THOMPSON RIVERS UNIVERSITY
OPEN LEARNING

THOMPSON RIVERS UNIVERSITY, OPEN LEARNING

MATH 2111 • CALCULUS III: MULTIVARIABLE CALCULUS

PRACTICE EXAMINATION

TIME ALLOWED: 3 Hours

TOTAL PAGES (Including This Page): 7

TOTAL MARKS: 100

MATERIALS PERMITTED:
- A basic scientific or graphing calculator

MATERIALS PROVIDED:
- Formula Sheet
- Two exam answer booklets

STUDENT: PLEASE COMPLETE THIS SECTION—*PRINT CLEARLY.*

Surname

____

First Name

____

Student Number

____

Open Learning Faculty Member's Name

____

Student's Signature (required)

____

Date

____

OPEN LEARNING FACULTY MEMBER: PLEASE COMPLETE THIS SECTION—*PRINT CLEARLY.*

Student's Mark________%

Name

____

I.D. No.

____

Signature

____

Date

____

Make and Model of Calculator Used

____

Calculator Checked □

Invigilator's Initials: ____________

Instructions

- Write your answers and complete solutions in the exam answer booklets provided.
- When you have finished, **return all papers, including the exam and all answer booklets (used and unused) in the envelope.** *Failure to do so may result in a fail grade.*

This Practice Exam is a sample final exam for the Math 2111 course. It follows the same format and includes the same instructions as in the final exam that you will write. At the end of this exam is a formula sheet. An exact copy of this formula sheet will be attached to the final exam you write. If a formula is not on this sheet, then you are responsible for knowing it.

At the end of each test question there is a reference to the text—in case you had difficulty with the problem. Please ignore this reference until after you have written the exam.

Exam Instructions:

- Give complete and detailed solutions to all problems. Marks may be deducted for insufficient details.
- Part marks are awarded for work that is substantially correct but contains errors.
- Incorrect work will be penalized, even if the final answer is correct.
- If you require space for rough work, please designate either the left page or right page in the exam booklet for this purpose and write "Rough Work" at the top of the page.
- Final numerical answers must be exact, unless otherwise specified.
- All graphs must be clearly presented with axes labeled, an appropriate scale identified on each axis and graphs labeled.
- Use correct symbolism and notation.
- Include units in the final answer, if applicable.
- Complete both parts of the examination.

Calculator: You will need a calculator in the exam. Either a scientific calculator or a non-symbolic graphing calculator is acceptable. Symbolic calculators or calculators with a computer algebra system are not allowed.

Good Luck!

PART A *(80 marks total)*

Attempt **all** eight questions in this section. *(10 marks each)*

1. Given the space curve $\vec{r}(t) = \langle t, \cos(at), \sin(at) \rangle$,

   a. Find the unit tangent vector $\vec{T}$ and the unit normal vector $\vec{N}$.
   (Reference: p. 872 and p. 882)

   b. Find the curvature κ.
   (Reference: pp. 879-880)

   c. Find the length of the curve from $t = 0$ to $t = 1$.
   (Reference: p. 878)

2. a. Find parametric equations for the line through $(1,1,-1)$ that is perpendicular to the plane $5x + 7y = 2 + 8z$.
   (Reference: p. 841)

   b. Find an equation for the plane determined by the parallel lines
   $\frac{x-1}{2} = y + 4 = \frac{z-1}{-1}$ and $\frac{x+2}{2} = y - 1 = \frac{z+2}{-1}$
   (Reference: p. 842 and p. 844)

   c. $\vec{u} = 2\vec{i} + 3\vec{j} + \vec{k}$, $\vec{v} = 4\vec{i} + \vec{j} - 2\vec{k}$ and $\vec{w} = 5\vec{i} + a\,\vec{j} + b\,\vec{k}$, a and b constants, are three vectors with their initial point or tail at the origin. Find a and b so that the heads of the three vectors lie along a straight line.
   (Reference: pp. 817-818)

3. a. Give detailed descriptions of all the horizontal and vertical traces of the surface $z^2 = x^2 + y^2$. Your descriptions must include the name and a general equation of the family of trace curves, and the coordinates of any special points on these trace curves. Sketch the graph of this surface and name it.
   (Reference: p. 854)

b. Given the surface, $z = e^x \sin(y)$, find an equation of the tangent plane to the surface at the point $(\ln(3), \frac{\pi}{2}, 3)$.

(Reference: p. 940)

4. a. Let $f(x,y) = x^2 + xy + y^2$. In what direction is $f(x,y)$ increasing most rapidly at the point $(1,1)$? What is the rate of change in this direction?

(Reference: p. 963)

b. At the point $(1,2)$ the function $w = g(x,y)$ has directional derivatives values of 2, in the direction towards $(2,2)$, and -2, in the direction towards $(1,1)$. Determine the value of the directional derivative at $(1,2)$ in the direction towards $(4,6)$.

(Reference: pp. 959-960)

5. a. Find and classify the critical points of $f(x,y) = 2x^3 - 24xy + 16y^3$.

(Reference: pp. 970-971)

b. Find the maximum and minimum values of $z = xy$ subject to $x^2 + y^2 = 1$.

(Reference: p. 982)

6. a. Carefully sketch the region in the plane bounded by the parabola $x = y - y^2$ and the line $x + y = 0$. Set up, but do not evaluate, an exact iterated integral expression for the area of this region

i. using the order $dx\,dy$

ii. using the order $dy\,dx$

(Reference: p. 1014 and p. 1018)

b. Evaluate $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} 2y^2\left(x^2+y^2\right)^2 \,dy\,dx$ by converting to polar coordinates.

(Reference: p. 1021 and p. 1023; section 10.3)

7. a. Carefully sketch the region between the surface $z=\sqrt{x^2+y^2}-1$ and $z=0$. Set up, but do not evaluate, exact triple integral expressions, in **both** rectangular and spherical coordinates, for the volume of this region.
(Reference: p. 854; pp. 1042-1043, p. 1046; pp. 1057-1059)

b. Find the mass of the solid tetrahedron in the first octant bounded by the plane $x+y+z=6$, if the density at any point in the region is proportional to the distance to the $xz-$plane.
(Reference: p. 1047)

8. a. Evaluate $\int_C (3x-2y+z)\,ds$, where C is the line segment from $(0,4,-4)$ to $(3,1,2)$.
(Reference: p. 1087 and p. 1091)

b. Use Green's Theorem to evaluate $\int_C \vec{F}\cdot d\vec{r}$, where $F(x,y)=\left\langle e^{-x}+y^2, e^{-y}+x^2\right\rangle$ and C is the closed curve consisting of the curve $y=\cos(x)$ from $(-\frac{\pi}{2},0)$ to $(\frac{\pi}{2},0)$ and the line segment from $(\frac{\pi}{2},0)$ to $(-\frac{\pi}{2},0)$.
(Reference: p. 1108)

PART B *(20 marks total)*

Attempt any two questions in this section. If you attempt more than two questions, be sure to indicate which two questions you want marked. Otherwise, the first two questions will be marked. *(10 marks each)*

9. a. Let $f(x, y) = g(r)$, where $r = \sqrt{x^2 + y^2}$. Show that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{d^2 g}{dr^2} + \frac{1}{r}\frac{dg}{dr}$$

(Reference: p. 950)

b. If $z^2 - \cos(x^2 z) = 2xy^2 + 3y$, find $\dfrac{\partial z}{\partial x}$.

(Reference: p. 953)

10. Determine if the vector field $F(x, y, z) = \left\langle ye^x, 2yz + e^x, y^2 \right\rangle$ is conservative. If so, find a potential function for F.
(Reference: pp. 1104-1105 (Example 5) and pp. 1116-1117)

11. Find the volume of the solid bounded above by the surface $z = 3 + r$ and below by the region in the $xy-$plane enclosed by the cardioid $r = 1 + \sin(\theta)$.
(Reference: p. 1046; pp. 1052-1053)

MATH 2111: Formula Sheet

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} \qquad s(t) = \int_a^t |\vec{r}\,'(u)|\,du = \int_a^t \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2 + \left(\frac{dz}{du}\right)^2}\,du$$

$$\kappa(t) = \frac{|\vec{T}\,'(t)|}{|\vec{r}\,'(t)|} = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3} \qquad \kappa(x) == \frac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{3/2}}$$

$$\vec{a} = v'\vec{T} + \kappa v^2 \vec{N} \qquad a_T = \frac{\vec{r}\,'(t) \cdot \vec{r}\,''(t)}{|\vec{r}\,'(t)|} \qquad a_N = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|}$$

$$L = \int_a^b \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\,d\theta \qquad A = \int_a^b \frac{1}{2} r^2\,d\theta = \int_a^b \frac{1}{2}\left(f(\theta)\right)^2\,d\theta$$

$$m = \iiint_E \rho(x, y, z)\,dV \qquad \bar{x} = \frac{M_{yz}}{m}, \quad \bar{y} = \frac{M_{xz}}{m}, \quad \bar{z} = \frac{M_{xy}}{m}$$

$$M_{yz} = \iiint_E x\,\rho(x, y, z)\,dV \qquad M_{xz} = \iiint_E y\,\rho(x, y, z)\,dV \qquad M_{xy} = \iiint_E z\,\rho(x, y, z)\,dV$$

$$I_x = \iiint_E \left(y^2 + z^2\right)\rho(x, y, z)\,dV \qquad I_y = \iiint_E \left(x^2 + z^2\right)\rho(x, y, z)\,dV$$

$$I_z = \iiint_E \left(x^2 + y^2\right)\rho(x, y, z)\,dV$$

$$A = \oint_C x\,dy = -\oint_C y\,dx = \frac{1}{2}\oint_C x\,dy - y\,dx$$