CIS 121—Data Structures and Algorithms with Java - Fall 2015

Lab 2 - TBD

Learning Goals

During this lab, you will:

- review Bachmann-Landau notation
- examine certain functions and their relative asymptotic growth rates
- examine the runtime complexity of code
- prove Bachmann-Landau relations

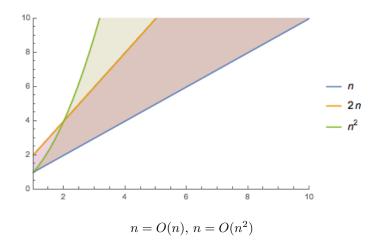
Big-Oh and Bachmann-Landau Notation

In class, you have started to discuss Big Oh and other ways of classifying functions and algorithms. These notations belong to what is commonly referred to as the *Bachmann-Landau* family of notations.

Big-Oh Notation

Definition. f(n) = O(g(n)) if there exist constants n_0 and c > 0 s.t. $f(i) \le cg(i)$ for all $i \ge n_0$.

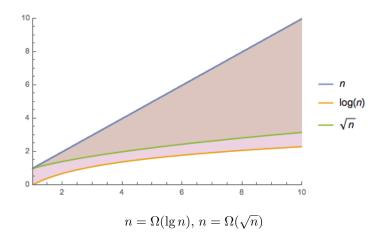
Simplified: If f(n) is O(g(n)), g(n) is an asymptotic upper bound for f(n).



Big-Omega Notation

Definition. $f(n) = \Omega(g(n))$ if there exist constants n_0 and c > 0 s.t. $f(i) \ge cg(i)$ for all $i \ge n_0$.

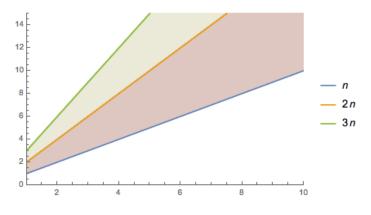
Simplified: If f(n) is $\Omega(g(n))$, g(n) is an asymptotic lower bound for f(n).



Big-Theta Notation

Definition. $f(n) = \Theta(g(n))$ if f(n) = O(g(n)) and $f(n) = \Omega(g(n))$.

Simplified: If f(n) is $\Theta(g(n))$, g(n) is an asymptotic tight bound for f(n).



 $n = \Theta(n)$. Every function is Big- Θ of itself.

As a protip, it is also good to note that the Bachmann-Landau notations refer to classes of functions. When you read f(n) = O(g(n)), this is equivalent to the statement:

$$f(n) \in O(g(n))$$

. Specifically, f(n) is in the class of functions which are asymptotically bounded above by g(n). Likewise, Big- Ω and Big- Θ both reflect classes of functions.

$$n! \sim (\frac{n}{e})^n \sqrt{2\pi n}$$

Lab Problems

Problem 1

Order the following functions such that if f precedes g, then f(n) is O(g(n)).

$$\sqrt{n}$$
, n , $n^{1.5}$, n^2 , $n \lg n$, $n \lg \lg n$, $n \lg n^2$, $2^{n/2}$, 2^n , $\lg(n!)$, $n^2 \lg n$, n^3 , 2^{2^n}

Problem 2

Provide a runtime analysis of the following loop:

```
for(int i = 0; i < n; i++)
for (int j = i; j <= n; j++)
   for (int k = i; k <= j; k++)
        sum++;</pre>
```

Problem 3

In this problem, you are **not** allowed to use the theorems about Big-Oh stated in the lecture notes. Your proof should follow exclusively from the definition of Big-Oh.

Prove or disprove the following statement:

$$f(n) + g(n)$$
 is $\Theta(\max\{f(n), g(n)\})$, where $f, g: R \to R^+$.

Problem 4

Prove or disprove the following statement:

$$2^n$$
 is $O(n!)$.

Problem 5

Provide a runtime analysis of the following loop:

```
for (int i = 2; i < n; i = i*i)
  for (int j = 1; j < Math.sqrt(i); j = j+j)
      System.out.println("*");</pre>
```

Problem 6

Prove or disprove the following statement:

$$\lg(n!)$$
 is $\Theta(n \lg n)$.

Logarithms Cheat Sheet

Exponential terms appear very frequently in the study of algorithms and their runtimes. Therefore, logarithms are very useful when manipulating exponential terms in Big-Oh proofs! It is therefore advised that you become very familiar with logs.

Here's a little cheat-sheet for you to refresh your memory!

Properties of Logarithms

$$\log(c \cdot f(n)) = \log c + \log f(n)$$
$$\log x^y = y \log x$$
$$\log a + \log b = \log(ab)$$
$$\log a - \log b = \log(a/b)$$
$$\log_b x = \frac{\log_c x}{\log_c b}$$
$$\log 2^n = n$$

$$\log n! = \log[n \cdot (n-1)...2 \cdot 1] = \log n + \log(n-1) + ... + \log 1 = \sum_{i=1}^{n} \log i$$