a

Jason Tang

May 2023

1 Introduction to Gradient Vectors

1. Computing the partial derivatives of the gradient,

$$
\begin{aligned}
f(x,y) &= -(\cos^2(x)+\cos^2(y))^2 \\
&= -(\cos(x)\cdot\cos(x)+\cos(y)\cdot\cos(y))^2 \\
\frac{\partial f}{\partial x}&(-(\cos^2(x)+\cos^2(-20))^2) \\
&= -2(\cos^2 x+\cos^2(-20))\cdot -\sin(2x) \\
\frac{\partial f}{\partial y}&(-(\cos^2(y)+\cos^2(-20))^2) \\
&= -2(\cos^2 y+\cos^2(-20))\cdot -\sin(2y) \\
\frac{\partial^2 f}{\partial x \partial y} &= \begin{pmatrix} \frac{\partial f}{\partial x} = -2(\cos^2 x+\cos^2(-20))\cdot -\sin(2x) \\ \frac{\partial f}{\partial y} = -2(\cos^2 y+\cos^2(-20))\cdot -\sin(2y) \end{pmatrix}
\end{aligned}
$$

2. Gradient at $(\mathbf{x,y}) = (-20,-20)$:

$$
\begin{aligned}
\frac{\partial f}{\partial x}&(-(\cos^2(x)+\cos^2(-20))^2) \\
&= -2(\cos^2 x+\cos^2(-20))\cdot -\sin(2x) \\
&= -0.5 \\
\frac{\partial f}{\partial y}&(-(\cos^2(y)+\cos^2(-20))^2) \\
&= -2(\cos^2 y+\cos^2(-20))\cdot -\sin(2y) \\
&= -0.5
\end{aligned}
$$

$$\therefore \nabla f(x,y) = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$$