“Rainbow” Storage Ring Isotope and Power Production

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Low-Z nuclear isotope pairs, say helium-3 (h) and deuterium (d), having more or less the same charge to momentum ratios, but different velocities, can co-circulate in the same “circular” storage ring provided that the (predominantly magnetic) bendng force is augmented by weak superimposed electric bending. This takes advantage of the different velocity dependence of magnetic and electric bending forces. With a helion to deuteron velocity ratio $v_h/v_d = 8/7$, a helion bunch on every eighth turn will “lap” (and pass through) a deuteron bunch that is just completing its seventh full turn. With judicious adjustment this will always occur at the same ring location and the rest mass of the h, d system will barely exceed, for example, the threshold of an $h + d \to \alpha + p$ channel. To the extent that only two body final states are produced, and spin dependence is insignificant, this output channel can be described as a “rainbow” circular ring (or rather cone) especially of the more massive (α-particles) emerging from, and centered on, the common beam axis. This phenomenon has not been observed previously in nuclear scattering since it requires a “rear end” collision in which a more massive but faster moving particle overtakes a lower mass but slower moving particle.

In spite of its large total cross section (greater than one barn) the $h + d \to \alpha$ channel proposed here is diffractive. Though the proton cone angles may be substantial, being “monochromatic” the output beam “emittances” (appropriately-defined) are “reasonably small”. In principle, the final stae beams can be separated and their energies recovered with high efficiency without violating Liouville’s principle. Of all the nuclear candidates for fusion power this $h+d$ channel is (marginally) the most exothermic as well as being “aneutronic”. Conventional thermonuclear power generation is restricted to the deuteron/triton channel $d + t \to n + \alpha$ process, because of its relativel low nuclear ignition temperature. Though almost as exothermic, this channel produces neutrons in the final state. These neutrons will inevitably cause unacceptably great radiation damage to the delicate apparatus needed to produce the requisite high ignition temperature.

This paper describes a practical and inexpensive conversion of the COSY storage ring into a form capable of investigating the practicality of the $d + h \to \alpha + p$ rainbow mechanism for the purpose of power generation.

Practical apparatus is described, with detailed kinematic parameters, for nuclear transmutation power production at the COSY lab in Juelich, Germany, using co-traveling 25 MeV deuteron and 49 MeV helion beams and capable of producing detectable two body $p+\alpha$ final states at exothermic power level of 26 KW. Though scarcely at the electric power level required to serve even a medium sized city, the exothermic electric power produced from the COSY storage ring could then serve a village of 26.

A sequential program at COSY is described, culminating in a test of time reversal invariance of nuclear physics by comparing the processes $d + h \to \alpha + p$ with its inverse $\alpha + p \to d + h$, both measured at COSY.

CONTENTS

I. INTRODUCTION

This paper is a companion to three papers with related subject matter [1][2][3].

The proposed novel storage ring approach is to have both "incident" beams travel in the same direction in the ring. This is made possible by superimposing electric field upon the magnetic bending at the radius required for beam storage. For two different particle types to travel simultaneously in the same direction relies on the different velocity dependence of electric and magnetic bending. In a storage ring it is easy to raise the "effective collision temperature", (i.e. center of mass energy) to a value far too high to be produced thermally because of a Maxwell-Boltzmann factor exceeding 10^{-6}. Since co-traveling beam bunches of different beam type have different revolution frequencies, fast bunches of one type periodically lap (i.e. pass through) slow bunches. By judicious tuning the incident center of mass energy can be adjusted to a value such as 300 KeV, just high enough to surmount the Coulomb barrier and cause resonant transmutation. Since the center of mass is moving forward in the laboratory, essentially all final state particles emerge in the forward hemisphere; here their detection is easy, and thir energy can, in principle, be efficiently recovered, possibly even in the form of electrical power.

The thermonuclear nuclear fusion process conventionally considered to be most promising for commercial power generation uses deuterium/tritium fuel. Recently, at the UK-based JET facility, Gibney et al.[4] have produced 59 MJ output power during a 5 s burst, with efficiency (meaning power out divided by power in) of 0.33. Other thermonuclear projects have been similarly promising.

Figure 1, copied from reference [5], shows parameters for nuclear process considered in this paper. It is the small value of the Lawson[6] ignition temperatures that has motivated the choice, by JET and others, of deuterium-tritium as the fuel of choice. Translating process labels to our notation, the exothermic energies available from these and the D-D channel[1] are,

$$ {}^2_1D - {}^3_1T: \qquad d + t \rightarrow n + \alpha, \quad (+17.6\,\mathrm{MeV}) \tag{1}$$
$$ {}^2_1D - {}^3_2He: \qquad d + h \rightarrow p + \alpha \quad (+18.3\,\mathrm{MeV}) \tag{2}$$
$$ {}^2_1D - {}^2_1D: \qquad d + d \rightarrow n + h \quad (+3.2\,\mathrm{MeV}) \tag{3}$$

The location of the minimum value of the lowest (blue) curve confirms the deuterium–tritium ignition temperature to be much lower than the deuterium-helion case.

Though seemingly disadvantaged by the data showing high ignition temperature and small cross section, this paper concentrates on the ${}^2_1D - {}^3_2He$ process.[2] The presence of neutrons as output of the process causes this advantage not to be dispositive. No matter how carefully these neutrons are controlled, they cause radiation damage to the power producing apparatus. There is no known solution to this problem. Radiation damage seems certain to limit the useful life of the apparatus unacceptably. Paradoxically, success of the deuteron-triton process in producing copious neutrons guarantees its failure as a commercially-realistic power source.

The $d + h \rightarrow p + \alpha$ process is referred to as "aneutronic", meaning it produces no neutrons. The secondary products are chemically inert and, being charged, are amenable to being steered and efficiently captured. The paper explains how deuterium-He^3 transmutation can be readily produced, detected and measured in a proposed COSY facility.

The seeming advantages of the D-T proces compared to the D-He3 process are entirely due the thermonuclear process, as contrasted to storage ring induced nuclear transmutation. The production is essentially non-relativistic in both cases. For storage ring nuclear fusion the characteristic energy is of order $1\,KeV$, at the right edge of Figure 1.

II. CO-TRAVELING NUCLEAR TRANSMUTATION KINEMATICS

The basic strategy for producing and detecting storage ring nuclear transmutation is for helion bunches to be traveling in the same direction and on the same orbit, but faster than, deuteron bunches. Each helion bunch is synchronized to pass through, on every 8-th turn, a deuteron bunch which has just completed 7 complete turns, always at the same longitudinal ring location. All collisions in this paper are "rear-end" collisions of this sort[3].

This is the only experimentally practical way of achieving incident co-traveling beams with (moving frame) energies in the 0.1 to 1.0 MeV energy range that is optimal

[1] Though the D-D case (3) is made impractical for power generation by its low exothermic energy, energy recoverability can be investigated using only deuteron beams presently available at COSY.

[2] Fuel availability for the D-T and D-He3 channels is more or less equivalent. He3 fuel is already available: lithium nuclei absorb neutrons and split into helium-4 and tritium, which decays into helium-3 with a half-life of 12.3 years. Helium-3 can therefore be produced by simply storing the tritium until it undergoes radioactive decay. This was not a channel Lawson favored; by his criterion it is strongly disfavored.

[3] To limit confusion, with unequal masses, for labeling convenience we assume beam 1 is the "fast" beam, $\beta_1 > \beta_2$. Curiously this also means $m_1 > m_2$; the faster particles are heavier. We also label the more massive final state particle as particle 3. With both incident beams travelling in the same (forward) direction, the center of mass (c.m) moves forward, and one expects all produced particles to move forward in the lab. But, for exothermic scatters, this is not necessarily the case; the leading (lighter) particle can emerge traveling backward in the lab. This can be the result of a kind of "slingshot" mechanism in which energy is temporarily packaged as rotational energy in a compound nuclear state.

Candidate reactions

Reaction	Ignition T [keV]	Cross-section $\langle\sigma v\rangle/T^2$ [$m^3/s/keV^2$]
${}^2_1D$-${}^3_1T$	13.6	1.24×10^{-24}
${}^2_1D$-${}^3_2He$	58	2.24×10^{-26}

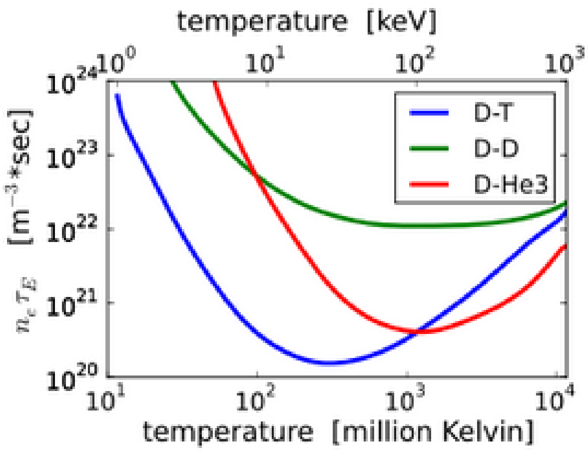

FIG. 1. Copied from the 2021 Wikipedia article, "Aneutronic fusion" for nuclear transmutation processes: **Top:** Comparison of potential thermonuculear fuels; **Bottom:** Plots of the "Lawson triple product" of density, temperature, and confinement time (an empirical expression for thermonuclear effectiveness) as a function of temperature, in Kelvin units (bottom scale) and KeV (top scale), for three nuclear fusion channels. All three of these channels can be studied in detail in storage rings. Though impractical for power generation, the d,d channel can be readily investigated at COSY.

for investigating nuclear collision processes. This capability has only been developed recently, with the invention of storage rings capapable of simultaneous storage of two different beam particle types. This capability is made possibe by the superposition of (weak) electric bending on dominant magnetic bending[1][2][3].

Kinematic details are shown in Table I. For these "rear-end collisions" both beams have conveniently high kinetic energy in the laboratory: K_h=49.5 MeV, K_d = 24.65 MeV. These energies have been adjusted just right for the center of mass (c.m) energy M^*=4.68430 MeV to surmount the Coulomb barrier and to resonantly produce the $d + h \rightarrow p + \alpha$ reaction. The c.m momenta are equal and opposite, the c.m velocity in the lab is β^*=0.1680.

For 2-particle generality we introduce subscripts 1,2→3,4;

$$d_1 + h_2 \longrightarrow p_3 + \alpha_4,$$

which enables convenient conversion to 2 particle collisions from any incident state pair drawn from proton (p), neutron (n), deuteron (d), triton (t), helion (h), alpha (α), or Li-6 (l) which, along with higher mass nuclei,

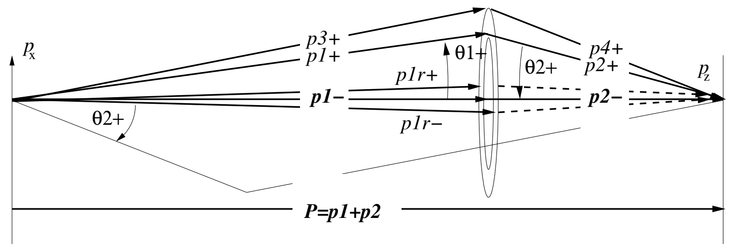

FIG. 2. Laboratory frame momentum vector diagram. The vector $\mathbf{P} = \mathbf{p_1} + \mathbf{p_2}$ is the sum of the lab momenta of one particle from beam 1(h) and one from beam 2(d). With increasing storage ring bending field and matching energy increase, starting with elastic Rutherford scattering, followed by the $\alpha + p \rightarrow$ inelastic exit channel, various inelastic thresholds are passed. Helion orbit directions are shown above the beam axis, deuteron directions could be displayed below. Instead they need to be inferred using the parallelogram construction. This description is purely classical, with no need even for relativity or Lorentz transformation. The incident state is one-dimensional. Any single scattering defines a two dimensional output plane; this space becomes three dimensional, with independence from azimuthal Φ-coordinate shown, for example, in Figure 3, guaranteed by symmetry. Output channels can be rotated azimuthally and arbitrarily around the beam axis as shown, with P held fixed, producing the conical surfaces referred to in the text; the cone angles increase with increasing energy. (Some *ad hoc* notation appearing in this figure is neither defined nor used in the text.)

are candidates for being stored simultaneously, with subsequent scattering into any other two-body final stete combinations.

Labeling center of mass momentum, energy and mass-squared (in natural units) as $P^* = p_1 + p_2$, $E^* = \epsilon_1 + \epsilon_2$, and $M^{*2} \equiv s = E^{*2} - P^{*2}$, for incident beam masses m_1 and m_2, the incident 4-vectors are $(\epsilon_1, \mathbf{p_1})$ and $(\epsilon_2, \mathbf{p_2})$ the center of mass velocity is given by

$$\beta^* = \frac{P^*}{E^*} = \frac{p_1 + p_2}{\epsilon_1 + \epsilon_2}, \tag{4}$$

and the gamma factor is

$$\gamma^* = \frac{1}{\sqrt{1 - \beta^{*2}}} = \frac{\epsilon_1 + \epsilon_2}{\sqrt{\big((\epsilon_1 + \epsilon_2)^2 - (p_1 + p_2)^2\big)}}. \tag{5}$$

Limiting considertaion to operation at low enough beam energy for two-body production to predominate, from these equations we obtain three critical energy values

$$\sqrt{s} = M^* = \frac{\epsilon_1 + \epsilon_2}{\gamma^*}, \tag{6}$$

$$Q_{1,2} = M^* - (M_1 + M_2), \tag{7}$$

$$Q_{3,4} = M^* - (M_3 + M_4). \tag{8}$$

with $s = M^{*2}$ being the c.m mass-squared, already introduced. $Q_{1,2}$ is the excess energy, over and above the sum

bm 1	beta1	Qs1	KE1 MeV	E0 MV/m	cB0 mT	beta2	Qs2	KE2 MeV	beta*	gamma*	M* GeV	Q12 KeV	7*bratio	bm 2
h	0.1769	-0.665	45.000	4.64395	-3.30871	0.1547	-1.096	22.864	0.16804	1.01443	4.68430	292.18399	8.00287	d
h	0.1788	-0.666	46.000	4.74965	-3.36417	0.1564	-1.097	23.373	0.16986	1.01475	4.68430	298.53366	8.00219	d
h	0.1807	-0.666	47.000	4.85547	-3.41966	0.1581	-1.097	23.882	0.17166	1.01507	4.68431	304.87722	8.00151	d
h	0.1826	-0.666	48.000	4.96139	-3.47518	0.1597	-1.097	24.391	0.17343	1.01539	4.68432	311.21468	8.00083	d
h	0.1844	-0.666	49.000	5.06742	-3.53074	0.1613	-1.098	24.901	0.17519	1.01571	4.68432	317.54605	8.00015	d
h	0.1862	-0.666	50.000	5.17355	-3.58633	0.1630	-1.098	25.410	0.17693	1.01603	4.68433	323.87133	7.99947	d
h	0.1880	-0.667	51.000	5.27980	-3.64198	0.1645	-1.099	25.920	0.17865	1.01635	4.68433	330.19053	7.99879	d
h	0.1898	-0.667	52.000	5.38616	-3.69767	0.1661	-1.099	26.429	0.18035	1.01667	4.68434	336.50366	7.99811	d
h	0.1916	-0.667	53.000	5.49263	-3.75342	0.1677	-1.099	26.939	0.18204	1.01699	4.68435	342.81073	7.99744	d

TABLE I. Kinematic parameters for the He3-D channel (same as D-He3 channel except the helion beam is treated as primary) fine-grain scan with the h beam spin tune Qs1 allowed to deviate from 0 while setting KE1=49 MeV helion energy and 25 MeV deuteron energy. The electric/magnetic field ratio then produces perfect β_h/β_d=8/7 velocity ratio so that, for every 7 deuteron turns, the helion makes 8 turns. Notice, also, the approximate match of Q12=317 KeV in this table, with $V_{d,He3}$=313.1 KeV evaluated using Eq. (15). This matches the available kinetic energy to the value required to surmount the repulsive Coulomb barrier.

bm 1	m1 GeV	G1	q1	beta1	Qs1	KE1 MeV	E0 MV/m	cB0 mT	m2 GeV	G2	q2	beta2	KE2 MeV	bratio	Qs2	bm
h	2.8084	-4.1842	2	0.18440	-6.662e-01	49.0000	5.0674	-3.5307	1.8756	-0.1430	1	-0.18144	31.6569	-0.98396	-8.936e-01	d
h	2.8084	-4.1842	2	0.18440	-6.662e-01	49.0000	5.0674	-3.5307	1.8756	-0.1430	1	0.16135	24.9007	0.87498	-1.098e+00	d

bm 3	m3 GeV	G3	q3	Q34_min MeV	KE3_eff MeV	KE4_eff MeV	E0 MV/m	cB0 mT	m4 GeV	G4	q4	bm
α	3.7274	0.0000	2	18.67	3.78	14.89	5.0674	-3.5307	0.9383	1.7928	1	p

TABLE II. **Top:** Table showing parameters for co- and contra-travelling helion and deuteron beams, for identical settings of magnetic and electric bending, appropriate, respectively, for head-on and rear-end collisions.
Bottom: Table showing matching parameters for final state α-particle and proton final state channel.

of incident rest mass energies. and $Q_{3,4}$ is the corresponding quantity for final state particles 3 and 4; oth need to be positive for both channels to be "open", but are bounded from above to the extent possible, to close undesirable higher energy channels.Subtracting these equations produces a measure of resonance separation.

$$\Delta M = Q_{1,2} - Q_{3,4} = (M_3 + M_4) - (M_1 + M_2). \quad (9)$$

One then anticipates two resonances, closely spaced by energy difference ΔM, with one corresponding to elastic scattering in the (1,2) channel, the other to resonant transmutation to the (3,4) channel.

Historically, with deuteron beam incident on Helium-3 target particles fixed in the laboratory, the threshold deuteron energy required to produce a massive α particle, would be far above the threshold of many competing channels. For most of these channels the exit direction of the proton originally belonging to the deuteron would be strongly peaked forward with the remaining system recoiling more or less transversely in the laboratory. As emphasized previously, it is not experimentally practical to measure ultra-low energy elastic scattering of two light nuclei with their c.m nearly at rest in the laboratory.

In the proposed $h + d \rightarrow p + \alpha$ configuration the c.m is moving directly forward, the incident deuteron energy is necessarily above the elastic scattering threshold, and minimally above the α-particle production threshold. This maximizes the exothermically available energy and minimizes the influence of undesirable channels.

III. DEUTERON STRIPPING PROCESSES

The $h + d \rightarrow \alpha + p$ process is perhaps the most favorable example of "stripping" or "direct nuclear" reactions, which are well described by semi-classical kinematic reasoning. This approach, which applies best to deuteron processes (because of their large radii and weakly coupled proton and neutron) was pioneered by Butler[7] in 1951, and 1957[11].

The stripping analysis bears directly on the recovery efficiency of energy transferred from helion to α-particle associated with the huge, 28 MeV, α-particle binding energy. To be commercially viable the exothermic energy released in the nuclear transmutation needs to be recovered with high efficiency. Most of this energy simply adds to the helion energy which can increase *at most*, from 49 MeV to 77 MeV, in any single event. For economically efffective electrical power generation it is essential to recover most of the 77 MeV of outgoing energy, rather than just the 18.3 MeV, of exothermically released energy. It is furthermore economically essential for most of this energy to be available as *electrical* power, rather than *thermal* energy. With energy recovery linac (ERL) technology, it is theoretically possible to recover most of this energy in the form of electrical energy for every captured α-particle. A significant fraction of the energy is carried by final state protons. It may be acceptable for this energy be recovered as heat.

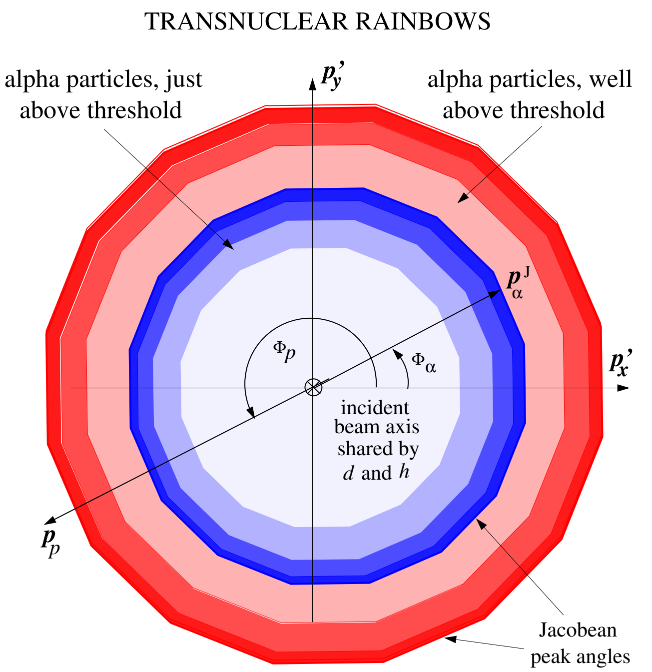

FIG. 3. Transnuclear "rainbows" produced in the reaction $d + h \to p + \alpha$. The shading intensity represents scattered beam intensities through a transverse plane orthogonal to the incident beam axis. Only the heavier exiting particle (α) produces a clean rainbow. As the incident beam energies increase above threshold the rainbow radius increases. Not drawn to scale, the cone angles should be labelled ϑ_3^J and be read from a table in the text. Azimuthal angle Φ_α or Φ_p orients the plane defined by the beam axis along with either one of the final state momenta $\mathbf{p}'^J_\alpha$ or $\mathbf{p}'_p$. The former is labelled with superscript "J" (for Jacobean peak) defining the rainbow cone limit. The subscript labels are specialized to particle types for the main final state channel under discussion. The axis labels are transverse Cartesian c.m momentum coordinates.

IV. QUANTUM MECHANICAL DETERMINATION OF NUCLEAR SCATTERING PATTERNS

Any calculation of the probabilites of conversion from one set of nuclear particle species into another is necessarily quantum mechanical (QM). But, for the initial state collinearity and two-body final state limitation we impose, this calculation is greatly simplified by symmetry and kinematic requirements. For the basic process described in this paper much sophisticated QM machinery is superfluous. Except for setting an overall cross section scale, which can be established eventually either experimentally or by sophisticated theory, only axiomatic quantum mechanic principles are required. The present paper amounts to a first cut design for performing spin-independent experimental measurements.

The proposed facility will have the capability of establishing incident state spin states and measuring final state polarizations. These can be investigated theoretically only by using the full QM machinery to calculate the relevant transfer matrices. None of this is covered in the present paper which, basically, accepts transfer matrices as being identity matrices. This reduces completion of the Fermi scattering formalism to a density of states calculation, with no need for more refined QM machinery.

As first formulated by Fermi[12], and clearly explained by Hagedorn[13], in a "The Statistical Theory" chapter, the distributions of multiparticle final states can be well approximated on a purely statistical basis, with no need for detailed transfer matrix determination.

With our two-body final state limitation, along with collinear incident beams, the two final state particle orbits lie in a plane containing the beam axis and defined by an azimuthal angle. By cylindrical symmetry it would be theoretically unnecesary even to label this azimuthal angle; in fact, in Figure 3, one such plane is labelled by either angle Φ_α or Φ_p.

Though the proposed apparatus will be capable of controlling the incident polarizations and measuring the scattered polarizations, for rate estimates it is adequate initially to treat all particles as unpolarized, assuming incident spins are unpolarized and final state spins are not measured. The only remaining purpose for any quantum mechanical wave function Ψ is to provide a probabilistic conversion from wave amplitude, via wave intensity, to "point" particle probability distributions. Such cross section distributions are illustrated by color intensities in Figure 3.

Within any one of the planes just defined, by momentum conservation, the two final state c.m particle momenta are labelled $\mathbf{p}'^J_\alpha$ and $\mathbf{p}'^J_p$ in the figure; their lengths appear different in this projection. The Fermi statistical model amounts to assuming that the angle of these two vectors (relative to the beam axis) is isotropic in the overall c.m. system.

Since the overall c.m is moving "forward" in the laboratory, these momenta will be forward-rotated, even to the extreme that the straight-backward c.m momentum will point straight-forward in the laboratory. Furthermore, the transverse components of every particle will be fairly small, meaning that all particles are contained within fairly acute "rainbow" cones, such as illustrated in the figure. The edges of these cones are determined by the Jacobean peaks corresponding to maximum transverse momentum in the c.m system[4]. (One cannot tell from the figure, how acute the cones are; only that the heavy mass cone angle is smaller than the lighter mass cone angle.)

There is a more serious issue associted with the discussion in the previous paragraph. To the extent the

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[4] There is one possible way in which the Fermi statistical assumption may not be quite correct. In principle, the maximum transverse momentum may not occur exactly at angle $\pi/2$ in the c.m system

radiation cones are "narrow", the trajectories will be more or less "paraxial", meaning, almost parallel. All resonantly-scattered, nuclear transmutated particles will certainly be sufficiently non-paraxial for easy extraction. But there is no possibility whatsoever, they will be sufficiently paraxial for direct capture into, for example, an energy recovery linac (ERL), which would be the natural way to recover the exothermically produced energy. In our case this energy is predominantly contained in the final state $\alpha-$particles.

Designers of *muon storage rings* are familiar with this problem. Experience with the muon g-2 anomalous magnetic experiment is especially relevant. The challenge is to transform the exciting $\alpha-$particle beam cone into a slender linear beam, while conserving emittaces without violating Liouville's theorem. If it were not for the fact that the *alpha*$-$particle energies and angles are correlated, this would be impossible; and perfect emittance conservation is experimentally unrealistic. Nevertheless, quite high capture efficiency may be achievable. If so, net positive energy extraction in the form of electrical power may be possible using ERL technology.

There is yet one further complication, not yet mentioned. Implicit in the transformation from c.m to laboratory discussed previously, is that the c.m backward and forward α-beams are superimposed (with different energies) in the laboratory. This will certainly complicate and reduce the energy recovery efficiency.

V. ESTIMATED RATE OF POWER GENERATION AT COSY

Important parameters, some discussed is subsequent sections, some obtained from Reva, et al.[8] using 1.7 GeV protons in the COSY ring, include

$$\begin{aligned}
f_{\rm sr} = &\ \text{storage ring revolution frequency} &= 10^6\,\text{Hz},\\
N_d, N_h = &\ \text{numbers of stored particles} &= 10^{10}\,,\\
A_b = &\ \text{beam area } = (0.1\,\text{cm}\times 0.1\,\text{cm})^2 &= 10^{-2}\,\text{cm}^2,\\
\sigma = &\ \text{nuclear cross section} &= 10^{-24}\,\text{cm}^2,
\end{aligned} \tag{10}$$

The (deuterium) "target bunch nuclear opacity" $O_{\rm N} = N_d\sigma/A_b = 10^{-12}$ gives the fraction of bunch passages which result in a nuclear transmutation event. For "deuteron stripping processes" the cross section σ can be anomalously large, as first suggested by Openheimer and Phillips[9] and discussed in detail by Blatt and Weisskopf[10]. Though taken here to be 1 barn, the actual cross section can be even larger.

For the proposed α-particle "rainbow" production channel the bunch/bunch passage rate ratio (8/7) denomonator is $\eta_{\rm denom.} = 7$. As given in Eq. (3), the maximum possible energy extraction is $Q_{D-He3} = 18.3\,\text{MeV}$ per storage ring collision. With the parameters given so far the total output power, accessible as some combination of heat and electrical power to be determined, is

$$\begin{aligned}
P_{\rm out} &= N_h O_{\rm N_d}\frac{f_{\rm sr}}{\eta_{\rm denom.}} Q_{D-He3}\\
&= 10^{10}\times 10^{-12}\times\frac{10^6}{7}\times 18.3\\
&= 26\,\text{KW}.
\end{aligned} \tag{11}$$

According to a CERN estimate[14] based of a comparison of their power consumption during operations with that of Geneva, Switzerland, this is the average power used by 26 city dwellers. At the time of writing, many of the parameters are uncertain. But, unlike similar estimates of thermonuclear power generation, these uncertainties can be reduced to reliable values within, say, a year of careful design. There are various ways that can be considered to increas the power some way, such as inreasing the number of circulating bunches, or reducing beam area A_b. On the other hand, the electrical power energy recovery efficiency will certainlly be less than its conceptually maximal value of 100%.

There is, however, a more optimistic way of assessing the magnitude of $P_{\rm out}$. Defining n_1^α to be the production rate per second of α particles, and P_1^α the electric energy delivered per α-particle, one has

$$\begin{aligned}
n_1^\alpha = \frac{P_{\rm out}}{P_1^\alpha} &= \frac{2.6\times 10^5\,[\text{W}]}{0.092\times 10^9\,[\text{eV/s}]}\\
&= \frac{2.6\times 10^5\times 0.624}{0.092}\approx 10^6/s,
\end{aligned} \tag{12}$$

which can be iterpreted either as the number of α-particles produced per second and available for collection, or as the number available for detection in a physics experiment studying the validity of the formulation. Since there is no serious world shortage of α-particles, the former application is useless. But one million scatters per second is about one million times higher than would be needed to confirm the validity of the formulation.

There are (at least) two ways this factor of one million "bonus" can be expended. One way is to greatly simplify the injection proccess. This will abbreviate the time scale for implementation at COSY. With the "fast" beam injected using stripper injection, the "slow" beam bunches can be injected directly from the cyclotron, or vice versa, without the need for bunch acculator ring (BA).

Far more important is the impact on the importance of the program as "pure physics". According to time reversal invariance the process $\alpha + p \to h + d$ is essentially equivalent to the inverse process $h + d \to \alpha + p$. To perform this test at COSY will require development of an α-paricle beam, which is, presumeably, easier and quicker than the He3 beam that will eventually be needed. Both of these processes can thhen be precisely measured at COSY. This will enable a test of the time reversal invariance of nuclear scattering of unprecidented precision and credability.

One can envisage a sequential nuclear transmutation program at COSY, starting almost immediately and proceeding step by step to the eventual time reversal goal. Possible immediately, once electric field inserts in every COSY bend one can compare $d + p \to p + d$ with $p + d \to d + p$, two processes that "notationlly" appear identical, but differ as to which incident beam is slow. This will provie a test of time reveral; but will be controversial since the distinction between "foraward" and " time-reversed" processes is purely kinematic. Other (of several) possible sequential variants have already been mentioned.

Tests of time reversal symmetry at COSY are being described in greater detail in a paper that is under preparation.

VI. THERMONUCLEAR/STORAGE RING NUCLEAR FUSION COMPARISON

Assumed to be spherical, the A dependence of all nuclear radii are tolerably well fit by the empirical formula

$$R(A) = 1.41 \times 10^{-15}\, A^{1/3}\, \mathrm{fm}. \tag{13}$$

For cases of interest in the present paper this formula produces spherical radii R_p=3.404, R_n=3.404, R_d=4.289, R_d_Butler=5.0, $R\_tritium$=4.909, R_He3=4.909, R_alpha=5.404, all in Fermi units. Included in this list is an alternative (significantly different) deuteron radius used by Butler[7] in a historically significant paper explaining the curiously strong nuclear transmutation induced with incident deuterons. Treating both incident particles as perfect spheres, the minimum separation of initial state particles 1 and two is

$$R = R_1 + R_2, \tag{14}$$

a result that can be used to evaluate the height of the repulsive electric potential preventing closer approach of the incident paricles, which is given by

$$V_{1,2} = Z_1 Z_2 \frac{e^2}{4\pi\epsilon_0\, R} = Z_1 Z_2 \frac{e^2}{4\pi\epsilon_0} \frac{1}{R_1 + R_2}, \tag{15}$$

a formula easily altered to obtain $V_{3,4}$ which provides the corresponding barrier heights for final state particle pairs. Four examples are $V_{p,p}$=211.5, $V_{d,He3}$=313.1, $V_{p,alpha}$=327.0, $V_{d,t}$=156.5, all in KeV.

These threshold energies are commensurate with the excess energy values $Q_{1,2}$ or $Q_{3,4}$ defined in Eqs. (7) and (8). This is convenient for a semi-quantitative interpretation of Figure 1; the energies are fairly close to the Lawson criterion "ignition temperatures" given in Table 1. The shapes of these curves are specific to the temperature-dependent Maxwell-Boltzmann factor influence on thermonuculear nuclear transmutation (as largely cancelled by the Gamow tunneling factor). The ignition temparature accounts for the "penalty" imposed by the Boltzmann statistical energy distribution.

A semi-classical estimate of the maximum possible cross section for any single transmutation process, for maximum possible stripping enhancement is.

$$\sigma_{\max} = \pi (R_1 + R_2)^2 \tag{16}$$

Qualitatively, the fact that KeV is the "natural" unit for expressing c.m kinetic energies provides an explanation for why collisions between beams of different energy, both traveling in the same direction in a storage ring (at multi-MeV energies) produce scattering events which, if produced with fixed target in the laboratory would require an incident beam of unmanageably low (KeV-scale) energy. Any such beam would dissipate its energy suddenly upon entry to any dense target material at rest in the laboratory while vaporizing any target material encountered and producing negligible nuclear transmutation. This accounts for the absence, to date, of the detection of the diffractive scattering described in the present paper.

In any comparison between thermonuclear and storage ring production many significant factors are implicated. The aneutronic radiation damage advantage of the D-He3 storage ring process has already been mentioned. The relative efficiences of energy recovery enter directly into the degree to which energy invested in producing transmutation can be recovered, and the relative achievable production rates is another.

The storage ring has the advantage (already mentioned) that the energy is made easier to recover, because the energy is carried in forward-travelling charged particles. Important also, though, is the fact, already implied, that much more energy will have been invested, event by event, in producing transmutation in the storage ring than with thermal generation in the laboratory. For example, in the D-He3 process, the 18.43 MeV exothermic energy is significantly smaller than the 75 Mev that has been "invested" in accelerating the deuteron and helion that disappear in the transmutation. This introduces a "handicap factor" of almost five by which storage ring energy reconery has to be more efficient than thermonuclear energy recovery in order to be competetive. However, this penalty is small compared to the Boltzmann factor accompanying the brute force heating used in producing KeV particle energies with magnetic or inertial confinement, such as JET, ITER or laser/pellet heating.

As regards inducing nuclear transmutation, the storage ring has the advantage that almost every incident deuteron can, in principle, eventually produce a transmutation. This is in contrast to thermonuclear transmutation for which only that tiny fraction of deuterons at the maximum of a Maxwell-Boltzman statistical distribution have comparably good odds of producing transmutation. during the brief period (5 seconds in the case of the JET facility) while the temperature is high enough for deuteron energies to be just right to induce transmutation in a medium containing candidate tritons of "known" number density. Though statistically quite complicated, these probabilities can be evaluted by sophisticated computer programs, enabling the number of induced transmutations to be calculated. Since this number can also be measured, one can be confident the modeling is more or less correct.

The current status of storage ring transmutation is very different. With a 50 year head start, thermonuclear fusion can be calculated and has been measured. Storage ring transmutation can, at present, only be calculated. Fortunately the storage ring estimation is quite simple, with the calculation already well begun in the present paper.

VII. RESURECTION OF COSY AS CO-TRAVELING BEAM COLLIDER

Conversion of the COSY ring, primarily consisting of superimposing weak electric bending on the existing strong magnetic bending has been discussed in detail in references [1][2][3]. This is summarized pictorially in Figure 4. Kinematic plots exhiting the rainbow scattering patterns quantitatively are shown in Figure 5 and 6.

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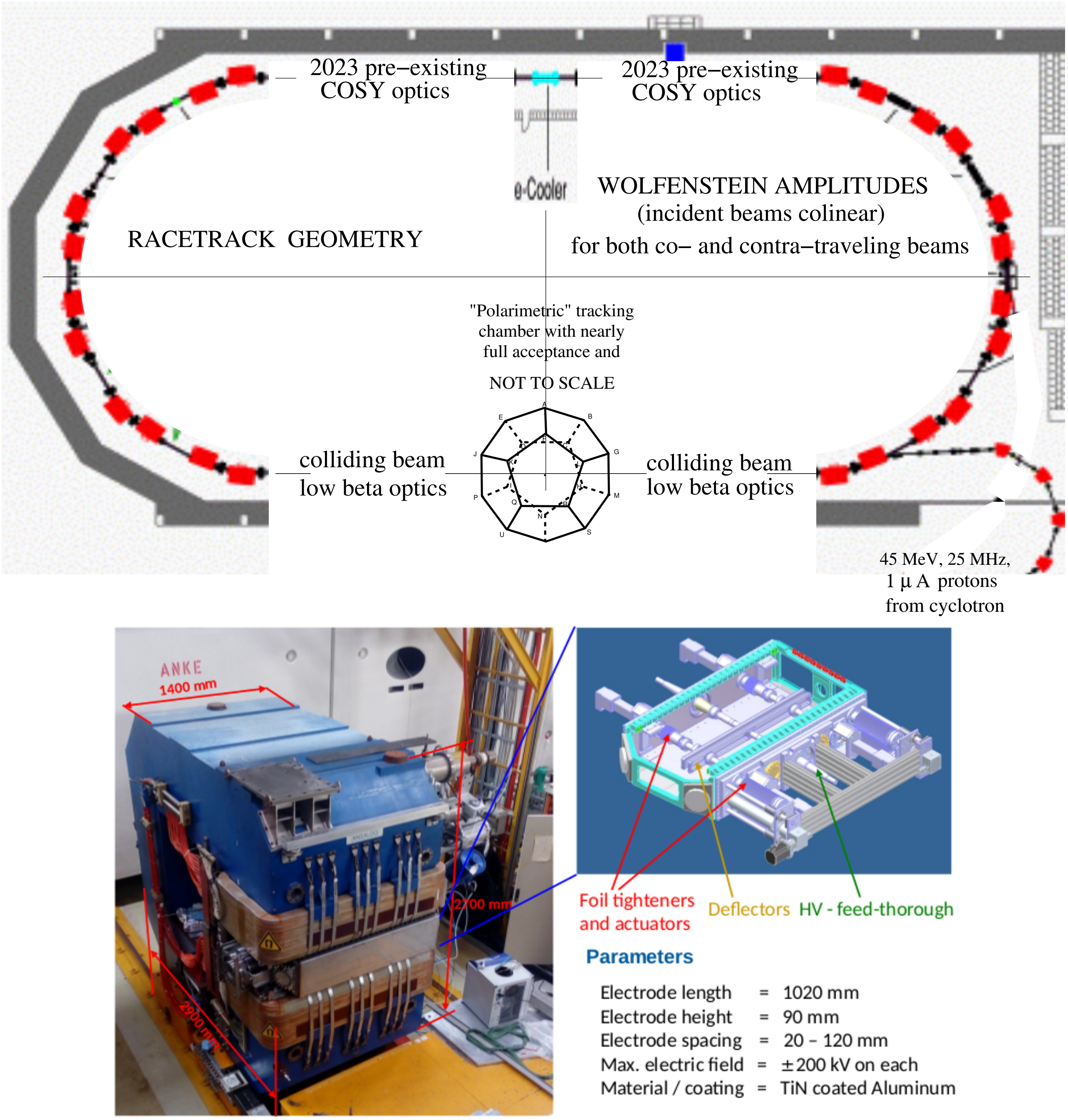

FIG. 4. **Left:** COSY layout after incorporation of the superimposed electric bending shown on the right. Otherwise the semicircular arcs are preserved as at present. Co-traveling beams of different velocity are stored simultaneously. **Right:** Grigoryev prototype electric field insert in COSY bending magnets up to 200 MeV “proton-equivalent” energy operation of COSY with electric bending superimposed on existing magnetic bending.

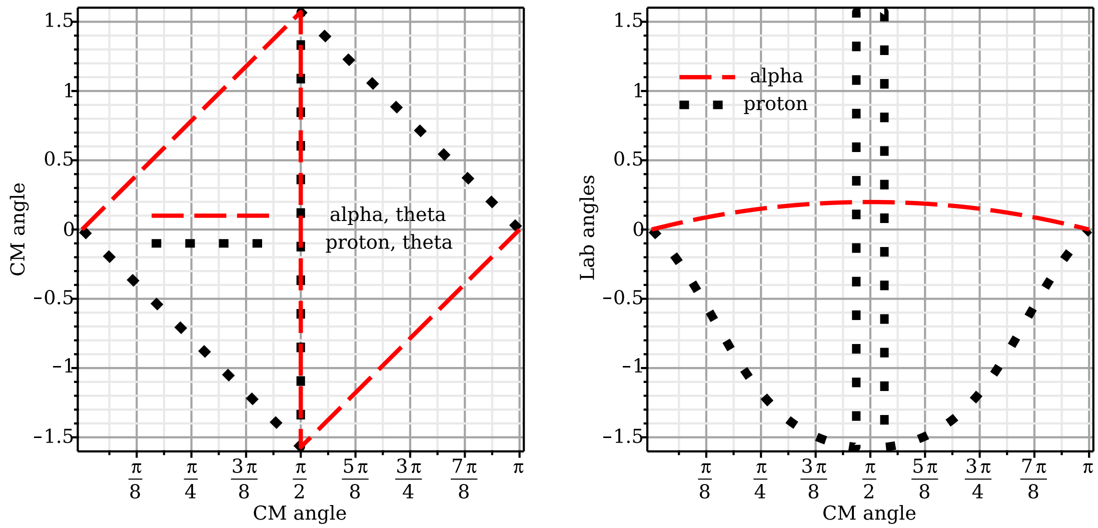

FIG. 5. **Left:** Kinematic plot for the $h + d \to \alpha + p$ channel: Center of mass (CM) angles for the $h + d \to \alpha + p$ channel, plotted "against themselves". Because the masses are unequal there is no symmetry about $\pi/2$. Directions below/above $\pi/2$ are "forward/backward". For just two-bodies in the c.m, if one particle type is forward the other must must be backward. **Right:** Plot showing relation between c.m angles and lab angles. The essential result is that produced α particles are contained within a cone half-angle of 0.2 radians which (at this energy) is the Jacobean maximum which establishes the outer edge of a rainbow shown in Figure 3.

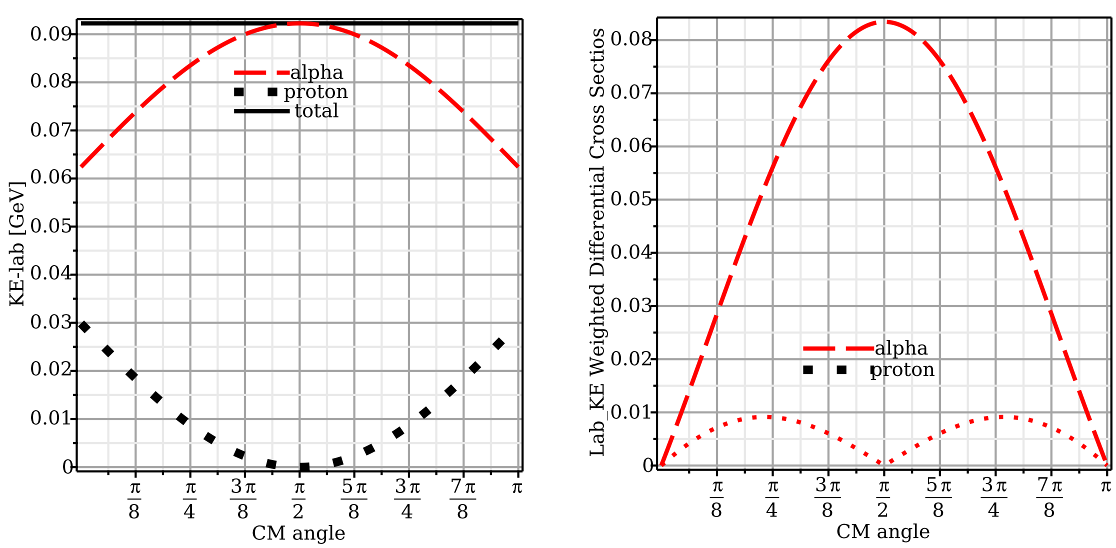

FIG. 6. **Left:** Kinematic plot for the $h + d \to \alpha + p$ channel: Plot showing relation between c.m angles and lab kinetic energies. Evidence that the process is exothermic is provided by the fact that the 90.2 GeV sum of kinetic energies exceeds the 49.0+24.9=73.9 GeV sum of incident energies by 16.3 GeV. **Right:**