Homework #3

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CECS 627: Digital Image Processing

Consider the images shown. The image on the right was obtained by:

a) **multiplying the image on the left by (-1)^(x+y)**
b) **computing the DFT**
c) **taking the complex conjugate of the transform**
d) **computing the inverse DFT**
e) **(3) multiplying the real part of the result by (-1)^(x+y).**

Explain mathematically why the image on the right appears as it does.

$$\mathcal{F}[F(u,v)^*] = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v)\, e^{-j2\pi(ux/M + vy/N)}$$

$$\Rightarrow \quad = \sum_{u}^{M-1} \sum_{v}^{N-1} F(u,v)\, e^{j2\pi(ux/M + vy/N)}$$

$$\Rightarrow \quad = \sum_{u}^{M-1} \sum_{v}^{N-1} F(u,v)\, e^{j2\pi(u(-x)/M + v(-y)/N)}$$

$\Rightarrow \mathcal{F}[F(u,v)^*] = f(-x,-y)$ ∴ This mirrors $f(x,y)$ about the origin.

Consider the sequence of images shown below. The image on the left is a segment of an X-ray image of a commercial printed circuit board. The images following it are, respectively the results of subjecting the image to 1, 10, and 100 passes of a gaussian highpass filter with D0 = 30. The images are of size 3030 x 334 pixels, with each pixel being represented by 8 bits of gray. The images were scaled for display, but this has no effect on the problem statement.

(a) It appears from the images that changes will cease to take place after a finite number of passes. Show whether or not this is the case. You may ignore computational round-off errors. Let cmin denote the smallest possible number representable in the machine in which the computations are conducted.

(b) If you determined in (a) that changes would cease after a finite number of iterations, determine the minimum value of that number.

a.)

Gaussian High Pass Filter

$$G(u,v) = H_{HP}(u,v)\, F_0(u,v)$$

$$= \left[1 - H_{LP}(u,v)\right] F(u,v)$$

$$= \left[1 - e^{-KD^2(u,v)/2D_0^2}\right] F(u,v)$$

→ after several passes, the filter will only pass one frequency centered at F(0,0). This is the average value of all pixels.

b.)

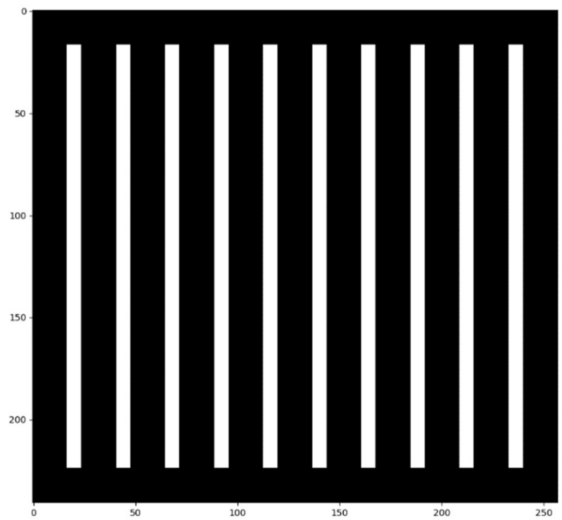
$$1 - e^{-KD^2(u,v)/2D_0^2} > 0.5c_{min}$$

$$\Rightarrow K > -\frac{2D_0^2 \ln(1-0.5c_{min})}{D^2(u,v)}$$

$$\Rightarrow K > -\frac{2D_0^2 \ln(0.5c_{min})}{D^2(u,v)}$$

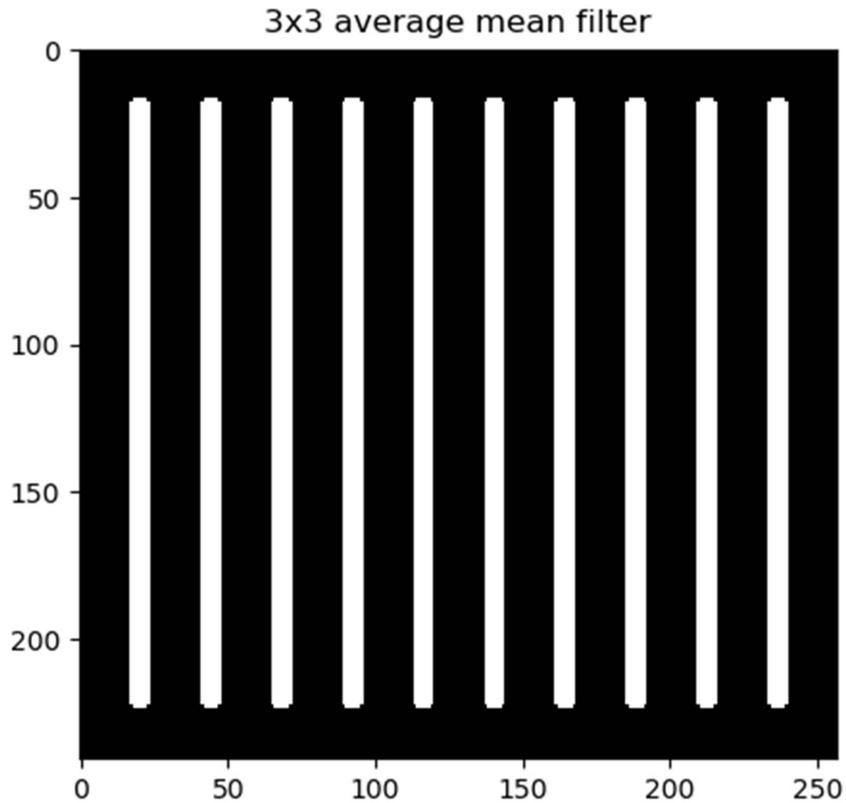
$$\Rightarrow K > -2D_0^2 \ln(0.5c_{min})$$

The white bars in the test pattern shown are 7 pixels wide and 210 pixels high. The separations between bars is 17 pixels.

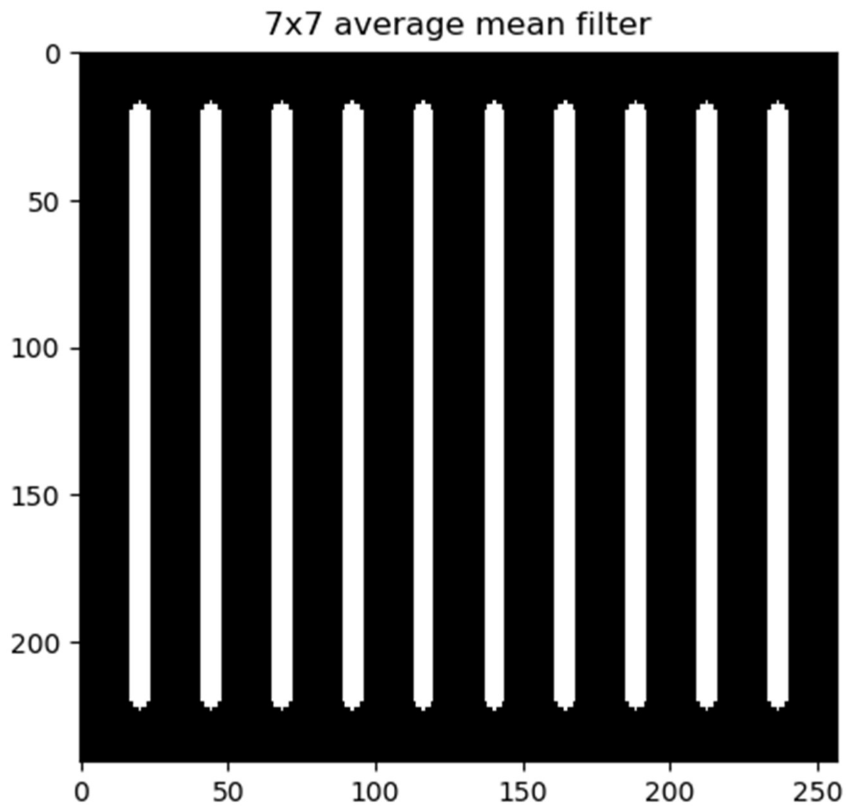


What would this image look like after application of:

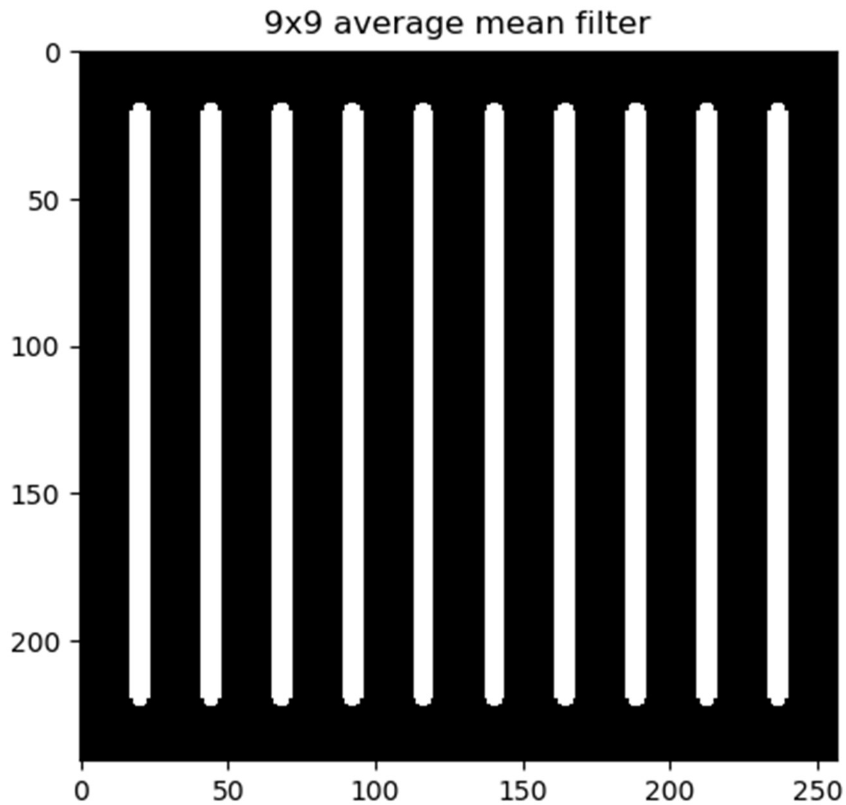
(a) A 3x3 arithmetic mean filter?



(b) A 7x7 arithmetic mean filter?



(c) A 9x9 arithmetic mean filter?



Consider an 8-pixel line of intensity data, {108, 139, 135, 244, 172, 173, 56, 99}. If it is uniformly quantized with 4-bit accuracy, compute the rms error and rms signal-to-noise ratios for the quantized data.

Intensity	Quantized Intensity	|Difference|
108	**96**	**12**
139	**128**	**11**
135	**128**	**7**
244	**240**	**4**
172	**160**	**12**
173	**160**	**13**
56	**48**	**8**
99	**96**	**3**

$$\text{Error}_{Rms} = \sqrt{\frac{1}{8}\Sigma\left(F-\hat{F}\right)^2}$$

$$= \sqrt{\frac{1}{8}\left(12^2 + 11^2 + 7^2 + 4^2 + 12^2 + 13^2 + 8^2 + 3^2\right)}$$

$$= \sqrt{\frac{1}{8}\{716\}}$$

$$\boxed{\text{Error}_{Rms} = 9.46}$$

$$SNR_{RMS} = \frac{\sqrt{\frac{1}{8}\Sigma(\hat{F})^2}}{\text{Error}_{Rms}} = \frac{\sqrt{96^2+128^2+128^2+240^2+160^2+160^2+48^2+96^2}}{9.46}$$

$$\boxed{SNR_{RMS} = 15.1}$$