

Assignment 1

Marks **Total marks for Assignment 1: 61**

1. Given the vectors $\vec{u} = \langle 1, -2, 3 \rangle$ and $\vec{v} = \langle 0, 5, -4 \rangle$, find :
 - [2] a) The angle (in degrees) between \vec{u} and \vec{v} . Express your answer correctly rounded to 2 decimal places.
 - [2] b) The vector projection of \vec{u} onto \vec{v} . Give an exact answer.
 - [2] c) A vector in the opposite direction to \vec{u} and of length 3 units. Give an exact answer.
 - [3] d) The exact area of the parallelogram determined by \vec{u} and \vec{v} .

- [5] 2. Let C be the point on the line segment AB that is twice as far from B as it is from A . Let $\vec{a} = \vec{OA}$, $\vec{b} = \vec{OB}$ and $\vec{c} = \vec{OC}$. Using vectors and the geometric definition of vector addition, show that $\vec{c} = \frac{2}{3}\vec{a} + \frac{1}{3}\vec{b}$.

- [7] 3. Find an equation in simplified form for the set of all points whose distance to $(-1, 4, -6)$ is twice the distance to $(2, -4, 0)$.
Name and give a precise description of this surface.

- [5] 4. Find the exact distance between the planes $2x - 4y - 6z = 9$ and $-x + 2y + 3z = 2$.

- [5] 5. Find the point of intersection of the two lines or show that no such point exists.

$$\frac{x}{3} = \frac{y-1}{-1} = \frac{z-2}{-1} ; \quad \frac{x-1}{-2} = \frac{y-3}{1} = \frac{z-5}{2}$$

- [5] 6. Find parametric equations of the line of intersection of the planes $x + 7y - 2z = 1$ and $-3x - y + 4z = 3$

- [5] 7. Find an equation, in simplified form, for the plane containing the line $x = 3 - 2t$, $y = t$, $z = 5 - 3t$ and passing through the point $(2, -4, 3)$.

8. Neatly sketch the surfaces described by the given equations. Be sure to label your axes and give an appropriate scale on each axis. Identify on your graph all intercepts and any special points.
Name the surface.
- [5] a) $y = -\sqrt{3x^2 + 2z^2}$
- [5] b) $x^2 + 3z^2 = 4y^2 - 9$
- [10] 9. Carefully sketch the surface with equation $y^2 + 4z^2 = 9$. Identify the coordinates of all intercepts. Give a detailed description of all the horizontal and vertical traces of this surface. Your description must include the name and a general equation of the family of trace curves, and the coordinates of any special points on these trace curves.