Control Systems HW 4

Jud Cassady

4 | 3 | 20

6.2 $P(s) = s^5 + 6s^3 + 5s^2 + 8s + 20$

			if $K \to 0^-$	if $K \to 0^+$
1	6	8	+	+
K	5	20	−	+
$\frac{6K-5}{K}$	$\frac{8K-20}{K}$	0	+	+
$\frac{-8K^2+50K-25}{6K-5}$	20		+	+
$\frac{8K-20}{K} + \frac{120K-100}{8K^2-50K+25}$			+	−
20			+	+

→ since there are two sign changes, there are two poles on the RHP. The other 3 poles are on LHP.

6.12 $G(s) = \frac{K(s+1)}{s(s-2)(s+3)}$

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K(s+1)}{s(s-2)(s+3)}}{1 + \frac{K(s+1)}{s(s-2)(s+3)} \cdot 1}$$

$$= \frac{K(s+1)}{s^3+s^2+(K-6)s+K}$$

		if $K \to 0^-$	if $K \to 0^+$
1	$K-6$	+	+
1	K	+	+
$\frac{K-6-K}{1} = -6$		−	−
K		−	+

since s^1 term is always negative, the system is unstable for all values of K.

6.13 $G(s) = \frac{84}{s^8+5s^7+12s^6+25s^5+45s^4+50s^3+82s^2+60s}$

$T(s) = \frac{G(s)}{1+G(s)H(s)}$ where $H(s) = 1$.

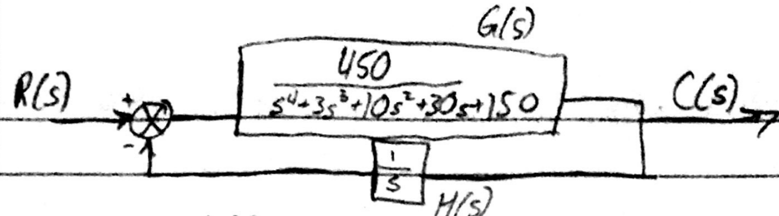
$$= \frac{84}{s^8+5s^7+12s^6+25s^5+45s^4+50s^3+82s^2+60s+84}$$

s^8	1	12	45	82	84	+
s^7	5	25	50	60		+
s^6	7	35	70	84		+
	$\cancel{0}\ 3$	$\cancel{0}\ 10$	$\cancel{0}\ 10$	0		+
	$\frac{35}{3}$	$\frac{140}{3}$	84			+
	-2	$-\frac{58}{5}$	0			−
	-21	84				−
	$-\frac{98}{5}$	0				−
	84					+

$$\frac{d}{ds}(7s^6+35s^4+70s^2+84) = 42s^5+140s^3+140s$$
$$= 3s^5+10s^3+10s$$

→ The system has two poles on $j\omega$-axis (1 row of 0), two poles on RHP (2 sign changes), and four LHP poles.

6.19



$$T(s) = \frac{G(s)}{1+G(s)\cdot H(s)} = \frac{450s}{s^5+3s^4+10s^3+30s^2+150s+450}$$

s^5	1	10	150
s^4	$\cancel{3}$ 1	$\cancel{30}$ 10	$\cancel{450}$ 150
	$\cancel{0}$ 1	$\cancel{0}$ 5	0
	5	150	
	-100	0	
	150		

$$\frac{d}{ds}(s^4+10s^2+150) = 4s^3+20s$$

→ There are two poles on RHP and three poles on LHP.

6.33 $G(s) = \frac{K(s+4)}{s(s+1.2)(s+2)}$

$P(s) = s^3 + 3.2s^2 + (2.4+K)s + 4K = 0$

1	2.4+K
3.2	4K
$\frac{7.8-0.8K}{3.2}$	
4K	

a) $\frac{7.8-0.8K}{3.2} > 0$

$0 < K < 9.6$ → system is stable

b) $K = 9.6$

c) $3.2s^2 + 4(9.6) = 0$

$s = \sqrt{-12} \Rightarrow j\omega = j\sqrt{12}$ ∴ frequency of oscillation $= \sqrt{12}$ ra

7.5 $G(s) = \frac{500}{(s+28)(s^2+8s+12)}$

- $20u(t)$

  → $K_p = \lim_{s\to 0} G(s) = \frac{500}{(28)(12)} = 1.488$

  $e(\infty) = \frac{20}{1+1.488} = 8.0386$

- $60tu(t)$

  → $K_v = \lim_{s\to 0} sG(s) = 0$

  $e_\infty = \frac{60}{0} = \infty$

- $81t^2u(t)$

  → $K_a = \lim_{s\to 0} s^2G(s) = 0$

  $e_\infty = \frac{81}{K_a} = \infty$

7.10 $G(s) = \frac{5000}{s(s+75)}$

a) $\omega_n = \sqrt{5000} \doteq 70.71$

$2\zeta\omega_n = 75 \Rightarrow \zeta = 0.531$

$\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \cdot 100\%$

$= 13.95\%$

b.) $T_s = \frac{4}{\zeta\omega_n} = 0.106s$

c) $K_p = \lim_{s\to0} G(s) = \frac{5000}{0} = \infty$

$e_\infty(5u(t)) = \frac{5}{1+K_p} = \frac{5}{\infty} = 0$

d) $K_v = \lim_{s\to0} sG(s) = \frac{5000}{75} = 66.\bar{6}$

$e_\infty(5tu(t)) = \frac{1}{66.\bar{6}} = 0.015$

e.) $K_a = \lim_{s\to0} s^2 G(s) = 0$

$e_\infty(5t^2u(t)) = \frac{1}{0} = \infty$