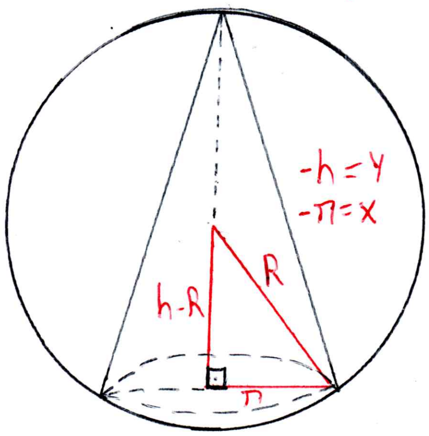
870 - - Observe o esquema:



- Observe o triângulo retângulo ao lado:

$$R^2 = (h-R)^2 + r^2 \Rightarrow x^2 = R^2 - (y-R)^2$$

- Para o volume do cone:

$$V(x,y) = \frac{1}{3}\pi y x^2$$

$$V(y) = \frac{1}{3}\pi y R^2 - \frac{1}{3}\pi y (y-R)^2$$

- Para o ponto extremal:

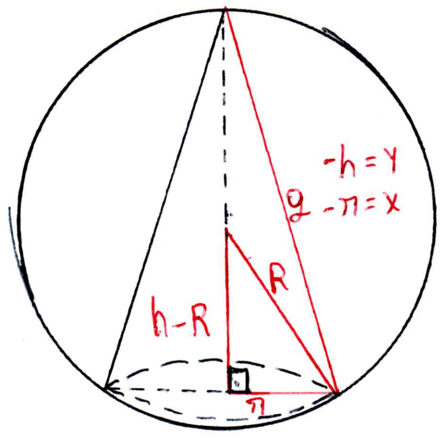
$$\frac{d}{dy}\left(V(y)\right) = 0$$

- Logo:

$$\frac{1}{3}\pi R^2 - \frac{1}{3}\pi\left[(y-R)^2 + y.2(y-R)\right] = 0 \Rightarrow \frac{1}{3}\pi R^2 - \frac{1}{3}\pi\left[y^2 - 2yR + R^2 + 2y^2 - 2yR\right] = 0$$

$$\Rightarrow \frac{1}{3}\pi . 3y^2 - 4yR \cdot \frac{1}{3}\pi = 0 \Rightarrow y(3y - 4R) = 0 \Rightarrow \boxed{y = \frac{4R}{3} \Rightarrow x = \frac{2\sqrt{2}}{3} \cdot R}$$

871 - - Observe o esquema:



- Observe o triângulo retângulo ao lado:

$$R^2 = (h-R)^2 + r^2 \Rightarrow x^2 = R^2 - (y-R)^2$$

$$\Rightarrow g^2 = y^2 + x^2$$

- Para a superfície lateral do cone:

$$A(x,y) = \pi x g \Rightarrow A(x,y) = \pi x . \sqrt{y^2 + x^2}$$

$$\Rightarrow A(y) = \pi\sqrt{R^2 - (y-R)^2} . \sqrt{y^2 + R^2 - (y-R)^2}$$

$$\Rightarrow A(y) = \pi\sqrt{2yR - y^2} . \sqrt{2yR}$$

$$\Rightarrow A(y) = \pi\sqrt{yR^2y^2 - 2Ry^3}$$

- Para o ponto extremal:

$$\frac{d}{dy}\left(A(y)\right) = 0$$

- Logo:

$$\pi\left[\frac{1}{2}(8R^2y - 6Ry^2)\right] \cdot \frac{1}{\sqrt{4R^2y^2 - 2Ry^3}} = 0 \Rightarrow 2Ry(4R - 3y) = 0$$

$$\Rightarrow \boxed{y = \frac{4R}{3} \Rightarrow x = \frac{2\sqrt{2}}{3} \cdot R}$$

Rosdrael Antonil de Araújo Freire //