Affective Computing Model Based on Rough Sets

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Abstract. The paper first builds a novel affective model based on rough sets, presents the static description of affective space. Meanwhile, the paper creatively combines rough sets with Markov chain, gives the dynamic forecast of human affective change. In this affective model, some concepts and states are defined such as affective description precision and so on. It is a fundamental work to more research. Simulations are done using Matlab software, and simulation results show that this affective model can well simulate the human emotion. The results of this paper are innovative. It is a new research direction of affective computing that rough sets and affective computing infiltrate each other.

1 Introduction

Emotion is an important part of psychology actions, psychologists are paying more attention to the research of emotion. So far, there are a lot of affective theories, such as the correlative theory of emotion and economy change, which is provided by James [1]. Thereafter, there are the affective motivation theory, the affective behavior theory, the affective cognizance theory and so on [2]. But there aren't lots of definite quantity researches on emotion. In 1997, Professor Picard punished her monograph 'Affective Computing'[3], in this book she defined 'Affective computing is computing that relates to, arises from, or deliberately influences emotions'. Thus, the conception 'Affective Computing'was put forward. Soon after, the academy paid attention to it and the enterprise was quickly in response to it. The British Telecom had built the expert group of affective computing research. IBM had exploited the 'affective mouse'. At present, the research of endowing the computer with emotion is becoming an important research direction. However, affective computing is a very difficult problem. Emotion is a psychological phenomenon, which has nonlinear character, that is to say, emotion has uncertainty, so affective computing belongs to the research of uncertainty question.

In 1982, Professor Z. Pawlak put forward Rough Sets theory [4], it generalized the classical sets on research uncertainty question and it had showed advantage

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and rationality in many fields. This paper analyzes human emotion, builds affective model and makes affective computing by using rough sets theory. We first apply the rough sets theory to the affective research, which is a new research direction of affective computing.

The rest of the paper is organized as follows: Section 2 introduces some terminology and presents preliminary results on psychology. Section 3 is devoted to the static description of emotion by using rough sets theory. In section 4, we combine rough sets and Markov chain to analyze the dynamic change of emotion. Section 5 simulates affective model by using Matlab software and gives some relative analyses. Section 6 describes some concluding remarks.

2 Affective Basic Conception

The human emotion is a quite complicated phenomenon and it has various contents. For the need of our research, we classify the emotion based on psychology theory [5].

Emotions are human's experiences for objective things, they reflect people' needs whether are satisfied or not, they conclude two aspects: (1) Affective process: The process of affective state's change is an important part of the affective process. From characters, affective states can be divided into happy, rage, dread, sad and so on. (2)Affective personality: It concludes needs, desire, motivation, interesting and so on, which are the personality orientation. It also concludes capability, temperament, character, attitudes and so on, which are the personality psychology characters.

In basic affective theory [6], the human emotions have many attribute dimensions, psychologists usually use strong or feeble to describe affective strength degree. For example, sad can be described as pity, despond, sorrow, melancholy, grief and despair; happy can be described as cozy, comfortable, pleasant, bright, cheer and spree, which are increasing in strength degree. Recently, psychologists have put forward more deliberate dimensions to describe emotion. From those, Izard [7] generalized four widely used dimensions, which are pleasant dimension, nervous dimension, impulse dimension and assurance dimension.

The intercourse of the human and the nature usually makes use of eyes, ears, mouth, skins and so on. The information received by the five sense organs is called affective information, viz. vision affective information, hearing affective information, scent affective information, gustatory affective information and feeling affective information. The research on affective information has already made home and abroad [8]. By researching the affective information, we can measure somebody's any emotions at sometime. Throughout the rest of the paper, we supposed the human emotions can be completely measured.

For the convenience of our research, we firstly give some conceptions:

Definition 1. *All of the human emotions constitute* ***affective space.***

Definition 2. *All kinds of signals and information to measure emotions are called* ***affective information.***

Definition 3. *At sometime, all of the emotions of somebody constitute* ***affective states set.***

Definition 4. *At sometime, the mostly or typical emotions of somebody constitute the* ***leading affective set.***

Generally speaking, the affective states set is the proper subset of the affective space. Affective experience is very complicated, such as 'not know whether to laugh or cry', 'pleasure and sadness are mixed'and so on. In this paper, we supposed several kinds of emotions can exist at the same time, but there are primary and secondary, for example, somebody's leading emotion is 'rage'at a time, but he may has the 'glorious feeling'.

Thus, from the analysis above and according to the literature [8], we regard, among the affective information, emotions and affective states (affective space), there is the following logic connection: *By collecting and disposing, we receive affective information, after analyzing, we can measure one person's emotions, finally, all kinds of emotions constitute affective states set.*

3 The Affective Static Description Based on Rough Sets

In this section we give the static description based on rough sets. The conception 'Affective space', 'Affective states set'have been given above. But we notice that, in actual psychology analysis and computing, we can't discriminate one emotion's ascription, that is to say, there is indiscernibility among emotions. For example, in [6], happy can be described as cozy, comfortable, pleasant, bright, cheer and spree, there are six different grades. It is supposed that somebody's happy emotion is considered as spree, if his happy emotion is stronger than spree, we have to consider that his happy emotion is also spree. That is to say, two different emotions in affective space have to be considered as the same, which embodies the indiscernibility of emotions and induces roughness of affective computing. So, we consider affective modeling based on rough sets and solve the indiscernibility in affective description.

Given affective space U, it is can also be called the affective universe of discourse, thus the human affective states set is a rough set, exact definition is as follows: Based on affective basic conceptions, we divide the affective space into finite affective equivalent class by using the affective class in psychology. Meanwhile, based on the affective attribute dimensions description above, we also can divide the affective space into different finite affective equivalent class. Thus we obtain a group of equivalent relations $\Re$ on U. $\forall R_i \in \Re$, according to the knowledge of rough sets[4], $\cap R_i$ is also an equivalent relation, which is denoted as $ind(R)$, so $U/ind(R)$ is another equivalent class division of affective space. Our affective model just based on this division. Then the human affective states set X is described as $(\underline{R}(X), \overline{R}(X))$, and

$$\underline{R}(X) = \cup\{Y \in U/ind(R) \mid Y \subseteq X\} \tag{1}$$

$$\overline{R}(X) = \cup\{Y \in U/ind(R) \mid Y \cap X \neq \phi\} \tag{2}$$

We notice that the human affective states set on time T is described as a rough set $A^T(X) = (\underline{R}(X), \overline{R}(X))^{(T)}$, then$\underline{R}(X)$ and $\overline{R}(X)$have definite actual meaning. $\underline{R}(X)$means the person's leading affective character on time T, $\overline{R}(X)$means all of the person's affective states on time T .

When we describe the human affective states set as a rough set, we allow the person can have many kinds of different affective states at the same time, so the relation between the number of elementary in the set and its equivalent class can tell us whether one's emotion is various or not. We can conveniently give the definition of the affective variety degree.

Definition 5. *Given the affective universe of discourse U and knowledge base$K = (U, \Re)$, $R \in \Re$ is an equivalent relation, $[x]_R$ is R equivalent class.Let affective states set $A^T(X)$, call $Q(X)$ the* ***affective variety degree****, and*

$$Q(X) = \sum \frac{|\ A^T(X) \cap [x]_R\ |}{|\ [x]_R\ |} \tag{3}$$

Obviously, variety degree describes how many emotions one person has and the bigger the value is, the more emotions the person has.

Another important thing is how to describe the affective description precision in different conditions. The next definition gives us such convenience that we can control the affective description precision based on the research require.

Definition 6. *Given the affective universe of discourse U and knowledge base$K = (U, \Re)$, $R \in \Re$ is an equivalent relation. Let affective states set $A^T(X)$, Call $\alpha_R(X)$ the* ***affective description precision****, and*

$$\alpha_R(X) = \frac{|\ \underline{R}(X)\ |}{|\ \overline{R}(X)\ |} \tag{4}$$

Here: $X \neq \phi$, $|\ A\ |$ denotes the number of elementary in set A.

In [9], there are two properties of $\alpha_R(X)$:

1. $\forall R \in \Re$ and $X \subseteq U$, there is $0 \leq \alpha_R(X) \leq 1$;
2. Let $R_1, R_2 \in \Re$, if $U/R_1 \subseteq U/R_2$, then $\alpha_{R_2}(X) \leq \alpha_{R_1}(X)$.

For property 2, its affective description meaning is: the finer the affective space is divided, the bigger the affective description precision is, so we can describe the emotion more exactly. That is obviously accord to our common sense. That is to say, our affective modeling based on rough sets and the precision definition are well accord to nature, which in favor of our more research.

4 The Affective Dynamic Description Based on Rough Sets

The simulation of affective states and affective dynamic forecast are the critical contents of affective computing. At present, many scientists[10, 11] simulated the affective dynamic change by using Markov chain. When there is no environment stimulus, affective states mostly behave as a spontaneous transition process, which usually can be simulated by Markov chain.

In affective space, there is one kind of emotion which has special meanings, it is the calm state. We denote it with Φ. When there is no environment stimulus, [5, 6] point that all kinds of human emotions will converge to the calm state. That is to say, it is the main trend of human affective transition that any one kind of emotion will transfer to the calm state. So we can regard the affective transition process as a Markov process and simulate affective transition by using Markov chain. Meanwhile, there is an ordinary return state, which is the calm state.

According to the psychology knowledge, the human affective transition is always a periodic process that from calm state to any other emotions and then back to calm state. This process repeats again and again, but their time intervals are random. Due to space limitations, we will discuss the cycle question of affective transition in another paper. In this paper we creatively combine rough sets and Markov chain to analyze affective dynamic change process.

Given the affective universe of discourse U_1 (nonempty finite set), R_1 is an equivalent relation on U_1, $[x]_{R_1}$ is R_1 equivalent class, $X_1 \subset U_1$. Based on rough sets, the affective states set on time T is $A^T(X_1) = (\underline{R_1}(X_1), \overline{R_1}(X_1))^{(T)}$, and

$$\underline{R_1}(X_1) = \cup[x]_{R_1} = \{x|x \in [x]_{R_1} \subseteq X_1\} \tag{5}$$

$$\overline{R_1}(X_1) = \cup[x]_{R_1} = \{x|x \in [x]_{R_1} \cap X_1 \neq \phi\} \tag{6}$$

For the affective states set$A^T(X_1)$, it is supposed that there are k kinds of affective states sets when it transfer from time T to time $T+1$, denoted as $A^{T+1}(X_1), \cdots, A^{T+1}(X_k)$. Let the transition probabilities of affective states sets are $P_{11}, P_{12}, \cdots, P_{1k}$ separately. Similarly, we can define the transition probability of other $k-1$ kinds of affective states sets $A^T(X_i), i = 2, \cdots, k$, when they transfer from time T to time $T+1$.

Let $a_1(T+1) = (\underline{R_1}(X_1), \overline{R_1}(X_1))^{(T+1)}, \cdots, a_k(T+1) = (\underline{R_k}(X_k), \overline{R_k}(X_k))^{(T+1)}$,

$$P = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1k} \\ P_{21} & P_{22} & \cdots & P_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ P_{k1} & P_{k2} & \cdots & P_{kk} \end{pmatrix}, a(T+1) = (a_1(T+1), \cdots, a_k(T+1))$$

Then we have

$$a(T+1) = a(T) \cdot P \tag{7}$$

Supposed a person has n kinds of emotions, which have m kinds of attribute dimensions. So there are l kinds of possible affective states, where

$$l = (C_n^1 + C_n^2 + \cdots + C_n^n)(C_m^1 + C_m^2 + \cdots + C_m^m) + 1 \tag{8}$$

then affective transition probability matrix is$(P_{ij})_{l\times l}$. There are many methods to confirm the transition probability of the transition matrix, generally, statistical method is widely used.

Markov chain contains various contents. Here, we make use of the absorption chain in Markov chain to simulate the process that affective transfer from an arbitrary emotion to the calm state for the first time. We firstly give the basic conception and properties of absorption chain [12].

Definition 7. [12] *The state i is called* ***the absorption state****, if its transition probability $P_{ii} = 1$.*

Definition 8. [12] *A Markov chain is called* ***the absorption chain****, if it at least has one absorption state and any unabsorption state can reach some absorption state by plus probability in finite transition steps.*

Let an absorption chain has r kinds of absorption states and $k - r$ kinds of unabsorption states, then the transition matrix of absorption chain can be written as follow:

$$P = \begin{pmatrix} Q & R \\ 0 & I_{r\times r} \end{pmatrix} \tag{9}$$

Here: Q is $(k-r)$ by $(k-r)$ square matrix and its eigenvalue $|\lambda(Q)| < 1$. I is the unit matrix.

Absorption chain has two properties[12]:

Property 1. In above standard form of P, $(I - Q)$ is reversible, and

$$M = (I - Q)^{-1} = \sum_{s=1}^{\infty} Q^s \tag{10}$$

Let column vector $e = (1, \cdots, 1)^T$, then the ith component of the vector $y = M \cdot e$ is the number of average transition step that from the ith unabsorption state to some absorption state.

Let unabsorption state is i , absorption state is j ,then $f_{ij}(n)$ is actually the probability that i transfer to j by n steps and $f_{ij} = \sum_{n=1}^{\infty} f_{ij}(n)$ is the probability that i transfer to j eventually. Let $F = \{f_{ij}\}_{(k-r)\times r}$, then we have

Property 2. Let P is above standard form, then

$$F = M \cdot R \tag{11}$$

Given affective state sets, supposed there are k kinds of states when emotions transfer from time T to time $T + 1$. For the given absorption chain, there are two cases of its transition process: 1. After a period of time (maybe including many transition steps), the above k kinds of states will be absorbed by the calm state. 2. The above k kinds of states will be absorbed by $l(l < k)$ kinds of states. In case 2, the l kinds of states are regarded as the typical affective states. For example, the mental disease sufferer can have any other typical affective states besides calm state, such as sad or ignorant and so on. For the l kinds of states, we can similarly give transition matrix P_{ll} and continue the above steps. Eventually, they will be absorbed by calm state, so we complete the description of an affective transition cycle.

5 Simulation

In this section, we give the simulation of the above process that emotions transfer from an arbitrary emotion to the calm state for the first time.

For the convenience of computing, we only consider three emotions: happy, sad, dread and two attribute dimensions: nervous dimension, impulse dimension. We suppose there is just one emotion at one time, so there are $C_3^1(C_2^1+C_2^2)+1=10$ kinds of affective possible states. Let somebody's affective states set $A^T(s)$ is $\{$(sad, nervous), (dread, not impulse)$\}$ at some time. We give the affective transition matrix by using the following psychology statistical rules: 1. For any affective state, it will keep its own state or transfer to calm state by bigger probability. The shorter the time interval t is, the bigger probability of keeping its own state. 2. The probability of transferring to any other states is relatively little, we suppose they are equal. The longer the time interval t is, the bigger probability of transferring to other state.

Using the above rules,we give a matrix

$$P_{10\times 10}=\begin{pmatrix} 0.17 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.13 \\ 0.075 & 0.2 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.2 \\ 0.0625 & 0.0625 & 0.25 & 0.0625 & 0.0625 & 0.0625 & 0.0625 & 0.0625 & 0.0625 & 0.25 \\ 0.0875 & 0.0875 & 0.0875 & 0.1 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.2 \\ 0.08 & 0.08 & 0.08 & 0.08 & 0.2 & 0.08 & 0.08 & 0.08 & 0.08 & 0.16 \\ 0.09 & 0.09 & 0.09 & 0.09 & 0.09 & 0.14 & 0.09 & 0.09 & 0.09 & 0.14 \\ 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.16 & 0.08 & 0.08 & 0.16 \\ 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.2 & 0.0875 & 0.1 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.2 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{12}$$

Here: We supposed the first affective states set is $A^T(s)$, the last one is the calm state, and $P_{ij}(i,j=1,\cdots,10)$ denotes the probability of state i transfer to state j .The matrix satisfies the property of absorption chain, so after computing, we have $M=(I-Q)^{-1}$, and

$$M=\begin{pmatrix} 1.6242 & 0.5602 & 0.6033 & 0.4964 & 0.5571 & 0.5160 & 0.5447 & 0.5523 & 0.5602 \\ 0.4802 & 1.6464 & 0.5423 & 0.4462 & 0.5007 & 0.4638 & 0.4895 & 0.4964 & 0.5035 \\ 0.4310 & 0.4519 & 1.7174 & 0.4004 & 0.4493 & 0.4162 & 0.4393 & 0.4455 & 0.4519 \\ 0.4964 & 0.5205 & 0.5606 & 1.4739 & 0.5176 & 0.4794 & 0.5061 & 0.5132 & 0.5205 \\ 0.5093 & 0.5340 & 0.5751 & 0.4732 & 1.6674 & 0.4919 & 0.5192 & 0.5265 & 0.5340 \\ 0.5307 & 0.5565 & 0.5993 & 0.4931 & 0.5534 & 1.5652 & 0.5411 & 0.5487 & 0.5565 \\ 0.4980 & 0.5222 & 0.5623 & 0.4627 & 0.5192 & 0.4810 & 1.6188 & 0.5148 & 0.5222 \\ 0.5523 & 0.5792 & 0.6237 & 0.5132 & 0.5759 & 0.5334 & 0.5631 & 1.6978 & 0.5792 \\ 0.4802 & 0.5035 & 0.5423 & 0.4462 & 0.5007 & 0.4638 & 0.4895 & 0.4964 & 1.6464 \end{pmatrix} \tag{13}$$

$$y=M\cdot e=\begin{pmatrix} 6.0145 & 5.5689 & 5.2029 & 5.5881 & 5.8307 & 5.9446 & 5.7011 & 6.2178 & 5.5689 \end{pmatrix}^T \tag{14}$$

The simulation shows that the first state will be absorbed by calm state after 7 time intervals. Similarly, we can obtain other states' transition time intervals. And

$$F = M \cdot R = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}^T \tag{15}$$

This shows the probability that the given affective states eventually transfer to the calm state is 1. All the above results show that our model based on rough sets and Markov chain can well simulate the human affective dynamic change.

6 Concluding Remarks

The paper presents the fundamental work about affective computing. Psychology conceptions are generalized and two hypothesizes are given: 1. A person can have several kinds of affective states at the same time and the human emotion has several attribute dimensions. 2. When there is no environment stimulus, all kinds of human emotions will converge to the calm state. That is to say, the main trend of human affective transition is from any one emotion to calm state. Then we give the static description of emotion based on rough sets theory. Furthermore we combine rough sets and Markov chain to analyze the dynamic change of emotion, which is a new point to more research. At last, we simulate our affective model by using Matlab software and give some relative analyses. When there is environment stimulus the affective change question and the affective transition cycle question, which are our research emphases in future.

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