0.2	1.0	2.5	0.059960662	0.00048	0.182403205	0.00026
1.0	0.2	1.0	3.0	0.045884673	0.00023	0.159998577	0.00401
1.0	0.2	1.5	0.0	0.375179652	0.00000	0.000000000	**********
1.0	0.2	1.5	0.5	0.290232390	-0.00000	0.235271409	0.00001
1.0	0.2	1.5	1.0	0.201837171	0.00001	0.224486038	0.00000
1.0	0.2	1.5	1.5	0.144811798	0.00001	0.203428803	0.00001
1.0	0.2	1.5	2.0	0.107145462	-0.00000	0.182187110	-0.00013
1.0	0.2	1.5	2.5	0.081523309	-0.00024	0.163043460	0.00013
1.0	0.2	1.5	3.0	0.063583441	-0.00526	0.146471802	0.00005

TABLE 6

ELECTRIC FIELD

SX	SY	X	Y	EX	% ERROR	EY	% ERROR
1.0	0.2	2.0	0.0	0.331732430	0.00000	0.000000000	-100.00000
1.0	0.2	2.0	0.5	0.275107760	-0.00000	0.142126698	0.00000
1.0	0.2	2.0	1.0	0.207615288	0.00000	0.159535304	0.00001
1.0	0.2	2.0	1.5	0.157885570	0.00000	0.158884136	0.00001
1.0	0.2	2.0	2.0	0.121865205	0.00007	0.151181333	-0.00003
1.0	0.2	2.0	2.5	0.095675363	-0.00011	0.141025458	-0.00004
1.0	0.2	2.0	3.0	0.076405819	-0.00005	0.130498294	-0.00103
1.0	0.2	2.5	0.0	0.269691672	-0.00001	0.000000000	-100.00000
1.0	0.2	2.5	0.5	0.238967635	0.00001	0.082203769	0.00001
1.0	0.2	2.5	1.0	0.195159279	0.00000	0.109829347	-0.00001
1.0	0.2	2.5	1.5	0.157365859	-0.00000	0.120587931	0.00000
1.0	0.2	2.5	2.0	0.126930660	0.00002	0.122245234	0.00002
1.0	0.2	2.5	2.5	0.103053002	0.00001	0.119197631	-0.00002
1.0	0.2	2.5	3.0	0.084457792	0.00051	0.113921503	-0.00002
1.0	0.2	3.0	0.0	0.216696629	0.00004	0.000000000	-100.00000
1.0	0.2	3.0	0.5	0.201836292	0.00004	0.048860894	-0.00005
1.0	0.2	3.0	1.0	0.175734343	-0.00001	0.075936386	-0.00005
1.0	0.2	3.0	1.5	0.149203114	-0.00002	0.090892429	-0.00000
1.0	0.2	3.0	2.0	0.125385121	-0.00001	0.097704130	0.00003
1.0	0.2	3.0	2.5	0.105170991	0.00002	0.099419196	0.00002
1.0	0.2	3.0	3.0	0.088464050	0.00001	0.098123311	0.00009
1.0	0.4	0.0	0.0	0.000000000	**************	0.000000000	**************
1.0	0.4	0.0	0.5	0.000000000	**************	0.369554408	0.00004
1.0	0.4	0.0	1.0	0.000000000	**************	0.376161270	0.00001
1.0	0.4	0.0	1.5	0.000000000	**************	0.299334873	0.00004
1.0	0.4	0.0	2.0	0.000000000	**************	0.242782483	0.00035
1.0	0.4	0.0	2.5	0.000000000	**************	0.203077208	-0.00166
1.0	0.4	0.0	3.0	0.000000000	**************	0.173981533	-0.01130

TABLE6

ELECTRIC FIELD

SX	SY	X	Y	EX	% ERROR	EY	% ERROR
1.0	0.4	0.5	0.0	0.187729778	-0.00000	0.000000000**********	
1.0	0.4	0.5	0.5	0.154067883	-0.00003	0.340442333	0.00004
1.0	0.4	0.5	1.0	0.100781805	-0.00001	0.352717906	0.00001
1.0	0.4	0.5	1.5	0.065895880	-0.00013	0.285791069	0.00003
1.0	0.4	0.5	2.0	0.045469597	-0.00150	0.234717447	0.00012
1.0	0.4	0.5	2.5	0.032884901	0.00880	0.197982065	-0.00012
1.0	0.4	0.5	3.0	0.024704531	0.06706	0.170598166	0.00212
1.0	0.4	1.0	0.0	0.306088880	-0.00000	0.000000000	-100.00000
1.0	0.4	1.0	0.5	0.255847246	-0.00002	0.268081192	0.00003
1.0	0.4	1.0	1.0	0.174199075	-0.00001	0.292826638	0.00001
1.0	0.4	1.0	1.5	0.118097225	-0.00006	0.249991478	-0.00001
1.0	0.4	1.0	2.0	0.083633440	-0.00039	0.212806962	-0.00016
1.0	0.4	1.0	2.5	0.061614353	0.00050	0.183855901	0.00100
1.0	0.4	1.0	3.0	0.046912260	-0.00963	0.161074309	0.00612
1.0	0.4	1.5	0.0	0.335636485	0.00000	0.000000000	-100.00000
1.0	0.4	1.5	0.5	0.288742293	-0.00001	0.184652478	0.00002
1.0	0.4	1.5	1.0	0.208941102	-0.00000	0.219788021	0.00000
1.0	0.4	1.5	1.5	0.149496051	-0.00001	0.203299914	-0.00002
1.0	0.4	1.5	2.0	0.110062176	0.00009	0.182674307	-0.00010
1.0	0.4	1.5	2.5	0.083382238	-0.00102	0.163655151	0.00013
1.0	0.4	1.5	3.0	0.064800801	-0.00635	0.147051930	-0.00211
1.0	0.4	2.0	0.0	0.305378694	-0.00000	0.000000000	-100.00000
1.0	0.4	2.0	0.5	0.272140883	-0.00001	0.115998820	0.00001
1.0	0.4	2.0	1.0	0.211583283	-0.00000	0.154257635	-0.00000
1.0	0.4	2.0	1.5	0.161175109	0.00000	0.157128783	-0.00000
1.0	0.4	2.0	2.0	0.124232950	0.00007	0.150570175	0.00001
1.0	0.4	2.0	2.5	0.097343840	-0.00010	0.140925752	-0.00021
1.0	0.4	2.0	3.0	0.077581483	0.00179	0.132617801	-0.00091

TABLE 6

ELECTRIC FIELD

SX	SY	X	Y	EX	% ERROR	EY	% ERROR
1.0	0.4	2.5	0.0	0.255380582	-0.00002	0.000000000	-100.00000
1.0	0.4	2.5	0.5	0.235696219	0.00001	0.070498214	-0.00000
1.0	0.4	2.5	1.0	0.196443699	-0.00000	0.105485694	-0.00002
1.0	0.4	2.5	1.5	0.159111839	-0.00000	0.118372085	0.00000
1.0	0.4	2.5	2.0	0.128500132	0.00000	0.121099091	0.00002
1.0	0.4	2.5	2.5	0.104321429	0.00009	0.118652820	-0.00001
1.0	0.4	2.5	3.0	0.085440407	0.00028	0.113704916	0.00037
1.0	0.4	3.0	0.0	0.209754810	-0.00000	0.000000000	**************
1.0	0.4	3.0	0.5	0.199112557	-0.00003	0.044057556	0.00029
1.0	0.4	3.0	1.0	0.175563185	-0.00002	0.072969967	-0.00007
1.0	0.4	3.0	1.5	0.149834080	-0.00003	0.088929371	0.00002
1.0	0.4	3.0	2.0	0.126241261	0.00001	0.096472539	0.00005
1.0	0.4	3.0	2.5	0.106009490	-0.00002	0.098689940	-0.00001
1.0	0.4	3.0	3.0	0.089196377	-0.00010	0.097716971	0.00002
1.0	0.6	0.0	0.0	0.000000000	**************	0.000000000	**************
1.0	0.6	0.0	0.5	0.000000000	**************	0.251874764	0.00007
1.0	0.6	0.0	1.0	0.000000000	**************	0.336072672	-0.00015
1.0	0.6	0.0	1.5	0.000000000	**************	0.302526407	0.00044
1.0	0.6	0.0	2.0	0.000000000	**************	0.249219066	-0.00009
1.0	0.6	0.0	2.5	0.000000000	**************	0.207429783	-0.00755
1.0	0.6	0.0	3.0	0.000000000	**************	0.176845603	-0.03087
1.0	0.6	0.5	0.0	0.164874770	0.00000	0.000000000	-100.00000
1.0	0.6	0.5	0.5	0.145901460	-0.00004	0.233884735	0.00005
1.0	0.6	0.5	1.0	0.106304471	0.00027	0.315178253	-0.00010
1.0	0.6	0.5	1.5	0.071498689	-0.00142	0.287705194	0.00015
1.0	0.6	0.5	2.0	0.048741173	0.00038	0.240139119	-0.00000
1.0	0.6	0.5	2.5	0.034724766	0.03444	0.201801321	0.00238
1.0	0.6	0.5	3.0	0.025790459	0.12483	0.173179973	0.01812

TABLE6

ELECTRIC FIELD

SX	SY	X	Y	EX	% ERROR	EY	% ERROR
1.0	0.6	1.0	0.0	0.271803241	0.00000	0.000000000	-100.00000
1.0	0.6	1.0	0.5	0.242906844	-0.00002	0.188703824	0.00003
1.0	0.6	1.0	1.0	0.181753941	0.00013	0.261939604	0.00000
1.0	0.6	1.0	1.5	0.126428332	-0.00035	0.249026138	-0.00017
1.0	0.6	1.0	2.0	0.088780843	0.00000	0.215778381	0.00005
1.0	0.6	1.0	2.5	0.064650071	-0.00768	0.186335372	0.00278
1.0	0.6	1.0	3.0	0.048770912	-0.05124	0.162924077	0.00039
1.0	0.6	1.5	0.0	0.303339440	-0.00000	0.000000000	-100.00000
1.0	0.6	1.5	0.5	0.275378373	-0.00001	0.135462219	0.00001
1.0	0.6	1.5	1.0	0.214739034	0.00003	0.197301241	0.00003
1.0	0.6	1.5	1.5	0.157115771	0.00006	0.199752389	-0.00011
1.0	0.6	1.5	2.0	0.115266566	-0.00003	0.183030311	0.00000
1.0	0.6	1.5	2.5	0.086707105	-0.00128	0.164575290	-0.00150
1.0	0.6	1.5	3.0	0.066963058	0.00956	0.147991123	-0.00467
1.0	0.6	2.0	0.0	0.282156579	-0.00001	0.000000000	-100.00000
1.0	0.6	2.0	0.5	0.261222310	0.00000	0.090068744	0.00001
1.0	0.6	2.0	1.0	0.214151127	0.00000	0.139561497	0.00001
1.0	0.6	2.0	1.5	0.166213837	0.00003	0.152477210	0.00001
1.0	0.6	2.0	2.0	0.128264532	0.00001	0.149130367	0.00000
1.0	0.6	2.0	2.5	0.100232132	0.00071	0.140592666	-0.00002
1.0	0.6	2.0	3.0	0.079622790	0.00015	0.130739950	0.00184
1.0	0.6	2.5	0.0	0.241345616	-0.00004	0.000000000	-100.00000
1.0	0.6	2.5	0.5	0.227984037	0.00003	0.058406584	-0.00011
1.0	0.6	2.5	1.0	0.196484279	-0.00002	0.096652159	-0.00003
1.0	0.6	2.5	1.5	0.161550406	-0.00001	0.114002284	0.00003
1.0	0.6	2.5	2.0	0.131027680	0.00002	0.118915859	0.00003
1.0	0.6	2.5	2.5	0.106438243	-0.00003	0.117594879	0.00006
1.0	0.6	2.5	3.0	0.087104185	-0.00034	0.113259116	-0.00023

TABLE 6

ELECTRIC FIELD

SX	SY	X	Y	EX	% ERROR	EY	% ERROR
1.0	0.6	3.0	0.0	0.201971633	0.00000	0.000000000	-100.00000
1.0	0.6	3.0	0.5	0.194069918	-0.00001	0.038752893	0.00002
1.0	0.6	3.0	1.0	0.174412869	0.00001	0.067916917	0.00001
1.0	0.6	3.0	1.5	0.150542149	-0.00004	0.085503160	0.00013
1.0	0.6	3.0	2.0	0.127529111	-0.00000	0.094297281	0.00003
1.0	0.6	3.0	2.5	0.107355661	-0.00001	0.097379142	-0.00000
1.0	0.6	3.0	3.0	0.090404848	0.00005	0.096973417	-0.00000

3 Recreated Table 1, WEXCT

4 URL link to refactored Fortran code

For more information about complex error function fortran code, click on the following link: FortranCode

Table 1: WEXCT

wr	wi	zr	zi
0.00	0.00	1.000000000000000	0.000000000000000
0.00	0.50	0.615690177904398	0.000000000000000
0.00	1.00	0.427583868928526	0.000000000000000
0.00	1.50	0.321584289440764	0.000000000000000
0.00	2.00	0.255402534123747	0.000000000000000
0.00	2.50	0.210849597766694	0.000000000000000
0.00	3.00	0.179119883122594	0.000000000000000
0.50	0.00	0.778800783071405	0.478925159573737
0.50	0.50	0.533156591133704	0.230488280661727
0.50	1.00	0.391234138767048	0.127202207682148
0.50	1.50	0.303355049877176	0.077851740528118
0.50	2.00	0.245273758511286	0.051516615437352
0.50	2.50	0.204695834494986	0.036175795560759
0.50	3.00	0.175013660760233	0.026623117998033
1.00	0.00	0.367879441171442	0.607157688945692
1.00	0.50	0.354900293344541	0.342871755762348
1.00	1.00	0.304744150133933	0.208218830492453
1.00	1.50	0.257128339312765	0.135242331276541
1.00	2.00	0.218490958441978	0.092999718081428
1.00	2.50	0.188143831076063	0.067039707747043
1.00	3.00	0.164303072898013	0.050209338851178
1.50	0.00	0.105399224561864	0.483227316693744
1.50	0.50	0.196636025659636	0.337720324630445
1.50	1.00	0.211836548282862	0.233170966384206
1.50	1.50	0.201115136771154	0.164348461581649
1.50	2.00	0.183335452017590	0.119298431859613
1.50	2.50	0.165137393190194	0.089217522723435
1.50	3.00	0.148606780170188	0.068580103542200
2.00	0.00	0.018315638888734	0.340026207603973
2.00	0.50	0.103358822002007	0.284785879309291
2.00	1.00	0.140239573816261	0.222213438219392
2.00	1.50	0.150415415022916	0.170371132299577
2.00	2.00	0.147952737100350	0.131179589036453
2.00	2.50	0.140219456205913	0.102329400148157
2.00	3.00	0.130759301196336	0.081113815128269
2.50	0.00	0.001930454136228	0.251723017570233
2.50	0.50	0.058437471245225	0.232420429753165
2.50	1.00	0.093750740957915	0.198307111906521
2.50	1.50	0.111233455711503	0.163236744644939
2.50	2.00	0.117238559619234	0.132719893761845
2.50	2.50	0.116737205072723	0.107908588497039
2.50	3.00	0.112877805272293	0.088282913440885
3.00	0.00	0.000123409804087	0.201157191681505
3.00	0.50	0.037126355895963	0.192983630876662
3.00	1.00	0.065317757536137	0.173918193589552
3.00	1.50	0.083209506894530	0.150879672266300
3.00	2.00	0.092710730944052	0.128316849682324
3.00	2.50	0.096393255412096	0.108249277644066
3.00	3.00	0.096402467057972	0.091236235688546