An investigative study on filtering

This is an [R Markdown](http://rmarkdown.rstudio.com) Notebook. When you execute code within the notebook, the results appear beneath the code.

Try executing this chunk by clicking the *Run* button within the chunk or by placing your cursor inside it and pressing *Ctrl+Shift+Enter*.

The whole lot of words are treated in english

Hereby I listed some functions using the markdown language for the filtering functions.

## The moving average equation

$$\hat{x} = \mathcal L(y)$$

$$\hat{x}\_t = \sum\_{i=-\infty}^{\infty} \mathcal L\_{t, t-i}y\_{t-i}$$

$$\hat{x}\_t = \sum\_{i=0}^{n-1} \mathcal L\_{i}y\_{t-i}$$

$$\mathcal L\_i = \frac{1}{n}\mathbf{1}\left\{
{i < n}
\right\}$$

$$\hat{x}\_t = \sum\_{i=0}^{n-1}\mathcal L\_i \mathbf{L^i} y\_t$$

with the lag operator is satisfying

## Measuring the trend and its derivative

$$\frac{d \mathcal S\_t}{\mathcal S\_t} = \mu\_t \mathop{d}t + \sigma\_t d W\_t$$

where

$$\hat{x}\_t = \sum\_{i=0}^{n-1} \mathcal L\_{i}y\_{t-i}$$

$$\hat{\mu}\_t \simeq \frac{1}{\Delta}\sum\_{i=0}^{n-1} \mathcal L\_{i}R\_{t-i}$$

$$\hat{\mu}\_t \simeq \frac{1}{\Delta}\sum\_{i=0}^{n-1} \mathcal l\_{i}y\_{t-i}$$

$$
\mathcal l\_i =
\begin{cases}
\mathcal L\_0 & if&i=0 \\
\mathcal L\_i - \mathcal L\_{i-1} & if&i=1, ..., n-1\\
-\mathcal L\_{n-1} & if&i=n
\end{cases}
$$

$$\mu\_t = \frac{1}{2}\sigma\_t^2 + \frac{1}{\Delta}\sum\_{i=0}^{n-1}\mathcal L\_i R\_{t-i}$$

## Moving average filters

$$\mathcal L\_i = \frac{1}{n}\mathbf{1}\left\{
{i < n}\right\}$$

$$\mathcal l\_i = \frac{1}{n\Delta}\left(\delta\_{i,0} - \delta\_{i,n} \right)$$

## Moving average crossovers

$$\mathcal l\_i = \frac{4}{n^2}sgn\left(\frac{n}{2}-i\right)$$

$$\mathcal L\_i = \frac{4}{n^2}\left(\frac{n}{2}-\left|i-\frac{n}{2}\right|\right)$$

$$\mathcal L\_i = \frac{2}{n^2}\left(n-i\right)\mathbf{1}\left\{{i < n}\right\}$$

$$\mathcal l\_i=\frac{2}{n}\left(\delta\_i - \mathbf{1}\left\{{i < n}\right\} \right)$$

$$\mathcal l\_i = \frac{12}{n^3}\left(\frac{n}{2}-i\right)\mathbf{1}\left\{{0 \leq i \leq n}\right\}$$

$$\mathcal L\_i = \frac{6}{n^3}i\left(n-i\right)\mathbf{1}\left\{{0 \leq i \leq n}\right\}$$

## The objective function:

where

The noise process is:

$$\begin{array}\\
\epsilon\_t = \mathcal N(0,1)\\
\eta\_t = \mathcal N(0,1)\\
\zeta\_t = \mathcal N(0,1)
\end{array}$$

## For nonlinear filtering.

$$\omega\_t \,=\,\mathcal K \left( \frac{\tau - t}{h} \right) $$

First we calculate the residual

The we compute with

Interval

with the condition that

another condition is because the optimum is reached for

## L1 filtering

The vectorial form is:

## Wavelet filtering

We note $y(\omega) = \mathcal F(y)$, so that

wavelat coefficients $\omega=\mathcal W(y)$

With a denoising rule :

$$x=\mathcal W^{-1}\left(\omega^\*\right)$$

Let , be two scalars with

* Hard Shrinkage

\* Soft Shrinkage

\* Semi-soft shrinkage

* Quantile shrinkage is a hard shrinkage method where is the quantile of the coefficients

## Multivariate filtering

where and is the signal and noise.

The average filter

We note and .

Let's now define

We note that

Error correction model:

where

$$\
\begin{align}
\hat{x}\_t & \,=\, \frac1m \sum\_{j=1}^{m} \sum\_{i=0}^{n-1} \mathcal{L}\_i y\_{t-i}^{(j)} \\
&\,=\, \sum\_{i=0}^{n-1} \mathcal{L}\_i \left( \frac1m \sum\_{j=1}^m y\_{t-i}^{(j)} \right) \\
&\,=\, \sum\_{i=0}^{n-1} \mathcal L\_i \bar{y}\_{t-i}
\end{align}$$

### Common Stochastic Trend Model

with $y\_t=(y\_t{(1)},...,y\_t{(m)}), \_t=(\_t^{(1)},...,\_t^{(m)}) (0, ) $ and

## Calibration Problem

### calibrate base on prediction error

We may estimate the set of parameters by maximizing the log-likelihood function.

where $v\_t=y\_t-*{t-1}[y\_t] $ is the innovation process and $F\_t =* {t-h}[v\_t^2 ] $

Another way is to consider the log-likelihood function in frequency domain analysis. The stationary formof is . The associated log-likelihood function is:

where is the periodogram of and is the spectral density:

because we have:

### Calibration based on benchmark estimator

### Trend detection vs trend filtering

We can show that:

Normalise the score we have

Obviously that

$$Z\_t^{(n)} \stackrel{\longrightarrow}{n\rightarrow\infty} \mathcal N(0,1) $$

with

## Kalman Filter notes

The process model is described as