Assignment 2

Marks **Total marks for Assignment 2: 68**

[5] 1. Neatly sketch the curve with the vector equation $\vec{r}(t) = \left\langle -t^2, 4, t \right\rangle$.

Identify any special points on its graph and indicate with an arrow the orientation of this curve.

[4] 2. At what point(s) (x, y, z) does the helix $\vec{r}(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$? Show all work and give an exact answer.

[4] 3. Find a single vector function, in simplified form, that represents the curve of intersection of the two surfaces $y^2 - z^2 = x - 2$ and $y^2 + z^2 = 9$.

4. Given $\vec{r}(t) = \left\langle e^t \sin t, e^t \cos t, 1 \right\rangle$, $t \geq 0$, find, in simplified form,

[5] a) The velocity function, the acceleration function and the speed function at time t.

[3] b) The exact arc length of the curve for $0 \leq t \leq \ln(4)$.

[5] c) A re-parametrization of the curve in terms of its arc length.

[5] 5. Find parametric equations for the tangent line to the curve $\vec{r}(t) = \left\langle t^2, \ln(2t), t \right\rangle$ when $t = \frac{1}{2}$.

6. Given $\vec{r}(t) = \langle \cos(\pi t), \sin(\pi t), t \rangle$, find exactly

[5] a) The unit tangent vector at $t = 2$.

[5] b) The unit normal vector at $t = 2$.

[3] c) The curvature at $t = 2$.

[15] 7. Determine the exact tangential component vector $a_T\vec{T}$ and the exact normal component vector $a_N\vec{N}$ of acceleration for the curve $\vec{r}(t)=\langle 2t^2,\ 2t-3\rangle$ at $t=2$ and verify that $\vec{a}=a_T\vec{T}+a_N\vec{N}$ at $t=2$.

8. Let $\vec{s}(t)=\langle 2, e^{2t}, 1\rangle$.

[5] a) Given $f(t)=\vec{r}(t)\cdot\vec{s}(t)$, where $\vec{r}(3)=\langle -1,9,9\rangle$, $\vec{r}'(3)=\langle 0,6,3\rangle$, find $f'(3)$ exactly.

[4] b) Find exactly: $\int_0^2 \vec{s}(t)\,dt$