Assignment 3

Marks **Total marks for Assignment 3: 79**

[3] 1. Using proper set notation, describe the domain of $f(x,y)=\cos\left(\frac{1}{\sqrt{y^2-x^2}}\right)$. Carefully sketch and shade this domain.

2. Given $g(x,y)=\begin{cases}\frac{x^3-x^2y}{x^2+y^2} & , \quad (x,y)\neq(0,0)\\ 0 & , \quad (x,y)=(0,0)\end{cases}$,

[2] a) Calculate $g_x(x,y)$ for all $(x,y)\neq(0,0)$. Simplify your answer.

[2] b) Use the limit definition of the partial derivative to determine $g_x(0,0)$.

[3] c) Using the definition of continuity at a point determine whether g is continuous at $(0,0)$. Show all work.

[5] d) Using the definition of continuity at a point determine whether g_x is continuous at $(0,0)$. Show all work.

[7] 3. Find the exact linearization of $h(s,t)=2st^3-s^2e^{st}-3$ at $(1,1)$ in simplified form and use it to estimate $h(0.95,1.03)$ to 5 decimal places.

[5] 4. The pressure, volume and temperature of an ideal gas are related by the equation $PV=8.31T$, where P is measured in kilopascals, V in liters and T in kelvins. Use differentials to find, to two decimal places, the approximate change in the volume if the pressure changes from $7kPa$ to $7.5kPa$ and the temperature changes from $300K$ to $295K$.

[5] 5. Let $z=f(2-3t,3+2t)$. Express $\frac{dz}{dt}$ in terms of partial derivatives of f.

[5] 6. Use the Implicit Function Theorem to find $\frac{\partial z}{\partial y}$, given $\sqrt{3y} - y\cos(xz) = 2x - 3z^2$. Give your answer in simplified form.

7. Given $f(x,y) = \frac{2y}{x+y}$,

[7] a) Find exactly all vectors of length 3 for which the directional derivative of f at $(-1, 2)$ in that direction is zero.

[2] b) Find the maximum rate of change of f at $(-1, 2)$ and the direction in which it occurs. Give an exact answer.

[3] c) Determine the exact rate of change of f at $(-1, 2)$ and in the direction towards the point $(3, 4)$.

8. Given $f(x,y) = x^3 + y^3 - 3xy + 1$,

[10] a) Find and classify the critical points of f.

[10] b) Find the absolute max and min of f on the triangular region with vertices $(0,2), (2,0), (2,2)$.

[10] 9. Use the method of Lagrange multipliers to find the max and min values of $g(s,t) = t^2 e^s$ given that $s^2 + t^2 = 3$. How are we assured that these extreme values exist?