1、Laplace Transform

Bilateral: $F_B(s) = \mathcal{L}_B[f(t)] = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt$

Unilateral: $F(s) = \mathcal{L}[f(t)] = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$ (默认是这个)

Feedback loop: overall TF $= \dfrac{G(s)}{1+G(s)H(s)}$ ($G(s)$为forward, $H(s)$为feedback)

ROC (region of convergence): 使 $\int_{-\infty}^{\infty} |f(t)|e^{-st}\,dt < \infty$ 的s的范围(s是复数)

Inverse Laplace Transform: $f(t) = \mathcal{L}^{-1}[F(s)]$

PFE (Partial Fraction Expansion): $F(s) = \dfrac{n(s)}{d(s)}$

poles: roots of $d(s)$, zeros: roots of $n(s)$

$$\frac{As+B}{(s+\sigma_i)^2+\omega_i^2} = A\mathcal{L}[e^{-\sigma_i t}\cos(\omega_i t)u(t)] + \frac{B-A\sigma_i}{\omega_i}\mathcal{L}[e^{-\sigma_i t}\sin(\omega_i t)u(t)]$$

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s) - f(0^-),\quad \mathcal{L}\left[\frac{d^nf(t)}{dt^n}\right] = s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \cdots - f^{(n-1)}(0^-)$$

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$\mathcal{L}[ZSR(t)] = H(s)X(s)$

$Y(t) = ZSR(t) + ZIR(t) = \mathcal{L}^{-1}[Y(s)]$

BIBO stable: $\int_{-\infty}^{\infty} |h(t)| < \infty$

Asymptotic stable: $\lim_{t\to\infty} ZIR(t) = 0$

ORHP: open right-half of complex plane ($Re[s] > 0$)

BIBO stable $\Leftrightarrow$ all poles of $H(s)$ in OLHP

asymptotically stable $\Leftrightarrow$ all poles of $H(s)$ in OLHP

marginally stable $\Leftrightarrow$ all poles of $H(s)$ in LHP and those on the imaginary axis are simple (they have multiplicity one in the denominator of $H(s)$)

(如果不stable就没有FRF)

$\int_{0^-}^{t} f(\tau)\,d\tau \overset{\mathcal{L}}{\Leftrightarrow} \frac{1}{s}F(s)$

$(-t)f(t) \overset{\mathcal{L}}{\Leftrightarrow} \dfrac{dF(s)}{ds}$, $(-t)^nf(t) \overset{\mathcal{L}}{\Leftrightarrow} \dfrac{d^nF(s)}{ds^n}$

$f(t-t_0)u(t-t_0)\ (t_0>0) \overset{\mathcal{L}}{\Longleftrightarrow} e^{-st_0}F(s)$

$e^{s_0t}f(t) \overset{\mathcal{L}}{\Leftrightarrow} F(s-s_0)$

$f(\alpha t)\ (\alpha>0) \overset{\mathcal{L}}{\Longleftrightarrow} \frac{1}{\alpha}F\left(\frac{s}{\alpha}\right)$

Initial Value Theorem: $f(0^+) = \lim_{s\to\infty} sF(s)$

Final Value Theorem: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Feedback Control

$H(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ $(\omega_n > 0, \zeta > 0)$ (ω_n是 underdamped natural frequency) (ζ是 damping ratio)

$\zeta > 1$: overdamped, $\zeta = 1$: critically damped, $\zeta < 1$: underdamped

也可考虑直接用 Δ, 若 $\zeta < \frac{1}{\sqrt{2}}$, 则有 overshoot

$u(t) \longrightarrow \dfrac{1}{s}$

$\delta(t) \longrightarrow 1$

$e^{-at}u(t) \longrightarrow \dfrac{1}{s+a}$

$u(t-a) \longrightarrow \dfrac{1}{s}e^{-as}$

$t^n e^{-at}u(t) \longrightarrow \dfrac{n!}{(s+a)^{n+1}}$

$t\cos(bt)u(t) \longrightarrow \dfrac{s^2-b^2}{(s^2+b^2)^2}$

$t\sin(bt)u(t) \longrightarrow \dfrac{2bs}{(s^2+b^2)^2}$

$\cos^2(bt)u(t) \longrightarrow \dfrac{s^2+2b^2}{s(s^2+4b^2)}$

$\sin^2(bt)u(t) \longrightarrow \dfrac{2b^2}{s(s^2+4b^2)}$

$\dfrac{1}{2b^3}[\sin(bt) - bt\cos(bt)]u(t) \longrightarrow \dfrac{1}{(s^2+b^2)^2}$

Butterworth filters: $|H(\omega)|^2 = \dfrac{1}{1+(\omega/\omega_c)^{2N}}$

N是 order, ω_c是 cut-off frequency

N越大, transition band 越小

exhibit no ripple in passband, shape of magnitude curve is monotonic in passband (also for stop band) maximally flat