Assignment 6

Marks **Total marks for Assignment 6: 67**

[5] 1. Use the transformation $x = \frac{1}{3}(u-v)$, $y = \frac{1}{3}(2u+v)$ to evaluate exactly the double integral $\iint_D e^{x+y}\,dA$, where $D = \{(x,y) \mid 1 \le x+y \le 4,\ -4 \le y-2x \le 1\}$.

2. Let D denote the region in the first quadrant bounded by $x^2 - y^2 = 2$, $x^2 - y^2 = -2$, $xy = 1$ and $xy = 5$.

[4] a) Carefully sketch and shade the region D.

[4] b) Carefully sketch and shade the image of D under the transformation described by $u = x^2 - y^2$, $v = xy$.

[7] c) Use the transformation described in part (b) to evaluate $\iint_D x^2 y^2 \left(x^2 + y^2\right) dA$.

3. Evaluate the following line integrals exactly:

[6] a) $\int_C \sqrt{1+25x^3 y}\,ds$, where C is the curve $y = x^5$ from $(0,0)$ to $(2,32)$.

[9] b) $\int_C \vec{F} \cdot d\vec{r}$, where $F(x,y) = y^2\vec{i} + xy\,\vec{j}$ and C is the semi-circle $y = \sqrt{4-x^2}$ oriented in the clockwise direction.

[10] 4. Find the exact work done by the force field $F(x,y) = \left(2xe^{x^2} - 2y\right)\vec{i} + (2y-2x)\vec{j}$ in moving an object from the point $(1,0)$ to $(2,3)$.

[5] 5. Let D be the region bounded by the closed curve C. Use Green's Theorem to show that if the region D has constant density, the coordinates of the center of mass of the region D are $\bar{x} = \frac{1}{2A}\oint_C x^2\,dy$ and $\bar{y} = -\frac{1}{2A}\oint_C y^2\,dx$, where A denotes the area of D.

6. Given the vector field $F(x,.y,z)=\left\langle -2xy, z^2\cos\left(yz^2\right)-x^2, 2yz\cos\left(yz^2\right)\right\rangle$,

[4] a) Determine its divergence.

[6] b) Determine if this vector field is conservative.

[7] 7. Use Green's Theorem to evaluate $\int_C \left(\ln(x+2)+2y\right)dx-3x^2dy$, where C is the triangle with vertices $(1,0)$, $(2,3)$ and $(0,2)$, traversed in a clockwise direction.