Assignment 4

Marks **Total marks for Assignment 4: 65**

1. Given $\iint_R \sin(y)\, dA$, where $R = [0,2]\times[0,\pi]$

[2] a) Sketch the solid region represented by this double integral.

[5] b) Use the Midpoint Rule with $m = n = 3$ equal subdivisions of each interval to estimate the value of this double integral.

[3] c) Use a geometric interpretation of the double integral and symmetry in the solid region to determine an exact value for this double integral. NOTE: No credit will be given for evaluating the double integral using an iterated integral expression.

[10] 2. Use a double integral to find the exact volume of the wedge cut from the cylinder $x^2 + y^2 = 4$ by the planes $z = 0$ and $z = 2 - y$.

[10] 3. Carefully sketch the domain of integration and then evaluate exactly $\int_0^1 \int_y^1 ye^{x^3}\, dx\, dy$.

[5] 4. Find the average value of $g(u,v) = \sin(u)\sqrt{v + \cos(u)}$ over the rectangle $R = [0, \frac{\pi}{2}] \times [0,1]$. Give an exact answer.

[10] 5. Evaluate exactly $\iint_R x\sqrt{x^2 + y^2}\, dA$ for the region R in the plane bounded above by line $y = -x$ and below by the circle $x^2 + y^2 - 2y = 0$.

[10] 6. Carefully sketch the graph of the polar curve $r = \theta^2$, $0 \le \theta \le \pi$, and determine the exact area of the region bounded by this curve and the $x-$axis.

[5] 7. Find the exact length of the equiangular spiral $r = e^{\theta}$, $0 \le \theta \le 2\pi$.

[5] 8. Set up only an exact iterated integral expression representing the area of the plane region bounded by $y^2 = x + 4$ and $x - y = 2$, using the order $dy\,dx$.