

**Problem Chosen**

**B**

**2024  
MCM/ICM  
Summary Sheet**

**Team Control Number**

**2422307**

---

**Title**

**Summary**

这里写美赛的摘要，美赛中摘要格外重要!! 官方对于摘要的说明：

*The summary is an essential part of your MCM/ICM paper and should appear as the first page of your solution report. The judges place considerable weight on the summary, and winning papers are often distinguished from other papers based on the quality of the summary.*

翻译：摘要是美赛论文的重要组成部分，应该放在论文的第一页展示。评委们对摘要相当重视，获奖论文和其他论文的区别往往就在于摘要的好坏！

标题下面的 Summary 可要可不要，如果你觉得你摘要写的不够多，页面下方留白太大的话就加上，这样看起来稍微好看点。

注意：美赛要求英文写作，很多同学英文写作水平不够，可以先写成中文的论文，然后再进行翻译！翻译可以使用有道翻译或者谷歌翻译，翻译后一定要人工修改，机器翻译的很生硬，对专有名词的翻译也不是很准确。.....

关键词记得加粗

**Keywords:** keyword1; keyword2; keyword3; keyword4

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# 1 Introduction

## 1.1 Problem Background

As the pearl of the Mediterranean, the Ionian Sea unites pristine waters with lush green islands, high culture and low-key authentic island lifestyle, beautiful beaches and top cuisine. With its breathtaking beaches and pristine ocean, this area is well-known to thousands of tourists all over the world!

To fully utilize local marine tourism resources, Maritime Cruises Mini Submarines (MCMS) launched an underwater sunken shipwreck adventure program. Their submersibles could take visitors to more challenging and colorful oceans, even the underwater worlds! Before the program was launched, it was significant to obtain the full confidence of the Greek regulatory authorities regarding the program's safety and dependability. Therefore, a well-established model that is specifically tailored to meet the requirements of undersea rescue operations must be formulated.

In this model, it is essential to take the reality of the situation into account, which includes not only potential issues with the submarine itself, such as mechanical defects and loss of connection but also the specific geography and marine environment of its location. In this way, we can ensure that our rescue team is ready to act at the first indication of an accident and maximize the chances of survival for the crew.

In this report, we intend to employ rigorous mathematical techniques to enhance the accuracy of our predictions for the whereabouts of lost vessels in diverse scenarios. Moreover, we will propose relevant rescue measures and devices corresponding to the predictions.

## 1.2 Restatement of the Problem

The Ionian Sea is known to have many unpredictable elements and complexities. Upon considering the contextual information and the specific constraints, the restatement of the objectives can be expressed as follows:

- \* **Objective 1:** Develop a predictive model that incorporates the uncertainties. And from a submersible's point of view, state which equipment should be installed to send regular messages to the host vessel to reduce these uncertainties.
- \* **Objective 2:** Establish an evaluation model to assess what type of equipment the host vessel is supposed to bring in the rescue. If necessary, consider and evaluate the device that should be carried on the rescue vessel to aid in rescuing individuals.
- \* **Objective 3:** Based on the previous two objectives, develop a model for recommending initial deployment points and search patterns, promising minimal rescue time. Furthermore, taking the time and accumulated search results as a variable, determine the probability of finding the submersible.
- \* **Objective 4:** Generalize our model to other attractions and consider changes in the model when multiple targets are present at the same time

\* **Objective 5:** Summarize the findings and prepare a 2-page memo to the Greek government to win approval.

### 1.3 Our Work

Based on the comprehensive review of the existing strategies, our work mainly includes the following :

**Strategy I:**

**Strategy II:**

**Strategy III:**

**Strategy IV:**

**Model I: Human-land Relationship Coupling Analysis for Objective 1**

- Coupling analysis model for human-land interaction analysis
- Rasterized MMNR model for grid classification

**Model II: Multi-objective Strategy Selection Model for Objective 2**

- Strategy evaluation model for assessment and ranking
- Multiple-objective optimization model for specific selection

In summary, the whole process can be expressed as follows:

**Figure1: flow chart of our work**

**Our Work:** 对分析思路和建模的框架简要介绍,类似于国赛论文中的问题分析部分。

大家如果看特等奖论文的话,会发现很多论文在这一部分都绘制了一个图形来介绍文章的思路,大家可以模仿模仿。我给大家看两个例子:

**例 1: 2019 特等奖论文 Team # 1916704**

## 2 Assumptions and Justifications

**Assumption 1:**

假设发生事故后,潜水艇的机械功能不会出现新的缺失 It is assumed that there will be no new deficiencies in the mechanical functioning of the submarine after an accident

**Justification:**

**Assumption 2: Assuming that the submarine has the same surface area on all sides**

**and that the submarine attitude does not affect the state of motion** 假设潜水艇各面迎水面面积相同, 潜水艇姿态不影响运动状态

**Justification:**

**Assumption 3: Assuming the hypothetical submarine loses power, movement is only affected by currents and buoyancy due to density differences** 假设假设潜水艇失去动力后, 移动只受海流和密度差导致的浮力影响

**Justification 3:**

**Assumption 4: Assuming that the direction of the currents at the location where the power is lost is not changing.** 假设失去动力后所处的位置洋流方向不在发生变化

**Justification 4:**

**Assumption 5: Considering uplift or sinking, it can be assumed that the process takes place at a constant acceleration** 考虑上浮或下沉，可以假设这个过程是以恒定加速度进行

**Justification 5:**

### 3 Definitions and Notations

#### 3.1 Definitions

The glossaries utilized in this paper are enumerated as follows:

A:

B:

C:

D:

#### 3.2 Notations

The key mathematical notations used in this paper are listed in Table 1.

**Table 1: Notations used in this paper.**

Symbol	Description	Unit
$V_m$	the velocity of the current at a particular depth z	m/s

本部分对应国赛论文中的符号（英文翻译为 notation 或者 symbol）说明部分。

另外，少数论文有一个 glossary 部分，这一部分主要是对文中出现的专有名词或者模糊的概念进行定义，这样可以帮助读者理解题目。

如果你需要写这一个部分的话，可以和 Notations 合并在一起变成一个大部分，该部分的命名为“Definitions and Notations”，写作时再把这个大的部分分成两个小的部分（用二级标题的方式）分别写即可。

### 4 Model Preparation—7 页

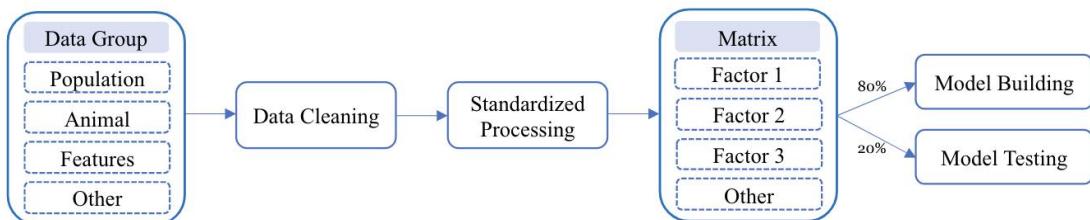
#### 4.1 Data collection

**Table 2 Data source collation(整理)**

Data Description	Data Resources	Types

## 4.2 Data Pre-processing/Data screening 也可以将数据可视化

--流程图

**Figure 4 Flowchart of data pre-processing**

## 5 Submersible prediction model

To make a more accurate prediction of the lost submersible's location, we divide it into two categories: lost totally and the presence of mechanical defects after diving. To help understanding, we have adopted a narrative model that progresses from simple to complex.

### 5.0 Lucky Submersible

Imagine a scenario where a submersible has gone missing, but it is lucky enough that its engines are not affected by ocean currents and maintain a steady speed. In such a situation, based on the latest data transmission, we can surmise where the submersible is now based on the most recent data., as there are no sudden changes in its motion. According to the velocity and displacement equation  $\vec{x}_0 = \vec{v}_0 t$ , it is relatively easy to predict the submersible's position in this scenario. However, due to the low frequency of such cases. As the name of the situation suggests, it is quite 'lucky' for the people in the submersible!

### 5.1 Awful mechanical device

The next area of focus pertains to the safety of individuals within the submersible, where every minute is of utmost value in the event of an actual occurrence. To more efficiently address the issue in the absence of engine power, we have subcategorized the problem into two areas: horizontal and vertical motion.

#### 5.1.1 Speed of ocean currents

Whether the submersible is moving horizontally or vertically, the effect of ocean currents is

always our primary goal to consider in the absence of engineer power. Therefore, in our following paper, we regard the speed of the ocean currents as a determining factor in our model.

According to the equations in the literature [1]:

$$V_m = A \sinh [a(H - z)] + B \cosh [a(H - z)] \quad (1)$$

Where,

$V_m$  represents the velocity of the current at a particular depth  $z$ ,

$H$  represents the depth of the sea at this point,

$a$  represents a constant related to the vertical scale of the flow velocity distribution,

$A$ ,  $B$  represent the parameters to be solved for this piece of ocean area

To simplify the model, we consider the average depth of the Ionian Sea 1400m as  $H$ , bring the surface flow velocity of this piece of seawater  $v=0.08\text{m/s}$  and the bottom boundary condition into the above equation,

Connecting the equations:

$$\begin{cases} 0.08 = A \sinh (aH) + B \cosh (aH) \\ 0 = A \sinh (0) + B \cosh (0) \end{cases} \quad (2)$$

Then we can get:

$$\begin{cases} A = 0.08 / \sinh(14000) \\ B = 0 \end{cases} \quad (3)$$

Therefore, the original equation, is expressed as:

$$V_m = 0.08 \cdot \sinh(14000-z) / \sinh(14000) \quad (4)$$

In this equation, we can quantitatively describe the relationship between ocean depth  $z$  and current flow velocity, which will set the stage for later calculations.

### 5.1.2 The relative ‘vagrancy’ at neural buoyancy points

In this case of levitation, we don't need to consider forces in the vertical direction.

In the horizontal direction, we can split the velocity into the velocity under the influence of ocean currents and the initial velocity and discuss their effects on the position of the submersible separately. Therefore, we can use vector splitting to map the velocity and force respectively to the  $x$ ,  $y$  direction in the Cartesian coordinate system. Considering that there may be an angle between the currents and the submarine, which means that different combinations of the current direction and submersible motion direction will produce different situations, we take the pitch angle and the horizontal velocity angle as a flexible variable quantity and use trigonometric functions to quantify the effect of different directions on the submarine motion state in the currents.

Along these lines, we can get:

$$\begin{cases} V_{mx} = \frac{dx_1}{dt} = V_m \cos(\psi) \cos(\theta) \\ V_{my} = \frac{dy_1}{dt} = V_m \sin(\psi) \sin(\theta) \end{cases} \quad (1)$$

Where  $V_{mx}$ ,  $V_{my}$  respectively represent the component velocities affected by the wave in the  $x$ ,  $y$  directions in the Cartesian coordinate system

$\psi$  represents the angle between the wave velocity and the initial velocity of the submersible

in the horizontal direction.

$\theta$  represents the angle between the submersible's head orientation and the direction of the current in the vertical direction.

With this formula we can forecast the situations that will occur in the change of submersible's practical situations under the influence of the ocean currents. However, at the same time we cannot ignore the effect that the initial speed of the ship will have on the displacement. Based on the previous results, we need to consider these two factors together:

$$\begin{cases} \vec{z_x} = \vec{x_1} + \vec{x_0} \\ \vec{z_y} = \vec{y_1} + \vec{y_0} \end{cases} \quad (2)$$

Where  $\vec{x_0}$ ,  $\vec{y_0}$  present, respectively, the displacements produced under the influence of the initial velocity of the submersible.

$\vec{z_x}$ ,  $\vec{z_y}$  respectively represent the total displacement of the submersible in the x, y directions.

Based on this formula, we can calculate the change in vessel displacement in the horizontal direction to better help prediction.

### 5.1.3 The absolute vagrancy

In the previous model we quantified the change in the motion of the submersible in the horizontal direction. In this subsection, we complicate it again. This time we will consider the change in the position of the submersible when it is not at the point of neutral buoyancy. In other words, on top of what we have done above, we are going to take the influence of multiple factors in the vertical direction into our accounts.

- ◆ Step1: The density of seawater surface

The surface density of seawater often shows a strong sensitivity to longitude: as longitude increases, the surface density of seawater usually increases. Based on Global Geographic Information System (GIS) documentation[2], we can get the surface density of the Ionian sea  $\rho_0=1.2$

- ◆ Step2: Density of ocean currents at depth z

我们利用数据点得到拟合的函数以表示出 z 与密度的关系式:

$$\rho_w = \rho_0 + Pz + \varphi z^2 + B(1 - e^{-cz}) \quad (1)$$

Where  $\rho_w$  and  $\rho_0$  represent the density of water at a given depth and at the sea-level

P ---- some linear effect of pressure on seawater density

$\varphi$ ---- the nonlinear effect with depth due to salinity, temperature and so on.

B----represents the influence of biogeochemical processes or other factors

c---- represents some unpredictable factors such as geochemical factors

根据文献资料与 matlab 函数拟合的结果

We can get the relationship between depth z and the corresponding current:

$$\begin{cases} \rho_w = \rho_0 + 0.015z - 0.000015z^2 + 2(1 - e^{-0.1z}) \quad (0 < z < 1000m) \\ \rho_w \approx \rho_0 + 0.003 \quad (z \geq 1000m) \end{cases} \quad (2)$$

进一步查阅资料[3]，we learned that for deep water, the physical and chemical properties of the ocean are relatively stable, so this further argues for the validity of our model.

◆ Step3:受力分析

Considering that the submarine is unable to float itself, it is reasonable to assume that the submarine's hull density will not change at this point, and for the hull itself, according to Newton's second law equation: $F=ma$ , 我们可以得到:

$$m \frac{d^2 z_1}{dt^2} = V(\rho_w - \rho_v)g - \frac{1}{2} C_d \cdot \rho_w \cdot A \left( \frac{dz_1}{dt} \right)^2 \quad (3)$$

Where

$z_1$  stands for the displacement due to the sum of forces other than currents

$t$  represents the lost time of shipwreck

$m$ ,  $V$ ,  $\rho_v$  represent the mass, volume and density of the submersible, respectively.,

$g$  represents the acceleration of gravity,

$C_d$  represents the drag coefficient of seawater,

To get more accurate data, we will use the data from one of the more popular submarines on the market today (Alicia[4]) for our survey. Taking  $A=10.93m^2$ ,  $m = 18t$ ,  $V = 17.56m^3$  into the above equation, we can obtain the curve about the depth of the submarine as a function of time with the effects of buoyancy and gravity :

$$z_1 = 2.1229t^2 + 2.5754t - 0.7535 \quad (4)$$

At the same time, we are also supposed to consider the effect of ocean currents on it in the vertical direction. Due to the diversity and complexity of the direction of ocean currents, we need to utilize the angle  $\theta$  between the submarine's head orientation and the direction of the ocean currents in the vertical direction, and we can simplify its motion as:

$$z_2 = z_m \sin \theta t \quad (5)$$

◆ Step4:阶段性总结，结合船体。。。和洋流的影响，我们便可算出船在垂直方向上的合位移，为：

$$z = z_1 + z_2 \quad (6)$$

## 5.2 The test of the Modl 1—检验模型的正确性

在直接将模型运用到实际中往往是不严谨的，所以，我们要先对模型进行检验.考虑到。。。因素，所以我们运用。。。方法进行检验

### 5.3 题目所述问题的实际解决方案

这里可以写第一个模型的求解，把实际问题归结为一定的数学模型后，就要利用数学模型求解所提出的实际问题了。一般需要借助计算机软件进行求解，例如常用的软件有 Matlab, Spss, Lingo, Excel, Stata, Python 等。求解完成后，得到的求解结果应该规范准确并且醒目，若求解结果过长，最好编入附录里。（注意：如果使用智能优化算法或者数值计算方法求解的话，需要简要阐明算法的计算步骤）

To have a better feeling  
of .....adgwkhhgkahsg

可以分为多个维度来解决问题，然后策略+详细解释

➤ Strategy 1: balabala

### 6 The name of model 2

和上一个部分类似的写法

**伪代码:**

---

**Algorithm:** Simulation and prediction of animal numbers and resident incomes.

---

**Input:**  $i, t, \alpha$

**Output:**  $A, x$

**Initialize the desired variables based on  $i, t$  and  $\alpha$ .**

**for**  $t = 1$  to **50** **do**

According to  $x_m, r, x$  simulate animal growth in the presence of environmental carrying capacity.

According to  $m$  mapped from  $\alpha$ , the quantized coefficients of each policy are determined for the next step of calculation.

**if** the relevant policy in  $i$  is selected, **execute** the relevant policy.

When the local people violate the relevant policies implemented, the punishment is imposed on the people, and the effect will react on  $A$  and  $x$  through the Equation of (11), (12), (13).

When the implementation of relevant policies is conducive to the economic development of local people, the value of  $x$  will be calculated through the Equation of (11), (12).

**end**

**end**

---

**Draw statistical graphs according to  $A, i$  and  $x$  for analysis.**

---

**Algorithm:** the name of the daima.....

---

**Input:**

**Output:**

**end**

**Function:** .....

---

**Algorithm :** Probability distribution of extreme fire events

---

**Input:**  $t, x_i (i=1, 2 \dots)$

**Output:**  $p(x, t, n)$

**for**  $k=1$  to **10000** **do**

According to  $x_i, t$ , the parameter  $\lambda$  of Poisson distribution can be calculated

Randomly select an integer  $a$  from the aggregate  $A=\{0, 1, 2 \dots 9, 10\}$

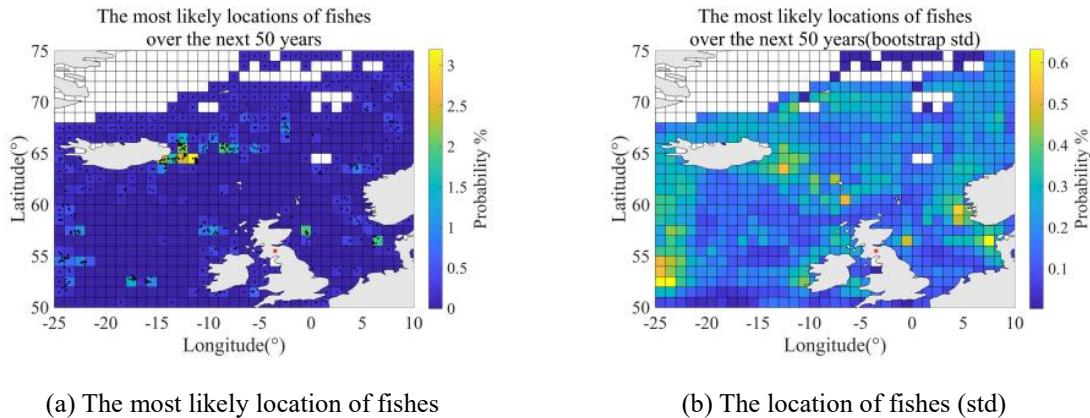
The probability of  $a$  in the entire dataset can be predicted based on the Poisson distribution

Then the probability is recorded as  $p(x, t, a)$

**end**

Draw the probability density distribution of extreme fire events

## 7 The name of model 3



(a) The most likely location of fishes

(b) The location of fishes (std)

**Figure 9: Prediction location of fishes over the next 50 years**

注意：大多数美赛优秀论文都是对每个问题或者每个模型作为一个一级标题，就像我们上面的这种布局；也有一部分论文建立一个大的一级标题，取名为“Models and Solutions”，然后在这个大的标题下设计每个问题或者模型对应的二级标题，这一种排版布局在国赛中用的更多

A	B	C	D
A	B	C	D

## 8 Model testing

### 8.1 Sensitivity Analysis

模型的分析：在建模比赛中模型分析主要有两种，一个是灵敏度(性)分析，另一个是误差分析。灵敏度分析是研究与分析一个系统（或模型）的状态或输出变化对系统参数或周围条件变化的敏感程度的方法。其通用的步骤是：控制其他参数不变的情况下，改变模型中某个重要参数的值，一个值改变多次，然后观察模型的结果的变化情况。误差分析是指分析模型中的误差来源，或者估算模型中存在的误差，一般用于预测问题或者数值计算类问题。--图

结果表明模型仍然可以保持他的高效率，换句话说，我们的模型具有高稳定性，高容

错率和广泛的适用性

## 8.2 Robustness Analysis

模型的检验：模型检验可以分为两种，一种是使用模型之前应该进行的检验，例如层次分析法中一致性检验，灰色预测中的准指数规律的检验，这部分内容应该放在模型的建立部分；另一种是使用了模型后对模型的结果进行检验，数模中最常见的是稳定性检验，实际上这里的稳定性检验和前面的灵敏度分析非常类似，等会大家看到例子就明白了。

在美赛的写作中，写的最多的就是灵敏度分析（Sensitivity Analysis），因此这里我们的标题就直接取得是灵敏度分析；如果你既要写灵敏度分析，又要写误差分析（Error Analysis），那么你可以把标题改成： Sensitivity Analysis and Error Analysis

# 9 Model Evaluation and Further Discussion

注：本部分的标题需要根据你的内容进行调整，例如：如果你没有写进一步讨论的话，就直接把标题写成模型的评价。（优缺点一定要写）

## 9.1 Strengths

➤ **Strength 1: The migration direction of population is predictable.**

⇒ **Explanation:** Although the swimming direction of each individual does not necessarily follow the law of migration, according to the law of large numbers, the behavior of the group will exclude the existence of unpredictable accidental factors, so we can predict the migration direction of fish by predicting the change of ocean temperature.

➤ **Strength 2:**

⇒ **Explanation:**

➤ **Strength 3:**

⇒ **Explanation:**

## 9.2 Weaknesses

➤ **Weakness 1:**

⇒ **Improvement:**

➤ **Weakness 2:**

⇒ **Improvement:**

## 9.3 Further Discussion

进行进一步的讨论，这里可以写模型的改进和拓展：

模型的拓展：将原题的要求进行扩展，进一步讨论模型的实用性和可行性。

## 10 Conclusion

结论部分，这个部分在国赛论文很少见到，但在美赛中出现的频率很高。

这个部分可以是论文中心思想的重申、研究结果或主要观点的归纳，也可以是某些启示性的解释或考虑。

有些论文把“Model Evaluation and Further Discussion”的内容放到了结论部分，这也是可以的，大家可以灵活调整。

**Article:**

## References

参考文献：所有引用他人或公开资料(包括网上资料)的成果必须按照科技论文的规范列出参考文献，并在正文引用处予以标注。

一般新起一页列出参考文献，如果上一个部分的下面有很多空白，那么就不用新起一页了。

美赛中不要出现中文，如果引用中文文献请翻译过来。

[1] Jing Zhenhua, A research on the calculation of sea current flow velocity, ACTA OCEANOLOGICAL SINICE, Vol 3, No.1(1981)

[2]

[3] B

[4] E

[5] F

[6]

## Appendices

### Appendix 1

Explanation: 这里放上附录 1 的介绍

### Appendix 2

Explanation: 这里放上附录 2 的介绍

本部分是附录部分，美赛对于附录不是特别看重，今年还限制了论文的页数（从第二页开始编号，不能超过 25 页）。

一般新起一页列出附录。

在不超过页数限制的条件下，附录中可以包括下面内容：

- ◆ 你们写的代码；某一问题的详细证明或求解过程；
- ◆ 自己在网上找到的数据；

- ◆ 比较大的流程图；
- ◆ 较繁杂的图表或计算结果。

## Report on Use of AI

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*OpenAI ChatGPT (Nov 5, 2023 version, ChatGPT-4)*

**1.Query:**

\* Purpose: polish our article

**Output:**

\* Analysis:

**2.Query:**

**Output:**

*OpenAI ChatGPT (Nov 5, 2023 version, ChatGPT-4)*

**1.Query:**

\* Purpose: polish our article

**Output**

\* Analysis:

**2.Query:**

**Output:**

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