SEMESTER 2
2020-2021

CS211FZ
Algorithms and Data Structures 2

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Time allowed: 2 hours

Answer ***five*** questions

Instructions

	Yes	No
Log Books allowed		■
Formula Tables allowed		■
Other allowed *(enter details)*		■

General *(Enter Details)*

QUESTION 1 [20 marks]

The hash function h(key) = key % 9 has been used to insert the following keys into an empty array of length 9:

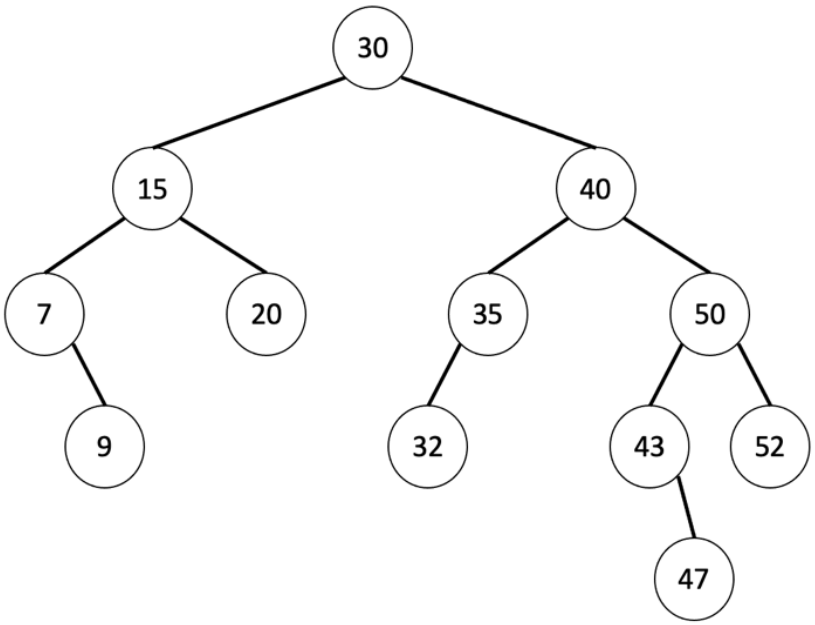
5, 28, 19, 15, 20, 33, 12, 17, 10

(i) Suppose that, for simplicity, we do not distinguish a key from its hashcode i.e. that no effort is made to avoid collisions. Determine the keys and slots for which collisions occur? (10 mark)

(ii) Now suppose you use linear probing. For each of the given keys, write down each step of the process of inserting that key, and show the slots used by each key once all of the keys have been inserted. (10 mark)

QUESTION 2 [20 marks]

(a) Draw the AVL tree that results when you delete 35 from the AVL tree given below. (10 marks)

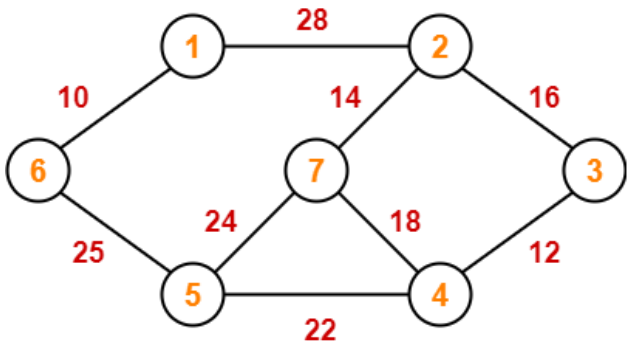


(b) What is the maximum number of black elements in a red-black tree of height 4? Note that the height of a tree is the number of edges on the longest path from the root to a leaf. (5 marks)

(c) What is the minimum number of black elements in a red-black tree of height 3? Draw an example to verify your answer. (5 marks)

QUESTION 3 [20 marks]

(a) Using Kruskal's Algorithm, show how you would construct the minimum spanning tree (MST) for the graph below. You should show all of your workings and you must draw all vertices and the state (edges) for each step. (10 marks)



(b) What is the worst-case time complexity of the following Algorithms?

(i) Prim's Algorithm with matrix representation (2 mark)

(ii) Prim's Algorithm with adjacent list representation and binary heap (2 mark)

(iii) Prim's Algorithm with adjacent list representation and Fibonacci heapheap (2 mark)

(iv) Breadth First Search (2 mark)

(v) Search an element in a binary search tree with n nodes (2 mark)

QUESTION 4 [20 marks]

(a) Draw a simple example of a directed graph with 5 vertices and negative-weight edges for which Dijkstra's algorithm produces incorrect answers. And explain why Dijkstra's algorithm cannot find the correct answer. (10 marks)

(b) Complete the following implementation in Java to find the minimum product possible with the subset of elements present in the array using the greedy approach. (10 marks)

```
static int minProductSubset(int a[], int n) {
    int negmax = Integer.MIN_VALUE;
    int posmin = Integer.MAX_VALUE;
    int count_neg = 0, count_zero = 0;
    int product = 1;

    …

}
```

QUESTION 5 [20 marks]

(a) Define the following types of problems, and give an example of each.

a. P problems (3 marks)

b. NP problems (3 marks)

c. NP-complete problems (3 marks)

(b) Prove the following statement. (11 marks)
"Suppose $Y \in$ **NP**-complete. Then $Y \in$ **P** if and only if **P = NP.**"