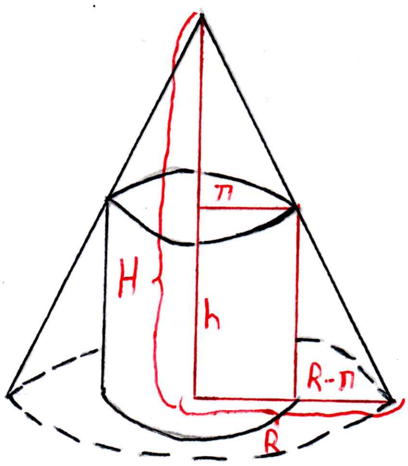
872 - -Observe o esquema:



-Temos a relação entre os triângulos:

$$* \ \frac{H}{h} = \frac{R}{R-r}$$

$$* \ \frac{H}{H-h} = \frac{R}{r}$$

$$\Rightarrow H = h \cdot \frac{R}{R-r}$$

-Seja o volume do cone:

$$V(R,H) = \frac{1}{3}\pi H R^2 \Rightarrow V(R) = \frac{1}{3}\pi \cdot \frac{R^3}{R-r} \cdot h$$

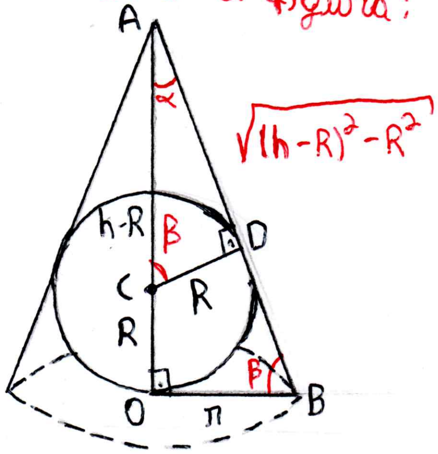
-Para o Ponto extremal:

$$\frac{d(V(R))}{dR} = 0$$

-Logo:

$$\frac{1}{3}\pi h \left[3R^2(R-r) - R^3\right] \cdot \frac{1}{(R-r)^2} = 0 \Rightarrow R^2(3R - 3r - R) = 0 \Rightarrow \boxed{R = \frac{3}{2}r}$$

873 - -Observe a figura:



- Relacionando os triângulos:

$$\frac{r}{h} = \frac{R}{\sqrt{(h-R)^2-R^2}}$$

$$\Rightarrow r^2 = h^2 \cdot \left[\frac{R^2}{h^2-2hR}\right]$$

- Para o volume do cone:

$$V(r,h) = \frac{1}{3}\pi h \cdot r^2$$

$$\Rightarrow V(h) = \frac{1}{3}\pi \cdot R^2 \cdot \left[\frac{h^3}{h^2-2hR}\right]$$

-Para o ponto extremal:

$$\frac{d}{dh}(V(h)) = 0$$

-Logo:

$$\frac{1}{3}\pi R^2\left[3h^2(h^2-2hR) - h^3(2h-2R)\right]\frac{1}{(h^2-2hR)^2} = 0 \Rightarrow h^2(3h^2 - 6hR - 2h^2 + 2hR) = 0$$

$$\Rightarrow h^3(h-4R) = 0 \Rightarrow \boxed{h = 4R}$$

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