Arbitrage Cycle Search Heuristic

Project Tycho

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Contents

1 Overview

The Tycho Searcher processes real-time blockchain data and identifies arbitrage opportunities. It receives block updates from a data feed, maintains an up-to-date graph of token exchanges, and applies a modified Bellman-Ford algorithm to detect negative cycles, which correspond to profitable arbitrage paths.

2 System Architecture

– **Tycho Feed:** An asynchronous task receives block updates and token data from an external source, and forwards them to the searcher.

– **Searcher:** The searcher maintains the exchange graph, updates it with new data, and runs the arbitrage detection algorithm.

The code is written in rust. It uses asynchronous channels (`tokio::sync::mpsc`) to communicate between the feed and the searcher.

3 Searcher Workflow

1. **Initialization:** The searcher initializes its graph structures and awaits block updates.
2. **Block Update Handling:** Upon receiving a new block, the searcher updates the graph with the latest exchange rates and liquidity data.
3. **Arbitrage Detection:** The searcher runs the modified Bellman-Ford algorithm to find negative cycles, which indicate arbitrage opportunities.
4. **Result Export:** The searcher processes the detected opportunities and can export or log them for further action.

4 Modified Bellman-Ford Algorithm

4.1 Purpose

The Bellman-Ford algorithm can detect negative cycles in a weighted graph. We have adapted the algorithm to search arbitrage cycles, in order to find cycles with positive profit in the exchange graph. We also consider that the internal state of an Automated Market Maker (AMM) changes after each exchange. We therefore restrict arbitrage cycles to traverse each AMM no more than once.

4.2 Data Structures

– `Graph`: This is a directed graph. Each node represents a token. And each edge corresponds to an AMM and represents the exchange rates between the corresponding tokens.

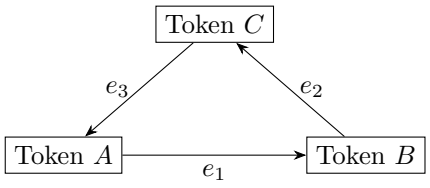
– `Edge Weights`: Each edge e is associated with two functions: $w_e(x_{\text{in}})$ and $g_e(x_{\text{in}})$. Here, $w_e(x_{\text{in}})$ returns the amount of tokens received after the swap, y_{out}. Meanwhile, $g_e(x_{\text{in}})$ returns the gas used. This is typically a constant function, because the gas cost is usually independent of x_{in}.

We assume that $w_e(x_{\text{in}})$ is a monotone non-decreasing function; that is, increasing the input amount x_{in} does not decrease the output amount y_{out}. Moreover, the exchange rate $\frac{y_{\text{out}}}{x_{\text{in}}}$ is a monotone non-increasing function of x_{in}.

Each query to $e()$ incurs a computational cost. In practice, a Rust call implements this to `get_amount_out`, which emulates the swap. Both x_{in} and y_{out} are stored as large integer values.

Fig. 1 illustrates an example of an arbitrage cycle. An arbitrage is defined by a closed path in the (exchange) graph and an input amount x_{in}, which is denominated in the start token. The cycle begins and ends at the same token, which we refer to as the *start node* or *start token*.

Arbitrage cycles have two key performance metrics:



If the arbitrage cycle traverses tokens $A \to B \to C \to A$ then it is profitable if and only if

$$w_3(w_2(w_1(x_{\text{in}}))) - x_{\text{in}} > p_A \cdot (g_1(x_{\text{in}}) + g_2(x_{\text{in}}) + g_3(x_{\text{in}}))$$

Figure 1: An illustration of arbitrage cycle. p_A is the exchange rate for Token A in ether.

1. Gas cost: the sum of the gas used by each pool along the arbitrage path, including inter-pool transfers;
2. Profit: the net gain in the amount of the start token.

Gas is paid in Ether. So determining whether an arbitrage is profitable requires knowing the start token's price p in ETH. If the start token is WETH, then $p = 1$.

It is important to note that the function $e()$ changes after each swap. Fig. 1 thus only holds if the three edges belong to different AMMs. For each edge, we therefore explicitly store the pool to which it belongs.

– `Pools`: For each edge e we store the associated pool $p(e)$.

We restrict the search space to cycles in which every edge belongs to a different pool; that is, for any cycle C and any distinct edges $e, f \in C$, we require $p(e) \neq p(f)$. Although some studies do not impose this restriction on the problem space [1], we find that in practice, it has only a minimal impact on solution quality.

4.3 Precomupations

As a first step, we decompose the graph into 2-edge-connected components. This enables us to search within each cycle independently. A 2-edge-connected component is a maximal set of nodes in which every pair of nodes is connected by at least two edge-disjoint paths. For any two nodes in the component, an equivalent cycle passes through each node – even if the cycle is not simple (e.g., it may consist of a figure-eight shape). This subdivision has several advantages: upon an update, only the affected components need to be recomputed; and the search is also easily parallelizable (although we have not implemented parallelization in our prototype).

In practice, we apply a slightly stronger decomposition, because we know the starting token (node) of the arbitrage cycles (e.g., WETH) in advance. As a first step, we consider the 2-edge-connected component of the graph that contains the start node. Tokens that are either unreachable from the start node through AMMs, or reachable only via a single AMM, are irrelevant for

Table 1: Performance of the algorithm for pools of various minimum TVL values.

AMMs types	TVL	full graph #nodes	full graph #edges	#components	largest component #nodes	largest component #edges	runtime
Uniswap v2,3,4 Ekubo Balancer curve	1000	111	322	54	47	190	4.2 s
	100	623	1662	435	166	574	12 s
	10	2467	6164	1980	462	1924	21 s
Uniswap v2, v3	1000	78	202	42	26	90	40 ms
	100	563	1324	420	126	398	271 ms
	10	2375	5470	1973	367	1258	359 ms

the purpose of cycle detection. Next, we temporarily remove the starting node (e.g., WETH) from the graph and compute the connected components of the remaining graph. When we re-add the starting node, each of these components becomes a subgraph that is only connected to the rest of the graph through the starting node. This effectively "splits" the search space into independent parts. It is easy to see that we can search cycles independently in these subgraphs, because any cycle that would traverse multiple components can only cross via the starting node. Any hypothetical cycle that spans multiple components must pass through the starting node. And such a cycle can always be decomposed into smaller cycles contained entirely within individual subgraphs.

Table 1 reports the graph size, the number of connected components, and the size of the largest component, as a function of the set of pools under consideration. The pools include Uniswap v2, v3, and v4, Ekubo v2, Balancer v2, and Curve. We further filter pools by their total value locked (TVL), measured in Ether. The table shows that this step yields a significant speedup. For instance, in a graph with 2 375 nodes, the largest component contains only 367 nodes. This illustrates how special the structure of the graph is.

See also Fig. 3, which shows the graph skeleton of the largest component. This is a simplified representation produced by first removing self-loops and iteratively pruning leaves (degree-1 nodes), then collapsing degree-2 chains into single edges marked as skeleton (and marking these edges as dashed lines). These transformations preserve the graph for a cycle search.

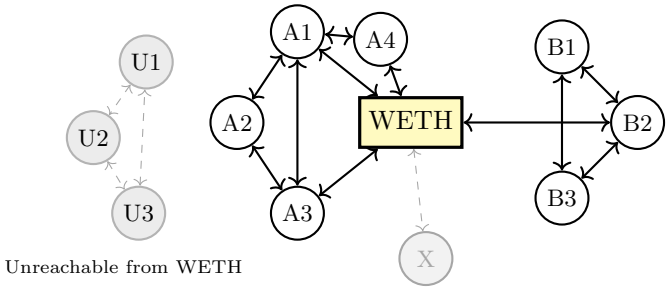
4.4 Modified Belmann-Ford

We now explain our most important subrutine. In this subrutine, we assume x_{in} is given, and a subgraph $G = (V, E)$. The task is to find candidate cycles that are profitable; that is, $w_3(w_2(w_1(x_{\text{in}}))) > x_{\text{in}}$ on Fig. 1.

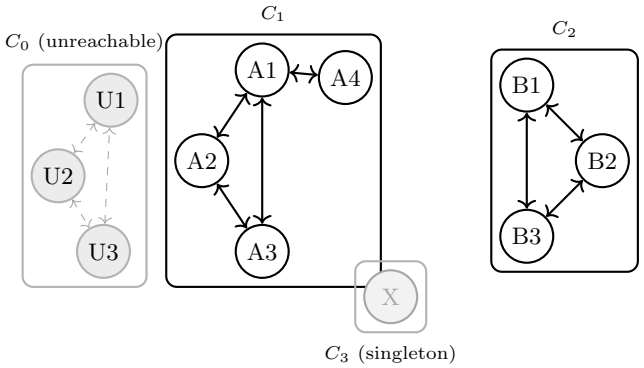
We define the following data structure:

– `Distance_without_loop`: For each node v, we store the maximum attainable output amount y_{out} (denoted as d_v) along a path that does not traverse the same node (token) more than once.

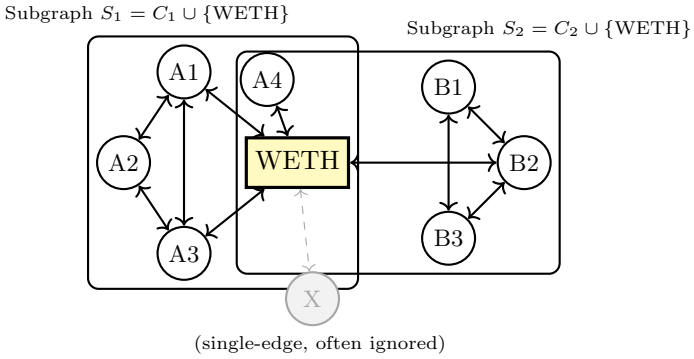
– `Path_without_loop`: For each node v, we store the path P_v toward the start node corresponding to d_v.



(a) Original graph with start node WETH

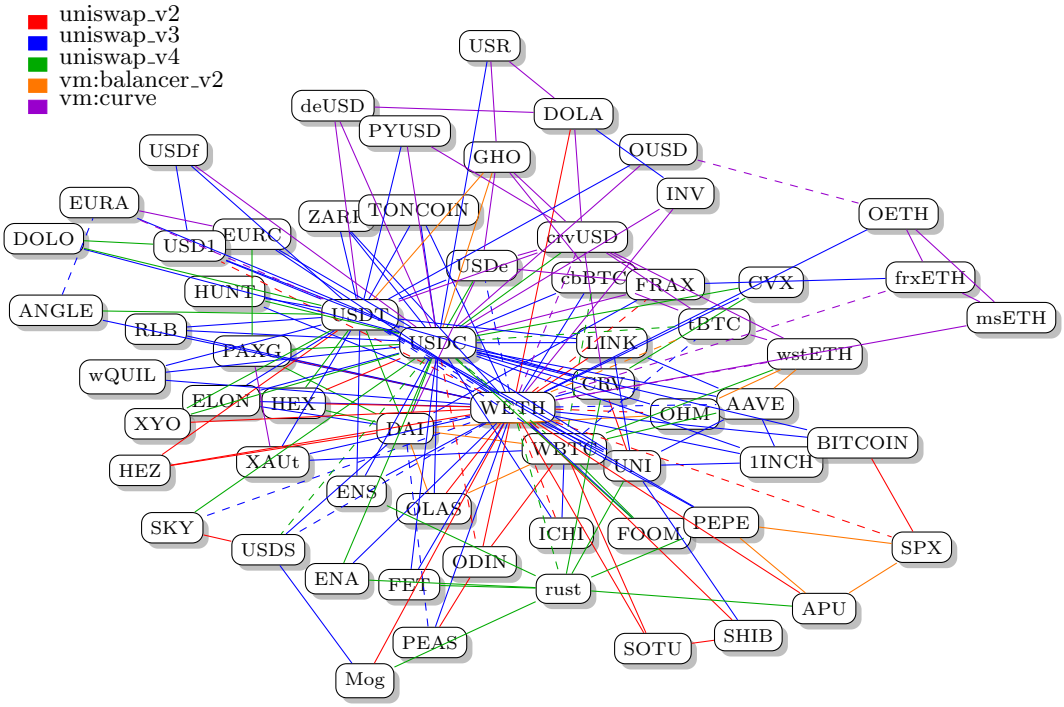


(b) Step1: Remove start node. Identify the connected components of $G \setminus \{\text{WETH}\}$

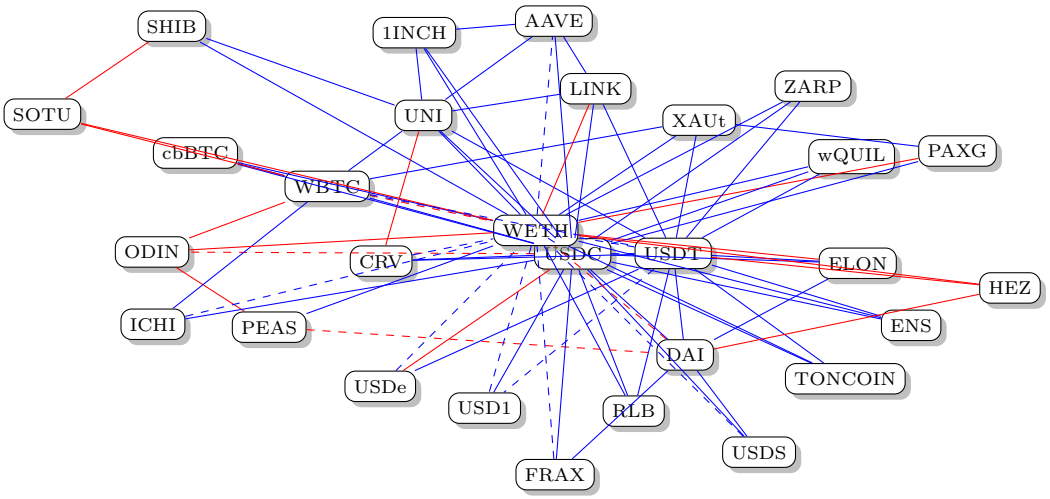


(c) Add start node back. Independent subgraphs via WETH

Figure 2: Decompose the graph into components for arbitrage cycle search



(a) All pools



(b) Uniswap v2 and v4 pools

Figure 3: The sceleton of the token graphs for pools with TVL$\geq$ 10 Ether, at 22nd of September 2025. Graph skeleton: the core network obtained by removing self-loops and peripheral leaves and collapsing chains of degree-2 nodes into single (dashed) edges, so the backbone connectivity between hubs is emphasized.

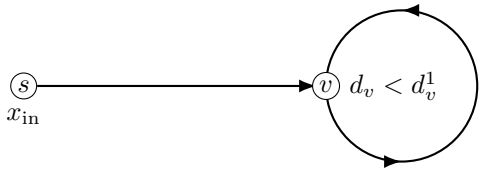


Figure 4: Illustration of an arbitrage path saved in `save_path`.

– `Distance_with_a_single_loop`: For each node v, we store a second value, the maximum attainable output amount y_{out} (denoted as d_v^1) along a path that traverses one node (token) twice, and all other nodes only once.

– `Path_with_a_single_loop`: For each node v, we store the corresponding path P_v^1 from the start node.

Algorithm 1 describes the pseudocode of the algorithm. It has the following steps:

1. Initialize distances from the source node to all other nodes as infinity, except the source itself (zero and empty path).

2. For each edge $u \to v$, if node u has a finite distance (i.e., $d_u \neq \infty$), attempt to extend the path P_u by adding the edge. See function `extend_path` for details. Our approach only allows this if the path has not previously used the pool of $e_{u \to v}$. If this results in more tokens than currently recorded at d_v, we need to update our data structures as follows: If the path P_u does not traverse token v, we update d_v and P_v; otherwise, we update d_v^1 and P_v^1. In the latter case, we have found a path from s and with a loop through v, which we save as a candidate. Finally, if node $v = s$, we have found a loop.

3. We perform a similar check if node u has a finite distance with loop (i.e., $d_u \neq \infty$). In this case, we do not deal with the possibility of multiple loops.

4. Repeat the previous step for k iterations, where k is the maximum path length allowed.

The function `extend_path`$(d_u, g_u, P_u, e_{u \to v})$ returns the token amount and gas usage that result from extending path P_u with edge $e_{u \to v}$. Note that this can handle cases where the same AMM is traversed multiple times along $P_u + e_{u \to v}$, by updating its internal state after each traversal.

The function `save_cycle`(x_{in}, P) saves a profitable arbitrage cycle with a given amount x_{in}. Similarly, the function `save_path`(x_{in}, P) saves a profitable path to a token, from which there is an arbitrage cycle. It also stores an amount x_{in} from the start node. See Fig. 4.

4.5 Arbitrage search

Next, we discuss how to find the optimal input amount x_{in}. We begin by running Algorithm 1 with $p = 0$ and a small initial input, for example, $x_{\text{in}} = 0.001$. This

typically yields multiple arbitrage cycles. For each identified cycle, we apply a black-box optimization method – the Golden Section Search (GSS) – to find the optimal x_{in}.

GSS is a derivative-free optimization technique for finding the maximum (or minimum) of a unimodal function over a bounded interval. It is particularly suitable when the function's analytic form is unknown or too expensive to compute, but function evaluations are available. GSS progressively narrows the interval of interest using the golden ratio, requiring only function values at strategically chosen points. In our setting, we use GSS to identify the input amount x_{in} that maximizes arbitrage profit along a given cycle.

4.6 Optimizations

To reduce computation time, we divide the graph into subgraphs and solve the problem independently for each subgraph. To compute the subgraphs, we remove the start node s and identify the resulting connected components. We then form each subgraph by reintroducing the start node s and connecting it to a component. We execute the algorithm independently within each subgraph.

To further reduce runtime, we implemented the following memory management strategy.

– **Reference Counting:** By using `Rc<Vec<EdgeIndex>>`, the algorithm avoids deep cloning of cycle paths. This is especially beneficial when we detect many cycles.

– **Efficient Graph Updates:** We only update relevant subgraphs per block. This reduces recomputation.

– **BigUint Arithmetic:** We ensure that all calculations are safe from overflow. This is critical for financial computations.

4.7 Summary:

We have presented a Bellman-Ford algorithm with the following modifications:

- We only consider cycles if they start and end at the designated start token (e.g., WETH).
- We use reference-counted vectors (`Rc<Vec<EdgeIndex>>`) to avoid unnecessary cloning.
- The algorithm skips cycles that revisit nodes, to avoid infinite loops.
- Profit calculation includes gas costs, and only exports cycles with net positive profit.
- The Golden Section Search adapts the input range if invalid points are encountered.

References

[1] Diamandis, T., Angeris, G., Edelman, A.: Convex network flows. arXiv preprint arXiv:2404.00765 (2024)

Algorithm 1: Modified Belmman-Ford

Input: Directed graph G, strat node s, amount x_{in}, price p, max iteration $k_{\max}$

$d_s = x_{\text{in}}$; $g_s = 0$; $d_v = \infty$ and $g_v = 0$ for $v \in V \setminus \{s\}$
for $j \in \{1, \ldots, k_{\max}\}$ **do**
  for $e_{u\to v} \in E$ **do**
    if $d_u \neq \infty$ *and pool of* $e_{u\to v}$ *is not in* P_u **then**
      $d', g' =$ `extend_path`$(d_u, g_u, P_u, e_{u\to v})$;
      if $v \neq s$ **then**
        if *token* $v \in P_u$ **then**
          if $d_v^1 + p \cdot g_v^1 > d' + p \cdot g'$ **then**
            $d_v^1 = d'$; $g_v^1 = g'$; $P_v^1 = P_u + e_{u\to v}$;
            `save_path`$(x_{\text{in}}, P_u + e_{u\to v})$;
        else
          if $d_v + p \cdot g_v > d' + p \cdot g'$ **then**
            $d_v = d'$; $g_v = g'$; $P_v = P_u + e_{u\to v}$;
      else
        if $d' > x_{in} + p \cdot g'$ **then**
          `save_cycle`$(x_{\text{in}}, P_u + e_{u\to v})$;
    if $d_u^1 \neq \infty$ *and pool of* $e_{u\to v}$ *is not in* P_u^1 **then**
      $d', g' =$ `extend_path`$(d_u^1, g_u^1, P_u^1, e_{u\to v})$;
      if $v \neq s$ **then**
        if *token* $v \notin P_u^1$ **then**
          if $d_v^1 + p \cdot g_v^1 > d' + p \cdot g'$ **then**
            $d_v^1 = d'$; $g_v^1 = g'$; $P_v^1 = P_u^1 + e_{u\to v}$;
      else
        if $d' > x_{in} + p \cdot g'$ **then**
          `save_cycle`$(x_{\text{in}}, P_u^1 + e_{u\to v})$;