Gazebo Factory Pick and Place Simulation

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The goal of this paper is to present the design and analysis of a pick and place simulation using ROS 2 Jazzy [1] and Gazebo Harmonic [2].

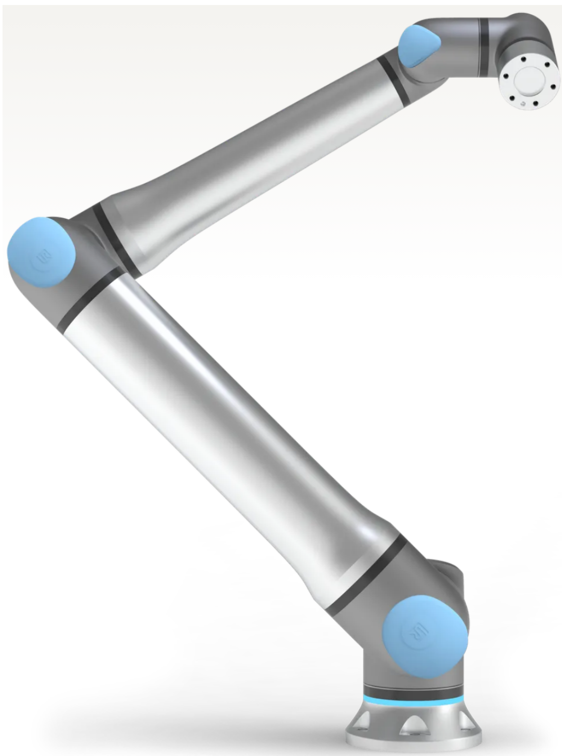
I. Introduction

The goal of our final project was to develop a factory robot. This robot would exist in a factory-like setting, and would be capable of grasping small objects and moving them around. In our specific factory, the robot, for which we chose a UR20, would pick a cube off of a table and place it onto an autonomous four-wheeled robot, specifically an Open Robotics Husky A300. This project would extend well in a real world setting, as robots like the UR20 and the Husky are increasingly becoming integral parts of day-to-day factory workflows. One can imagine a UR20 picking up any number of items and placing them on a four-wheeled robot, and then that robot moving autonomously to transport its goods to either the next stage of production or to be shipped. It was the real-world applicability of this project that inspired us to pursue it.

[3]

II. Robot Design

A. Hardware



(a) UR20 [4]

(b) Robotiq 85 Gripper [5]

Fig. 1 Side-by-side view of UR20 and Robotiq 85 Gripper.

B. Equations of Motion

A python script was created to calculate the equations of motion for the UR20 robot. The full script is included in the appendix while some important sections have been included when pertinent.

1. Denavit-Hartenberg Parameters

Denavit-Hartenberg parameters for the UR20 robot were obtained from Universal Robots documentation [6]. A diagram of the frame for each link is shown in figure 2 and a table of the Denavit-Hartenberg parameters is shown in Table 1. The diagram is the same for every UR robot, but the values of d_i and a_i are dependent on the model. Note: although figure 2 labels their links as Link 0 to Link 5, this paper will refer to them as Link 1 to Link 6.

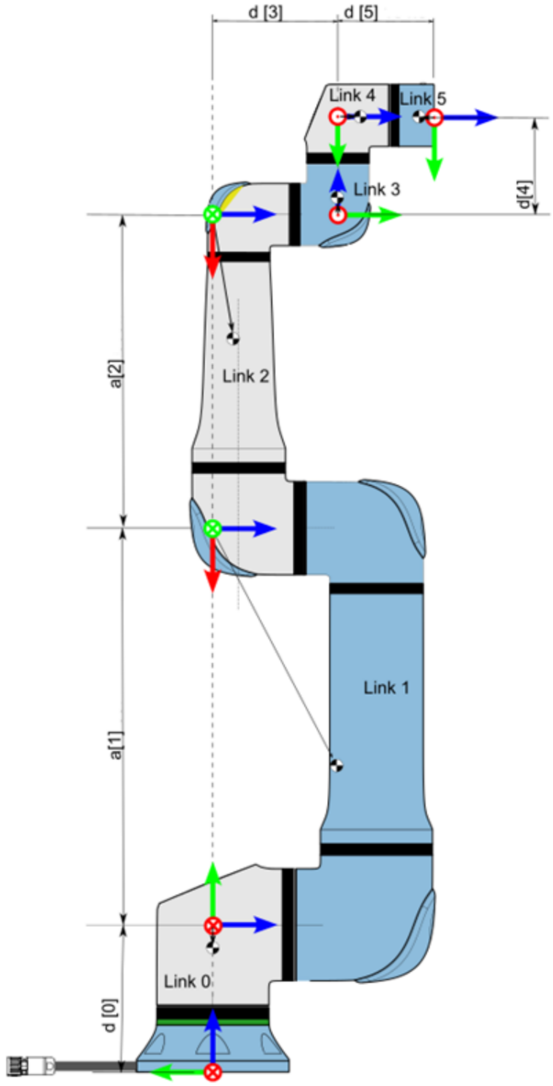


Fig. 2 UR20 DH Diagram [6]

Link	θ_i	d_i	a_i	α_i
1	θ_1^*	0.2363	0	$\frac{\pi}{2}$
2	θ_2^*	0	−0.8620	0
3	θ_3^*	0	−0.7287	0
4	θ_4^*	0.2010	0	$\frac{\pi}{2}$
5	θ_5^*	0.1593	0	$-\frac{\pi}{2}$
6	θ_6^*	0.1543	0	0

Table 1 DH Table [6]

2. *Homogenous Transforms*

Homogenous Transformation matrices were calculated using the Denavit-Hartenberg parameters and equation 1 from Spong [7].

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$A_1 = \begin{bmatrix} c_{\theta_1^*} & 0 & s_{\theta_1^*} & 0 \\ s_{\theta_1^*} & 0 & -c_{\theta_1^*} & 0 \\ 0 & 1 & 0 & 0.2363 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_{\theta_2^*} & -s_{\theta_2^*} & 0 & -0.862c_{\theta_2^*} \\ s_{\theta_2^*} & c_{\theta_2^*} & 0 & -0.862s_{\theta_2^*} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} c_{\theta_3^*} & -s_{\theta_3^*} & 0 & -0.7287s_{\theta_3^*} \\ s_{\theta_3^*} & c_{\theta_3^*} & 0 & -0.7287s_{\theta_3^*} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_{\theta_4^*} & 0 & s_{\theta_4^*} & 0 \\ s_{\theta_4^*} & 0 & -c_{\theta_4^*} & 0 \\ 0 & 1 & 0 & 0.201 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} c_{\theta_5^*} & 0 & -s_{\theta_5^*} & 0 \\ s_{\theta_5^*} & 0 & c_{\theta_5^*} & 0 \\ 0 & -1 & 0 & 0.1593 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c_{\theta_6^*} & -s_{\theta_6^*} & 0 & 0 \\ s_{\theta_6^*} & c_{\theta_6^*} & 0 & 0 \\ 0 & 0 & 1 & 0.1543 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_0^1 = A_1$

$$T_0^1 = \begin{bmatrix} c_{\theta_1^*} & 0 & s_{\theta_1^*} & 0 \\ s_{\theta_1^*} & 0 & -c_{\theta_1^*} & 0 \\ 0 & 1 & 0 & 0.2363 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_0^2 = A_1 A_2$

$$T_0^2 = \begin{bmatrix} c_{\theta_1^*}c_{\theta_2^*} & -s_{\theta_2^*}c_{\theta_1^*} & s_{\theta_1^*} & -0.862c_{\theta_1^*}c_{\theta_2^*} \\ s_{\theta_1^*}c_{\theta_2^*} & -s_{\theta_1^*}s_{\theta_2^*} & -c_{\theta_1^*} & -0.862s_{\theta_1^*}c_{\theta_2^*} \\ s_{\theta_2^*} & c_{\theta_2^*} & 0 & 0.2363 - 0.862s_{\theta_2^*} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_0^3 = A_1 A_2 A_3$

$$T_0^3 = \begin{bmatrix} c_{\theta_1^*}c_{\theta_2^*+\theta_3^*} & -s_{\theta_2^*+\theta_3^*}c_{\theta_1^*} & s_{\theta_1^*} & -c_{\theta_1^*}(0.862c_{\theta_2^*} + 0.7287c_{\theta_2^*+\theta_3^*}) \\ s_{\theta_1^*}c_{\theta_2^*+\theta_3^*} & -s_{\theta_1^*}s_{\theta_2^*+\theta_3^*} & -c_{\theta_1^*} & -s_{\theta_1^*}(0.862c_{\theta_2^*} + 0.7287c_{\theta_2^*+\theta_3^*}) \\ s_{\theta_2^*+\theta_3^*} & c_{\theta_2^*+\theta_3^*} & 0 & 0.2363 - 0.862s_{\theta_2^*} - 0.7287s_{\theta_2^*+\theta_3^*} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_0^4 = A_1 A_2 A_3 A_4$

$$T_0^4 = \begin{bmatrix} c_{\theta_1^*} c_{\theta_2^*+\theta_3^*+\theta_4^*} & s_{\theta_1^*} & s_{\theta_2^*+\theta_3^*+\theta_4^*} c_{\theta_1^*} & -c_{\theta_1^*}(0.862 c_{\theta_2^*} + 0.7287 c_{\theta_2^*+\theta_3^*}) - 0.201 s_{\theta_1} \\ s_{\theta_1^*} c_{\theta_2^*+\theta_3^*+\theta_4^*} & -c_{\theta_1^*} & s_{\theta_1^*} s_{\theta_2^*+\theta_3^*+\theta_4^*} & -s_{\theta_1^*}(0.862 c_{\theta_2^*} + 0.7287 c_{\theta_2^*+\theta_3^*}) - 0.201 c_{\theta_1} \\ s_{\theta_2^*+\theta_3^*+\theta_4^*} & 0 & -c_{\theta_2^*+\theta_3^*+\theta_4^*} & 0.2363 - 0.862 s_{\theta_2^*} - 0.7287 s_{\theta_2^*+\theta_3^*} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_0^5 = A_1 A_2 A_3 A_4 A_5$

$$T_0^5 = \begin{bmatrix} r_{11}^5 & r_{12}^5 & r_{13}^5 & r_{14}^5 \\ r_{21}^5 & r_{22}^5 & r_{23}^5 & r_{24}^5 \\ r_{31}^5 & r_{32}^5 & r_{33}^5 & r_{34}^5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11}^5 = s_{\theta_1^*} s_{\theta_5^*} + c_{\theta_1^*} c_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{12}^5 = -c_{\theta_1^*} s_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{13}^5 = s_{\theta_1^*} c_{\theta_5^*} - c_{\theta_1^*} s_{\theta_5^*} c_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{14}^5 = -c_{\theta_1^*}(0.862 c_{\theta_2^*} + 0.7287 c_{\theta_2^*+\theta_3^*}) - 0.201 s_{\theta_1} + 0.1593 c_{\theta_2^*+\theta_3^*\theta_4^*} c_{\theta_1^*}$$

$$r_{21}^5 = s_{\theta_1^*} c_{\theta_2^*+\theta_3^*+\theta_4^*} - s_{\theta_1^*} s_{\theta_5^*}$$

$$r_{22}^5 = -s_{\theta_1^*} s_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{23}^5 = -c_{\theta_1^*} c_{\theta_5^*} - s_{\theta_1^*} s_{\theta_5^*} c_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{24}^5 = -s_{\theta_1^*}(0.862 c_{\theta_2^*} + 0.7287 c_{\theta_2^*+\theta_3^*}) - 0.201 c_{\theta_1} + 0.1593 c_{\theta_2^*+\theta_3^*\theta_4^*} s_{\theta_1^*}$$

$$r_{31}^5 = s_{\theta_2^*+\theta_3^*+\theta_4^*} c_{\theta_5^*}$$

$$r_{32}^5 = c_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{33}^5 = -s_{\theta_5^*} s_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{34}^5 = 0.2363 - 0.862 s_{\theta_2^*} - 0.7287 s_{\theta_2^*+\theta_3^*} - 0.1593 c_{\theta_2^*+\theta_3^*\theta_4^*}$$

$T_0^6 = A_1 A_2 A_3 A_4 A_5 A_6$

$$T_0^6 = \begin{bmatrix} r_{11}^6 & r_{12}^6 & r_{13}^6 & r_{14}^6 \\ r_{21}^6 & r_{22}^6 & r_{23}^6 & r_{24}^6 \\ r_{31}^6 & r_{32}^6 & r_{33}^6 & r_{34}^6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11}^6 = c_{\theta_6^*}(s_{\theta_1^*} s_{\theta_5^*} + c_{\theta_1^*} c_{\theta_5^*} c_{\theta_2^*+\theta_3^*+\theta_4^*}) - c_{\theta_1^*} s_{\theta_6^*} s_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{12}^6 = c_{\theta_6^*}(c_{\theta_1^*} c_{\theta_5^*} c_{\theta_2^*+\theta_3^*+\theta_4^*} + s_{\theta_1^*} s_{\theta_5^*}) - c_{\theta_1^*} s_{\theta_6^*} s_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{13}^6 = -s_{\theta_6^*}(c_{\theta_1^*} c_{\theta_5^*} c_{\theta_2^*+\theta_3^*+\theta_4^*} - s_{\theta_1^*} s_{\theta_5^*}) - c_{\theta_1^*} c_{\theta_6^*} s_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{14}^6 = -c_{\theta_1^*}(0.862 c_{\theta_2^*} + 0.7287 c_{\theta_2^*+\theta_3^*}) - 0.201 c_{\theta_1} + 0.1593 s_{\theta_2^*+\theta_3^*\theta_4^*} c_{\theta_1^*} - 0.1543 c_{\theta_1^*} s_{\theta_5^*} c_{\theta_2^*+\theta_3^*\theta_4^*} + 0.1543 s_{\theta_1^*} c_{\theta_5^*}$$

$$r_{21}^6 = c_{\theta_6^*}(-c_{\theta_1^*} s_{\theta_5^*} + s_{\theta_1^*} c_{\theta_5^*} c_{\theta_2^*+\theta_3^*+\theta_4^*}) - s_{\theta_1^*} s_{\theta_6^*} s_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{22}^6 = c_{\theta_6^*}(s_{\theta_1^*} c_{\theta_5^*} c_{\theta_2^*+\theta_3^*+\theta_4^*} - c_{\theta_1^*} s_{\theta_5^*}) - s_{\theta_1^*} s_{\theta_6^*} s_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{23}^6 = -s_{\theta_6^*}(c_{\theta_1^*} s_{\theta_5^*} c_{\theta_2^*+\theta_3^*+\theta_4^*} - c_{\theta_1^*} s_{\theta_5^*}) - s_{\theta_1^*} c_{\theta_6^*} s_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{24}^6 = -s_{\theta_1^*}(0.862 c_{\theta_2^*} + 0.7287 c_{\theta_2^*+\theta_3^*}) - 0.201 c_{\theta_1} + 0.1593 s_{\theta_2^*+\theta_3^*\theta_4^*} s_{\theta_1^*} - 0.1543 s_{\theta_1^*} s_{\theta_5^*} c_{\theta_2^*+\theta_3^*\theta_4^*} - 0.1543 c_{\theta_1^*} c_{\theta_5^*}$$

$$r_{31}^6 = c_{\theta_6^*} s_{\theta_2^*+\theta_3^*+\theta_4^*} c_{\theta_5^*} + s_{\theta_6^*} c_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{32}^6 = -s_{\theta_6^*} s_{\theta_2^*+\theta_3^*+\theta_4^*} c_{\theta_5^*} + c_{\theta_6^*} c_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{33}^6 = -s_{\theta_5^*} s_{\theta_2^*+\theta_3^*+\theta_4^*}$$

$$r_{34}^6 = 0.2363 - 0.862 s_{\theta_2^*} - 0.7287 s_{\theta_2^*+\theta_3^*} - 0.1543 s_{\theta_5^*} s_{\theta_2^*+\theta_3^*+\theta_4^*} - 0.1593 c_{\theta_2^*+\theta_3^*+\theta_4^*}$$

3. Jacobians for $c_1 \ldots c_6$

Center of Mass values were obtained from the UR20 documentation [6] and are shown in Table 2. Note these values are relative to the link frames not the base frame. To find the center of mass relative to the base frame, the homogenous transformation matrix for each link was multiplied by the center of mass vector $O_{c_i} = T_0^i COM$.

Link	Mass [kg]	Center of Mass	Inertia Matrix
1	16.343	$\begin{bmatrix} 0 \\ -0.0610 \\ 0.0062 \end{bmatrix}$	$\begin{bmatrix} 0.0887 & -0.0001 & -0.0001 \\ -0.0001 & 0.0763 & 0.0072 \\ -0.0001 & 0.0072 & 0.0842 \end{bmatrix}$
2	29.632	$\begin{bmatrix} 0.5226 \\ 0 \\ 0.2098 \end{bmatrix}$	$\begin{bmatrix} 0.1467 & 0.0002 & -0.0516 \\ 0.0002 & 4.6659 & 0.0000 \\ -0.0516 & 0.0000 & 4.6348 \end{bmatrix}$
3	7.879	$\begin{bmatrix} 0.3234 \\ 0 \\ 0.0604 \end{bmatrix}$	$\begin{bmatrix} 0.0261 & -0.0001 & -0.0290 \\ -0.0001 & 0.75763 & 0 \\ -0.0290 & 0 & 0.7533 \end{bmatrix}$
4	3.054	$\begin{bmatrix} 0 \\ -0.0026 \\ 0.0393 \end{bmatrix}$	$\begin{bmatrix} 0.0056 & 0 & 0 \\ 0 & 0.0054 & 0.0004 \\ 0 & 0.0004 & 0.0040 \end{bmatrix}$
5	3.126	$\begin{bmatrix} 0 \\ 0.0024 \\ 0.0379 \end{bmatrix}$	$\begin{bmatrix} 0.0059 & 0 & 0 \\ 0 & 0.0058 & -0.0004 \\ 0 & -0.0004 & 0.0043 \end{bmatrix}$
6	0.846	$\begin{bmatrix} 0 \\ -0.0003 \\ -0.0318 \end{bmatrix}$	$\begin{bmatrix} 0.0009 & 0 & 0 \\ 0 & 0.0009 & 0 \\ 0 & 0 & 0.0012 \end{bmatrix}$

Table 2 Link Physical Properties [6]

J_{c_1}

$$\begin{bmatrix} \hat{z}_0 \times (\hat{O}_{c_1} - \hat{O}_0) & 0 & 0 & 0 & 0 & 0 \\ \hat{z}_0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

J_{c_2}

$$\begin{bmatrix} \hat{z}_0 \times (\hat{O}_{c_2} - \hat{O}_0) & \hat{z}_0 \times (\hat{O}_{c_2} - \hat{O}_1) & 0 & 0 & 0 & 0 \\ \hat{z}_0 & \hat{z}_1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} 0.0062\cos(\theta_1) & 0.061\cos(\theta_1) & 0 & 0 & 0 & 0 \\ 0.0062\sin(\theta_1) & 0.061\sin(\theta_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sin(\theta_1) & 0 & 0 & 0 & 0 \\ 0 & -\cos(\theta_1) & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

J_{c_3}

$$\begin{bmatrix} \hat{z}_0 \times (\hat{O}_{c_3} - \hat{O}_0) & \hat{z}_1 \times (\hat{O}_{c_3} - \hat{O}_1) & \hat{z}_2 \times (\hat{O}_{c_3} - \hat{O}_2) & 0 & 0 & 0 \\ \hat{z}_0 & \hat{z}_1 & \hat{z}_2 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} 0.3394\sin(\theta_1)\cos(\theta_2) + 0.2098\cos(\theta_1) & 0.3394\sin(\theta_2)\cos(\theta_1) & -0.5226\sin(\theta_2)\cos(\theta_1) & 0 & 0 & 0 \\ 0.2098\sin(\theta_1) - 0.3394\cos(\theta_1)\cos(\theta_2) & 0.3394\sin(\theta_1)\sin(\theta_2) & -0.5226\sin(\theta_1)\sin(\theta_2) & 0 & 0 & 0 \\ 0 & -0.3394\cos(\theta_2) & 0.5226\cos(\theta_2) & 0 & 0 & 0 \\ 0 & \sin(\theta_1) & \sin(\theta_1) & 0 & 0 & 0 \\ 0 & -\cos(\theta_1) & -\cos(\theta_1) & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

J_{c_4}

$$\begin{bmatrix} \hat{z}_0 \times (\hat{O}_{c_4} - \hat{O}_0) & \hat{z}_1 \times (\hat{O}_{c_4} - \hat{O}_1) & \hat{z}_2 \times (\hat{O}_{c_4} - \hat{O}_2) & \hat{z}_3 \times (\hat{O}_{c_4} - \hat{O}_3) & 0 & 0 \\ \hat{z}_0 & \hat{z}_1 & \hat{z}_2 & \hat{z}_3 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} 0.862\sin(\theta_1)\cos(\theta_2) + 0.4053\sin(\theta_1)\cos(\theta_2+\theta_3) + 0.0604\cos(\theta_1) & (0.862\sin(\theta_2) + 0.4053\sin(\theta_2+\theta_3))\cos(\theta_1) & 0.4053\sin(\theta_2+\theta_3)\cos(\theta_1) & -0.3234\sin(\theta_2+\theta_3)\cos(\theta_1) & 0 & 0 \\ 0.0604\sin(\theta_1) - 0.862\cos(\theta_1)\cos(\theta_2) - 0.4053\cos(\theta_1)\cos(\theta_2+\theta_3) & (0.862\sin(\theta_2) + 0.4053\sin(\theta_2+\theta_3))\sin(\theta_1) & 0.4053\sin(\theta_1)\sin(\theta_2+\theta_3) & -0.3234\sin(\theta_1)\sin(\theta_2+\theta_3) & 0 & 0 \\ 0 & -0.862\cos(\theta_2) - 0.4053\cos(\theta_2+\theta_3) & -0.4053\cos(\theta_2+\theta_3) & 0.3234\cos(\theta_2+\theta_3) & 0 & 0 \\ 0 & \sin(\theta_1) & \sin(\theta_1) & \sin(\theta_1) & 0 & 0 \\ 0 & -\cos(\theta_1) & -\cos(\theta_1) & -\cos(\theta_1) & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

J_{c_5}

$$\begin{bmatrix} \hat{z}_0 \times (\hat{O}_{c_5} - \hat{O}_0) & \hat{z}_1 \times (\hat{O}_{c_5} - \hat{O}_1) & \hat{z}_2 \times (\hat{O}_{c_5} - \hat{O}_2) & \hat{z}_3 \times (\hat{O}_{c_5} - \hat{O}_3) & \hat{z}_4 \times (\hat{O}_{c_5} - \hat{O}_4) & 0 \\ \hat{z}_0 & \hat{z}_1 & \hat{z}_2 & \hat{z}_3 & \hat{z}_4 & 0 \end{bmatrix}$$

$$\begin{matrix} (0.862\cos(\theta_2) + 0.7287\cos(\theta_2+\theta_3))\sin(\theta_1) - 0.0393\sin(\theta_1)\sin(\theta_2+\theta_3+\theta_4) + 0.1984\cos(\theta_1) & (0.862\sin(\theta_2) + 0.7287\sin(\theta_2+\theta_3) + 0.0393\cos(\theta_2+\theta_3+\theta_4))\cos(\theta_1) & (0.7287\sin(\theta_2+\theta_3) + 0.0393\cos(\theta_2+\theta_3+\theta_4))\cos(\theta_1) & 0.0393\cos(\theta_1)\cos(\theta_2+\theta_3+\theta_4) & 0.00259\cos(\theta_1)\cos(\theta_2+\theta_3+\theta_4) & 0 \\ -(0.862\cos(\theta_2) + 0.7287\cos(\theta_2+\theta_3))\cos(\theta_1) + 0.1984\sin(\theta_1) + 0.0393\sin(\theta_2+\theta_3+\theta_4)\cos(\theta_1) & (0.862\sin(\theta_2) + 0.7287\sin(\theta_2+\theta_3) + 0.0393\cos(\theta_2+\theta_3+\theta_4))\sin(\theta_1) & (0.7287\sin(\theta_2+\theta_3) + 0.0393\cos(\theta_2+\theta_3+\theta_4))\sin(\theta_1) & 0.0393\sin(\theta_1)\cos(\theta_2+\theta_3+\theta_4) & 0.00259\sin(\theta_1)\cos(\theta_2+\theta_3+\theta_4) & 0 \\ 0 & 0.0393\sin(\theta_2+\theta_3+\theta_4) - 0.862\cos(\theta_2) - 0.7287\cos(\theta_2+\theta_3) & 0.0393\sin(\theta_2+\theta_3+\theta_4) - 0.7287\cos(\theta_2+\theta_3) & 0.0393\sin(\theta_2+\theta_3+\theta_4) & 0.00259\sin(\theta_2+\theta_3+\theta_4) & 0 \\ 0 & \sin(\theta_1) & \sin(\theta_1) & \sin(\theta_1) & \sin(\theta_2+\theta_3+\theta_4)\cos(\theta_1) & 0 \\ 0 & -\cos(\theta_1) & -\cos(\theta_1) & -\cos(\theta_1) & \sin(\theta_1)\sin(\theta_2+\theta_3+\theta_4) & 0 \\ 1 & 0 & 0 & 0 & -\cos(\theta_2+\theta_3+\theta_4) & 0 \end{matrix}$$

J_{c_6}

$$\begin{bmatrix} \hat{z}_0 \times (\hat{O}_{c_6} - \hat{O}_0) & \hat{z}_1 \times (\hat{O}_{c_6} - \hat{O}_1) & \hat{z}_2 \times (\hat{O}_{c_6} - \hat{O}_2) & \hat{z}_3 \times (\hat{O}_{c_6} - \hat{O}_3) & \hat{z}_4 \times (\hat{O}_{c_6} - \hat{O}_4) & \hat{z}_5 \times (\hat{O}_{c_6} - \hat{O}_5) \\ \hat{z}_0 & \hat{z}_1 & \hat{z}_2 & \hat{z}_3 & \hat{z}_4 & \hat{z}_5 \end{bmatrix}$$

$$\begin{matrix} (0.862\cos(\theta_2) + 0.7287\cos(\theta_2+\theta_3))\sin(\theta_1) + 0.0379\sin(\theta_1)\sin(\theta_5)\cos(\theta_2+\theta_3+\theta_4) - 0.1569\sin(\theta_1)\sin(\theta_2+\theta_3+\theta_4) + 0.0379\cos(\theta_1)\cos(\theta_5) + 0.201\cos(\theta_1) & (0.862\sin(\theta_2) + 0.0379\sin(\theta_5)\sin(\theta_2+\theta_3+\theta_4) + 0.7287\sin(\theta_2+\theta_3) + 0.1569\cos(\theta_2+\theta_3+\theta_4))\cos(\theta_1) & (0.0379\sin(\theta_5)\sin(\theta_2+\theta_3+\theta_4) + 0.7287\sin(\theta_2+\theta_3) + 0.1569\cos(\theta_2+\theta_3+\theta_4))\cos(\theta_1) & (0.0379\sin(\theta_5)\sin(\theta_2+\theta_3+\theta_4) + 0.1569\cos(\theta_2+\theta_3+\theta_4))\cos(\theta_1) & -0.0379\sin(\theta_1)\sin(\theta_5) - 0.0379\cos(\theta_1)\cos(\theta_5)\cos(\theta_2+\theta_3+\theta_4) & -0.0024\sin(\theta_1)\sin(\theta_5) - 0.0024\cos(\theta_1)\cos(\theta_5)\cos(\theta_2+\theta_3+\theta_4) \\ -(0.862\cos(\theta_2) + 0.7287\cos(\theta_2+\theta_3))\cos(\theta_1) + 0.0379\sin(\theta_1)\cos(\theta_5) + 0.201\sin(\theta_1) - 0.0379\sin(\theta_5)\cos(\theta_1)\cos(\theta_2+\theta_3+\theta_4) + 0.1569\sin(\theta_2+\theta_3+\theta_4)\cos(\theta_1) & (0.862\sin(\theta_2) + 0.0379\sin(\theta_5)\sin(\theta_2+\theta_3+\theta_4) + 0.7287\sin(\theta_2+\theta_3) + 0.1569\cos(\theta_2+\theta_3+\theta_4))\sin(\theta_1) & (0.0379\sin(\theta_5)\sin(\theta_2+\theta_3+\theta_4) + 0.7287\sin(\theta_2+\theta_3) + 0.1569\cos(\theta_2+\theta_3+\theta_4))\sin(\theta_1) & (0.0379\sin(\theta_5)\sin(\theta_2+\theta_3+\theta_4) + 0.1569\cos(\theta_2+\theta_3+\theta_4))\sin(\theta_1) & -0.0379\sin(\theta_1)\cos(\theta_5)\cos(\theta_2+\theta_3+\theta_4) + 0.0379\sin(\theta_5)\cos(\theta_1) & -0.0024\sin(\theta_1)\cos(\theta_5)\cos(\theta_2+\theta_3+\theta_4) + 0.0024\sin(\theta_5)\cos(\theta_1) \\ 0 & -0.0379\sin(\theta_5)\cos(\theta_2+\theta_3+\theta_4) + 0.1569\sin(\theta_2+\theta_3+\theta_4) - 0.862\cos(\theta_2) - 0.7287\cos(\theta_2+\theta_3) & -0.0379\sin(\theta_5)\cos(\theta_2+\theta_3+\theta_4) + 0.1569\sin(\theta_2+\theta_3+\theta_4) - 0.7287\cos(\theta_2+\theta_3) & -0.0379\sin(\theta_5)\cos(\theta_2+\theta_3+\theta_4) + 0.1569\sin(\theta_2+\theta_3+\theta_4) & -0.0379\sin(\theta_2+\theta_3+\theta_4)\cos(\theta_5) & -0.0024\sin(\theta_2+\theta_3+\theta_4)\cos(\theta_5) \\ 0 & \sin(\theta_1) & \sin(\theta_1) & \sin(\theta_1) & \sin(\theta_2+\theta_3+\theta_4)\cos(\theta_1) & \sin(\theta_1)\cos(\theta_5) - \sin(\theta_5)\cos(\theta_1)\cos(\theta_2+\theta_3+\theta_4) \\ 0 & -\cos(\theta_1) & -\cos(\theta_1) & -\cos(\theta_1) & \sin(\theta_1)\sin(\theta_2+\theta_3+\theta_4) & -\sin(\theta_1)\sin(\theta_5)\cos(\theta_2+\theta_3+\theta_4) - \cos(\theta_1)\cos(\theta_5) \\ 1 & 0 & 0 & 0 & -\cos(\theta_2+\theta_3+\theta_4) & -\sin(\theta_5)\sin(\theta_2+\theta_3+\theta_4) \end{matrix}$$

4. Inertia Matrices

The Inertia matrix $D(q)$ was found following equation 2. Below is a snippet from the python code used to solve the equations of motion. Some of the inertia matrices are shown below while others are omitted for brevity.

$$D(q) = \sum_{i=1}^{n} \{ m_i J_{v_i}^T J_{v_i} + J_{\omega_i}^T R_i I_i R_i^T J_{\omega_i} \} \tag{2}$$

```
D_0 = simplify(m_1 * Jvc0[:3, :].T @ Jvc0[:3, :] + Jwc0[:3, :].T @ R_1 @ I_1 @ R_1.T @ Jwc0[:3, :])
D_1 = simplify(m_2 * Jvc1[:3, :].T @ Jvc1[:3, :] + Jwc1[:3, :].T @ R_2 @ I_2 @ R_2.T @ Jwc1[:3, :])
D_2 = simplify(m_3 * Jvc2[:3, :].T @ Jvc2[:3, :] + Jwc2[:3, :].T @ R_3 @ I_3 @ R_3.T @ Jwc2[:3, :])
D_3 = simplify(m_4 * Jvc3[:3, :].T @ Jvc3[:3, :] + Jwc3[:3, :].T @ R_4 @ I_4 @ R_4.T @ Jwc3[:3, :])
D_4 = simplify(m_5 * Jvc4[:3, :].T @ Jvc4[:3, :] + Jwc4[:3, :].T @ R_5 @ I_5 @ R_5.T @ Jwc4[:3, :])
D_5 = simplify(m_6 * Jvc5[:3, :].T @ Jvc5[:3, :] + Jwc5[:3, :].T @ R_6 @ I_6 @ R_6.T @ Jwc5[:3, :])

D = simplify(D_0 + D_1 + D_2 + D_3 + D_4 + D_5)
```

$i = 1$

$$\begin{bmatrix} 0.0763 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$i = 2$

$$\begin{bmatrix} 0.0002\sin(2\theta_2) + 2.26\cos(2\theta_2) + 2.407 & 0.01121 - 0.0516\sin(\theta_2) & 0 & 0 & 0 & 0 \\ 0.01121 - 0.0516\sin(\theta_2) & 4.745 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$i = 3$

$$\begin{bmatrix} -0.0001\sin(2\theta_2 + 2\theta_3) + 0.4492\cos(2\theta_2) + 0.3658\cos(2\theta_2 + 2\theta_3) + 1.184 & 0.5554\sin(\theta_2) - 0.029\sin(\theta_2 + \theta_3) & -0.8552\sin(\theta_2) - 0.029\sin(\theta_2 + \theta_3) & 0 & 0 & 0 \\ 0.5554\sin(\theta_2) - 0.029\sin(\theta_2 + \theta_3) & 1.652 & -0.6302 & 0 & 0 & 0 \\ -0.8552\sin(\theta_2) - 0.029\sin(\theta_2 + \theta_3) & -0.6302 & -2.22 \cdot 10^{-16}\sin^4(\theta_1) + 2.22 \cdot 10^{-16}\sin^2(\theta_1) + 2.884 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$i = 4$

$$\begin{bmatrix} 1.135\cos(2\theta_2) + 1.067\cos(\theta_3) + 1.067\cos(2\theta_2 + \theta_3) + 0.2508\cos(2\theta_2 + 2\theta_3) - 0.0008\cos(2\theta_2 + 2\theta_3 + 2\theta_4) + 1.401 & 0.159\sin(\theta_2) + 0.07476\sin(\theta_2 + \theta_3) - 0.0004\cos(\theta_2 + \theta_3 + \theta_4) & 0.07476\sin(\theta_2 + \theta_3) - 0.0004\cos(\theta_2 + \theta_3 + \theta_4) & -0.05965\sin(\theta_2 + \theta_3) - 0.0004\cos(\theta_2 + \theta_3 + \theta_4) & 0 & 0 \\ 0.159\sin(\theta_2) + 0.07476\sin(\theta_2 + \theta_3) - 0.0004\cos(\theta_2 + \theta_3 + \theta_4) & 2.22 \cdot 10^{-16}\cos(2\theta_2) + 2.134\cos(\theta_3) - 1.11 \cdot 10^{-16}\cos(2\theta_2 - \theta_3) + 1.11 \cdot 10^{-16}\cos(2\theta_2 + \theta_3) + 5.551 \cdot 10^{-17}\cos(2\theta_2 + 2\theta_3) + 2.776 & 7.806 \cdot 10^{-18}\cos(2\theta_1) + 1.067\cos(\theta_3) + 0.5071 & -0.8514\cos(\theta_3) - 0.3949 & 0 & 0 \\ 0.07476\sin(\theta_2 + \theta_3) - 0.0004\cos(\theta_2 + \theta_3 + \theta_4) & 7.806 \cdot 10^{-18}\cos(2\theta_1) + 1.067\cos(\theta_3) + 0.5071 & 0.5071 - 1.648 \cdot 10^{-17}\sin^2(\theta_1) & -0.3949 & 0 & 0 \\ -0.05965\sin(\theta_2 + \theta_3) - 0.0004\cos(\theta_2 + \theta_3 + \theta_4) & -0.8514\cos(\theta_3) - 0.3949 & -0.3949 & 0.3248 - 1.561 \cdot 10^{-17}\sin^2(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$i = 5$ See attached python script for this calculation as the text is too long to fit in the document.

$i = 6$ See attached python script for this calculation as the text is too long to fit in the document.

5. *Christoffel Symbols*

Equation 3 from Spong et al. [7] was used to calculate the Christoffel symbols. Since the values are very long, the code is shown below.

$$C_{ijk} = \frac{1}{2}\left(\frac{\partial D_{ij}}{\partial q_k} + \frac{\partial D_{ik}}{\partial q_j} - \frac{\partial D_{jk}}{\partial q_i}\right) \tag{3}$$

```
c = zeros(6, 6)
thetas = [theta_1, theta_2, theta_3, theta_4, theta_5, theta_6]
theta_dots = symbols('theta_dot_1:7')
for i in range(6):
    for j in range (6):
        c_ij = 0
        for k in range(6):
            c_ijk = (D[i, j].diff(thetas[k]) + D[i, k].diff(thetas[j]) - D[j, k].diff(thetas[i])) / 2
            c_ij += simplify(c_ijk * theta_dots[k])
        c[i, j] = c_ij
```

6. *Potential Terms*

The potential term was calculated using center of masses described in section II.B.3 and equation 4.

$$P = \sum_{i=1}^{n} m_i g^T O_{c_i} \tag{4}$$

$g(q)$ was then found by differentiating the potential term with respect to the joint angles. The full code for this term in shown below.

```
P_1 = m_1 * 9.81 * COM_1[2]
P_2 = m_2 * 9.81 * COM_2[2]
P_3 = m_3 * 9.81 * COM_3[2]
P_4 = m_4 * 9.81 * COM_4[2]
P_5 = m_5 * 9.81 * COM_5[2]
P_6 = m_6 * 9.81 * COM_6[2]
P = P_1 + P_2 + P_3 + P_4 + P_5 + P_6


g = Matrix([ simplify(P.diff(theta)) for theta in thetas ])
```

The final equations of motion are given by equation 5.

$$\tau = D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) \tag{5}$$

III. Simulation Design

industrial warehouse [8]

Husky [9]

IV. Conclusion

A video of the simulation running a successful pick and place operation can be found at `https://finalvideolink`. The video shows the UR20 robot approaching the cube, grasping it using the Robotiq 85 gripper, and then placing it ontop of the Husky A300 robot before opening the gripper and returning to a ready state. As stated, MoveIt was leveraged for implementation of equations of motion. Future work includes adding a camera to the system and detecting the position of the cube with computer vision as well as adding force feedback control.

Appendix

Equations of Motion Python Code

```
from typing import Dict

from numpy import cross

from sympy import symbols, Matrix, pprint, simplify, cos, sin, pi, zeros, latex


def solve():

    theta, theta_1, theta_2, theta_3, theta_4, theta_5, theta_6, d, a, alpha = symbols('theta theta_1 th

    A_definition: Matrix = Matrix([

        [cos(theta), -sin(theta) * cos(alpha), sin(theta) * sin(alpha), a * cos(theta)],

        [sin(theta), cos(theta) * cos(alpha), -cos(theta) * sin(alpha), a * sin(theta)],

        [0, sin(alpha), cos(alpha), d],

        [0, 0, 0, 1]

    ])

    local_homogenous_transforms: Dict[str, Matrix] = {

        'A_1': A_definition.subs({theta: theta_1, d: 0.2363, a: 0, alpha: pi / 2.}),

        'A_2': A_definition.subs({theta: theta_2, d: 0, a: -0.8620, alpha: 0}),

        'A_3': A_definition.subs({theta: theta_3, d: 0, a: -0.7287, alpha: 0}),

        'A_4': A_definition.subs({theta: theta_4, d: 0.2010, a: 0, alpha: pi / 2.}),

        'A_5': A_definition.subs({theta: theta_5, d: 0.1593, a: 0, alpha: -pi / 2.}),

        'A_6': A_definition.subs({theta: theta_6, d: 0.1543, a: 0, alpha: 0}),

    }

    T_1  = simplify(local_homogenous_transforms['A_1'])

    T_2  = simplify(T_1 @ local_homogenous_transforms['A_2'])

    T_3  = simplify(T_2 @ local_homogenous_transforms['A_3'])

    T_4  = simplify(T_3 @ local_homogenous_transforms['A_4'])

    T_5  = simplify(T_4 @ local_homogenous_transforms['A_5'])

    T_6  = simplify(T_5 @ local_homogenous_transforms['A_6'])
```

```
R_1 = T_1[:3, :3]
R_2 = T_2[:3, :3]
R_3 = T_3[:3, :3]
R_4 = T_4[:3, :3]
R_5 = T_5[:3, :3]
R_6 = T_6[:3, :3]

# Center of mass are taken from documentation
O_c_1: Matrix = Matrix([[0], [-0.0610], [0.0062], [1]])
O_c_2: Matrix = Matrix([[0.5226], [0], [0.2098], [1]])
O_c_3: Matrix = Matrix([[0.3234], [0], [0.0604], [1]])
O_c_4: Matrix = Matrix([[0], [-0.0026], [0.0393], [1]])
O_c_5: Matrix = Matrix([[0], [0.0024], [0.0379], [1]])
O_c_6: Matrix = Matrix([[0], [-0.0003], [-0.0318], [1]])

COM_1: Matrix = Matrix([[0], [0], [0.2363/2], [1]])[:3, :]
COM_2 = (T_1 @ O_c_1)[:3, :]
COM_3 = (T_2 @ O_c_2)[:3, :]
COM_4 = (T_3 @ O_c_3)[:3, :]
COM_5 = (T_4 @ O_c_4)[:3, :]
COM_6 = (T_5 @ O_c_5)[:3, :]

O_0 = Matrix([0, 0, 0])
O_1 = T_1[0:3, 3]
O_2 = T_2[0:3, 3]
O_3 = T_3[0:3, 3]
O_4 = T_4[0:3, 3]
O_5 = T_5[0:3, 3]

z_0 = Matrix([0, 0, 1])
z_1 = T_1[0:3, 2]
z_2 = T_2[0:3, 2]
z_3 = T_3[0:3, 2]
```

```
z_4 = T_4[0:3, 2]
z_5 = T_5[0:3, 2]

Jvc0_0 = simplify(Matrix(cross(z_0.T, COM_1.T - O_0.T)))
Jvc0 = Matrix([
    Jvc0_0,
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3)
]).T

Jvc1_0 = simplify(Matrix(cross(z_0.T, COM_2.T - O_0.T)))
Jvc1_1 = simplify(Matrix(cross(z_1.T, COM_2.T - O_1.T)))
Jvc1 = Matrix([
    Jvc1_0,
    Jvc1_1,
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3)
]).T

Jvc2_0 = simplify(Matrix(cross(z_0.T, COM_3.T - O_0.T)))
Jvc2_1 = simplify(Matrix(cross(z_1.T, COM_3.T - O_1.T)))
Jvc2_2 = simplify(Matrix(cross(z_2.T, COM_3.T - O_2.T)))
Jvc2 = Matrix([
    Jvc2_0,
    Jvc2_1,
    Jvc2_2,
    zeros(1, 3),
    zeros(1, 3),
```

```
    zeros(1, 3)
]).T

Jvc3_0 = simplify(Matrix(cross(z_0.T, COM_4.T - O_0.T)))
Jvc3_1 = simplify(Matrix(cross(z_1.T, COM_4.T - O_1.T)))
Jvc3_2 = simplify(Matrix(cross(z_2.T, COM_4.T - O_2.T)))
Jvc3_3 = simplify(Matrix(cross(z_3.T, COM_4.T - O_3.T)))
Jvc3 = Matrix([
    Jvc3_0,
    Jvc3_1,
    Jvc3_2,
    Jvc3_3,
    zeros(1, 3),
    zeros(1, 3)
]).T

Jvc4_0 = simplify(Matrix(cross(z_0.T, COM_5.T - O_0.T)))
Jvc4_1 = simplify(Matrix(cross(z_1.T, COM_5.T - O_1.T)))
Jvc4_2 = simplify(Matrix(cross(z_2.T, COM_5.T - O_2.T)))
Jvc4_3 = simplify(Matrix(cross(z_3.T, COM_5.T - O_3.T)))
Jvc4_4 = simplify(Matrix(cross(z_4.T, COM_5.T - O_4.T)))
Jvc4 = Matrix([
    Jvc4_0,
    Jvc4_1,
    Jvc4_2,
    Jvc4_3,
    Jvc4_4,
    zeros(1, 3)
]).T

Jvc5_0 = simplify(Matrix(cross(z_0.T, COM_6.T - O_0.T)))
Jvc5_1 = simplify(Matrix(cross(z_1.T, COM_6.T - O_1.T)))
Jvc5_2 = simplify(Matrix(cross(z_2.T, COM_6.T - O_2.T)))
```

```
Jvc5_3 = simplify(Matrix(cross(z_3.T, COM_6.T - O_3.T)))
Jvc5_4 = simplify(Matrix(cross(z_4.T, COM_6.T - O_4.T)))
Jvc5_5 = simplify(Matrix(cross(z_5.T, COM_6.T - O_5.T)))
Jvc5 = Matrix([
    Jvc5_0,
    Jvc5_1,
    Jvc5_2,
    Jvc5_3,
    Jvc5_4,
    Jvc5_5
]).T

Jwc0 = Matrix([
    z_0.T,
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3)
]).T

Jwc1 = Matrix([
    z_0.T,
    z_1.T,
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3)
]).T

Jwc2 = Matrix([
    z_0.T,
    z_1.T,
```

```
    z_2.T,
    zeros(1, 3),
    zeros(1, 3),
    zeros(1, 3)
]).T

Jwc3 = Matrix([
    z_0.T,
    z_1.T,
    z_2.T,
    z_3.T,
    zeros(1, 3),
    zeros(1, 3)
]).T

Jwc4 = Matrix([
    z_0.T,
    z_1.T,
    z_2.T,
    z_3.T,
    z_4.T,
    zeros(1, 3)
]).T

Jwc5 = Matrix([
    z_0.T,
    z_1.T,
    z_2.T,
    z_3.T,
    z_4.T,
    z_5.T
]).T
```

```
Jc0 = Matrix([
    Jvc0,
    Jwc0
])

Jc1 = Matrix([
    Jvc1,
    Jwc1
])

Jc2 = Matrix([
    Jvc2,
    Jwc2
])

Jc3 = Matrix([
    Jvc3,
    Jwc3
])

Jc4 = Matrix([
    Jvc4,
    Jwc4
])

Jc5 = Matrix([
    Jvc5,
    Jwc5
])

for Jc_index, Jc in enumerate([Jc0, Jc1, Jc2, Jc3, Jc4, Jc5]):
    print(f"\nJc_{Jc_index+1}:")
    pprint(Jc)
```

```
    with open(f"Jc_{Jc_index+1}.tex", "w") as f:
        f.write(r"\documentclass{article}" + "\n")
        f.write(r"\usepackage{amsmath}" + "\n")
        f.write(r"\begin{document}" + "\n")
        f.write(r"\[\text{Transformation Matrix: }" + latex(Jc) + r"\]" + "\n")
        f.write(r"\end{document}" + "\n")

# kg
m_1 = 16.343
I_1 = Matrix([
    [0.0887, -0.0001, -0.0001],
    [-0.0001, 0.0763, 0.0072],
    [-0.0001, 0.0072, 0.0842]
])

m_2 = 29.632
I_2 = Matrix([
    [0.1467, 0.0002, -0.0516],
    [0.0002, 4.6659, 0.0000],
    [-0.0516, 0.0000, 4.6348]
])

m_3 = 7.8
I_3 = Matrix([
    [0.0261, -0.0001, -0.0290],
    [-0.0001, 0.75763, 0],
    [-0.0290, 0, 0.7533]
])

m_4 = 3.054
I_4 = Matrix([
    [0.0056, 0, 0],
```

```
    [0, 0.0054, 0.0004],
    [0, 0.0004, 0.0040]
])

m_5 = 3.126
I_5 = Matrix([
    [0.0059, 0, 0],
    [0, 0.0058, -0.0004],
    [0, -0.0004, 0.0043]
])

m_6 = 0.846
I_6 = Matrix([
    [0.0009, 0, 0],
    [0, 0.0009, 0],
    [0, 0, 0.0012]
])

D_0 = simplify(m_1 * Jvc0[:3, :].T @ Jvc0[:3, :] + Jwc0[:3, :].T @ R_1 @ I_1 @ R_1.T @ Jwc0[:3, :])
D_1 = simplify(m_2 * Jvc1[:3, :].T @ Jvc1[:3, :] + Jwc1[:3, :].T @ R_2 @ I_2 @ R_2.T @ Jwc1[:3, :])
D_2 = simplify(m_3 * Jvc2[:3, :].T @ Jvc2[:3, :] + Jwc2[:3, :].T @ R_3 @ I_3 @ R_3.T @ Jwc2[:3, :])
D_3 = simplify(m_4 * Jvc3[:3, :].T @ Jvc3[:3, :] + Jwc3[:3, :].T @ R_4 @ I_4 @ R_4.T @ Jwc3[:3, :])
D_4 = simplify(m_5 * Jvc4[:3, :].T @ Jvc4[:3, :] + Jwc4[:3, :].T @ R_5 @ I_5 @ R_5.T @ Jwc4[:3, :])
D_5 = simplify(m_6 * Jvc5[:3, :].T @ Jvc5[:3, :] + Jwc5[:3, :].T @ R_6 @ I_6 @ R_6.T @ Jwc5[:3, :])

D = simplify(D_0 + D_1 + D_2 + D_3 + D_4 + D_5)
print("D:")
pprint(D.evalf(4))

c = zeros(6, 6)
thetas = [theta_1, theta_2, theta_3, theta_4, theta_5, theta_6]
theta_dots = symbols('theta_dot_1:7')
for i in range(6):
```

```
    for j in range (6):
        c_ij = 0
        for k in range(6):
            c_ijk = (D[i, j].diff(thetas[k]) + D[i, k].diff(thetas[j]) - D[j, k].diff(thetas[i])) /
            c_ij += simplify(c_ijk * theta_dots[k])
        c[i, j] = c_ij
        print(f"c[{i}, {j}] = {c[i, j]}")
print("c:")
pprint(c)

P_1 = m_1 * 9.81 * COM_1[2]
P_2 = m_2 * 9.81 * COM_2[2]
P_3 = m_3 * 9.81 * COM_3[2]
P_4 = m_4 * 9.81 * COM_4[2]
P_5 = m_5 * 9.81 * COM_5[2]
P_6 = m_6 * 9.81 * COM_6[2]
P = P_1 + P_2 + P_3 + P_4 + P_5 + P_6
print("P:")
pprint(P)

g = Matrix([ simplify(P.diff(theta)) for theta in thetas ])

print("\n\n\ng(q):")
pprint(g)

with open("robot_dynamics.tex","w") as f:
    f.write(r"\documentclass{article}"+"\n")
    f.write(r"\usepackage{amsmath}"+"\n")
    f.write(r"\begin{document}"+"\n")
    f.write(r"\section*{Inertia Matrix}"+"\n")
    f.write(r"\[ D = " + latex(D.evalf(4)) + r"\]"+"\n")
    f.write(r"\section*{Coriolis Matrix}"+"\n")
    f.write(r"\[ C = " + latex(c.evalf(4)) + r"\]"+"\n")
```

```
        f.write(r"\section*{Gravity Terms}"+"\n")
        f.write(r"\[ g(q) = " + latex(g.evalf(4)) + r"\]"+"\n")
        f.write(r"\end{document}")

if __name__ == "__main__":
    solve()
```

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