Random Walks & Percolation

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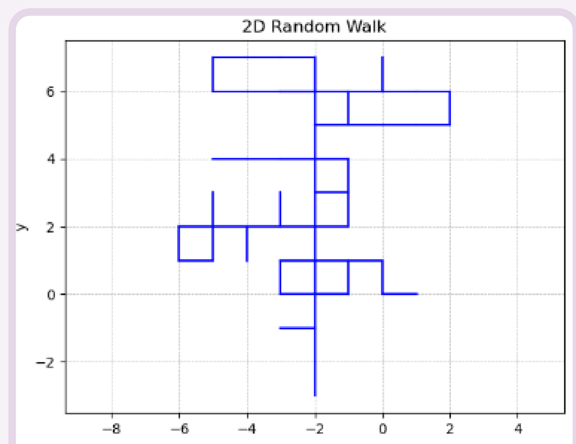
Background

We use Monte Carlo simulations, which are random sampling methods, to model complex systems with inherent uncertainty. Our project investigates various types of random walks on lattices, including non-reversing, self-avoiding, and biased walks, in up to five dimensions. We analyze key properties such as mean squared displacement, loop formation, and step count to understand how dimensionality and walk constraints influence spatial behavior.

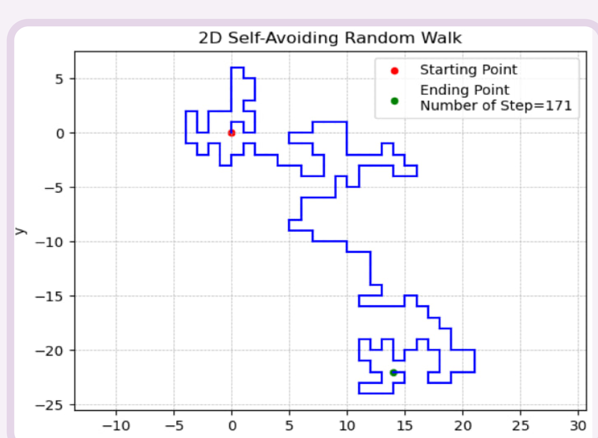
Using Monte Carlo simulations, we also model bond and site percolation on lattices to explore phase transitions, critical probabilities, and cluster formation as system size and occupation probability vary. This is the percolation theory, which studies the emergence of connectivity in random systems, such as fluid flow.

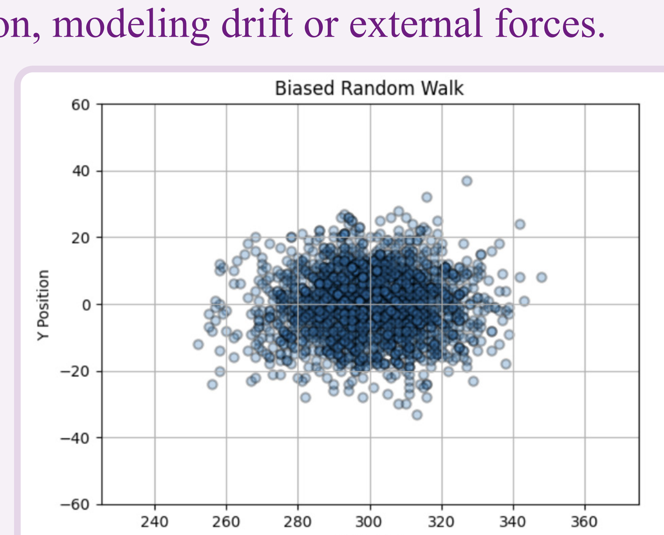


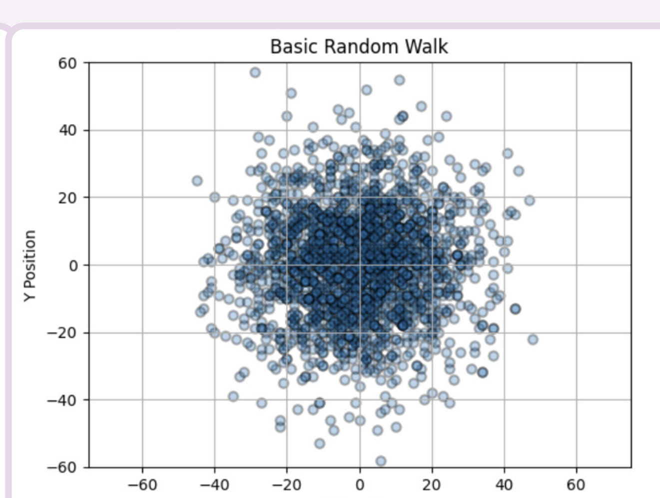
Random Walks

Types of Walks:

We simulate different types of random walks using Monte Carlo simulations. In the basic walk, a “four-sided die”chooses a direction at each step. A non-reversing walk remembers its last move and avoids backtracking. A self-avoiding walk has full memory and never revisits the same site. In a biased walk, we use a weighted “ten-sided die” to favor movement in one direction, modeling drift or external forces.





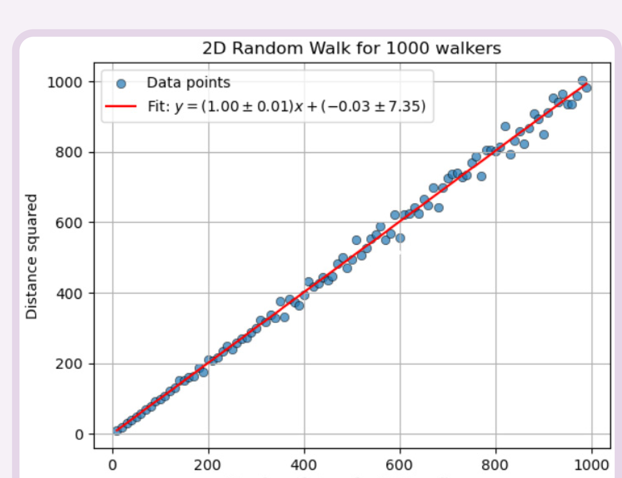


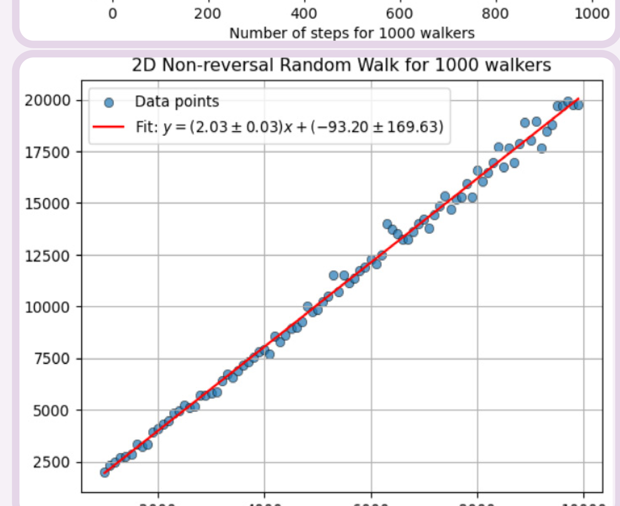
Results

Basic Walk - 1D: 1.000 ± 0.007. 2D: 1.000 ± 0.005. 3D: 1.000 ± 0.004. 4D: 0.999 ± 0.003. 5D: 1.000 ± 0.003.

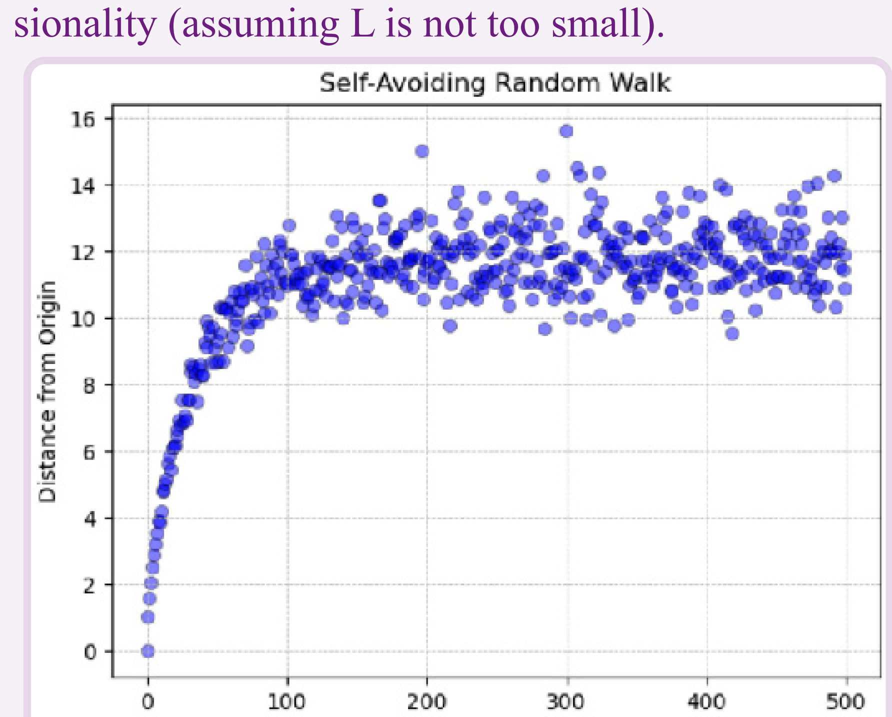
Non-Reversing -

Dimension	Slope (m)	Uncertainty
1	1	0
2	2.0331	0.0281
3	1.4887	0.0195
4	1.34	0.02
5	1.25	0.01





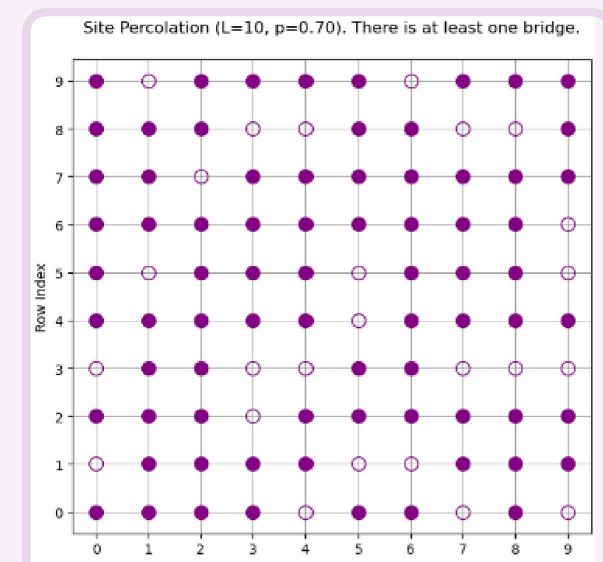
Self-Avoiding - The end-to-end distance grows non-linearly with step count and eventually plateaus, with the saturation value depending on the dimensionality (assuming L is not too small).



Lattice Percolation

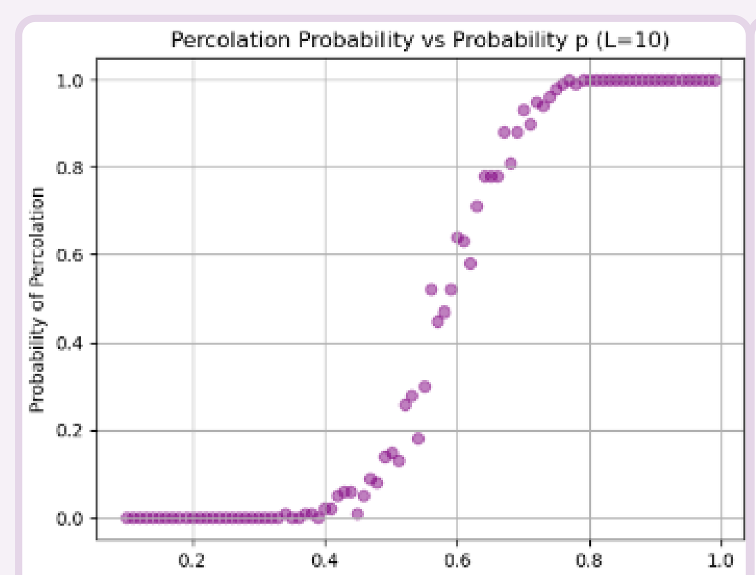
Methods

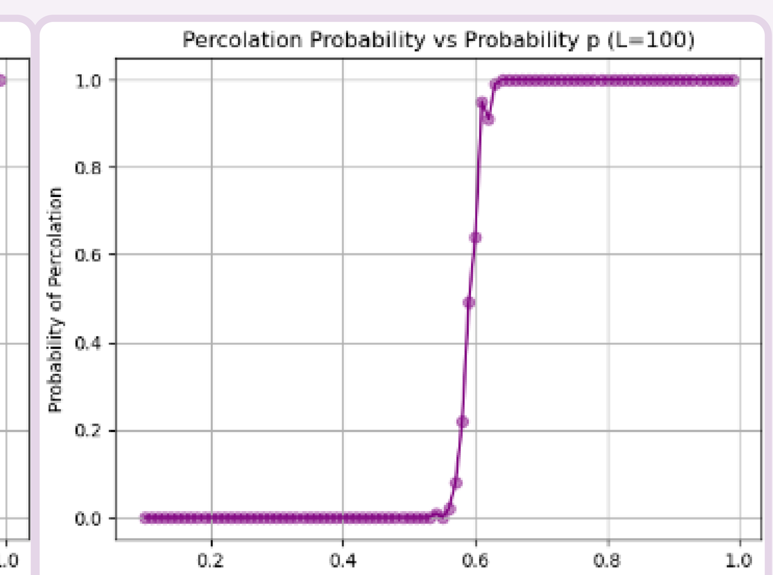
- For percolation we start off with creating a square grid.
- With chosen probability, P, we sweep through the grid and make every junction, or site, either occupied or unoccupied.
- There is a chance a bridge will be created by these occupied sites connecting one side of the grid to the other.
- We also tested the relationship between the number of clusters and the probability P.



Results (Percolation Probability vs Occupation Probabilty)

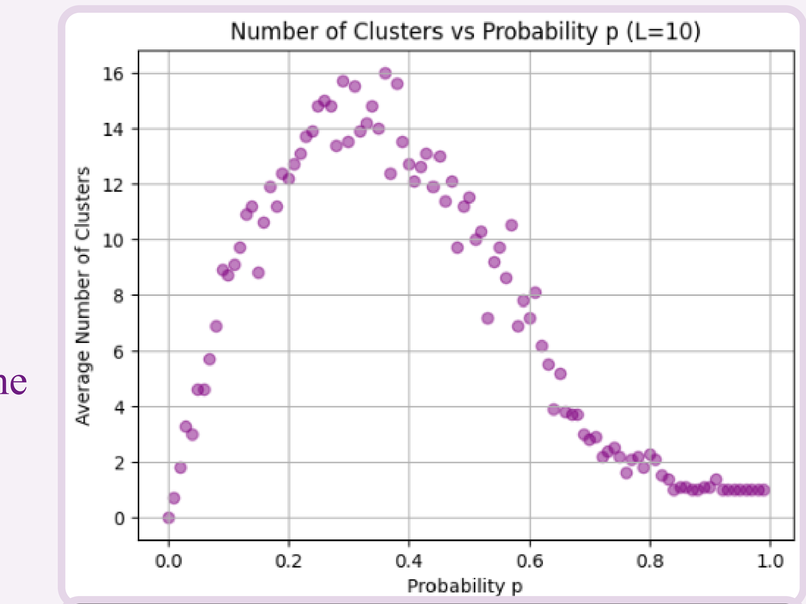
We that around P = 60% ± 5% there is a phase transition and the lattice has a full almost every time where as P < 60% there is almost never a bridge. This transition gets sharper as L increases

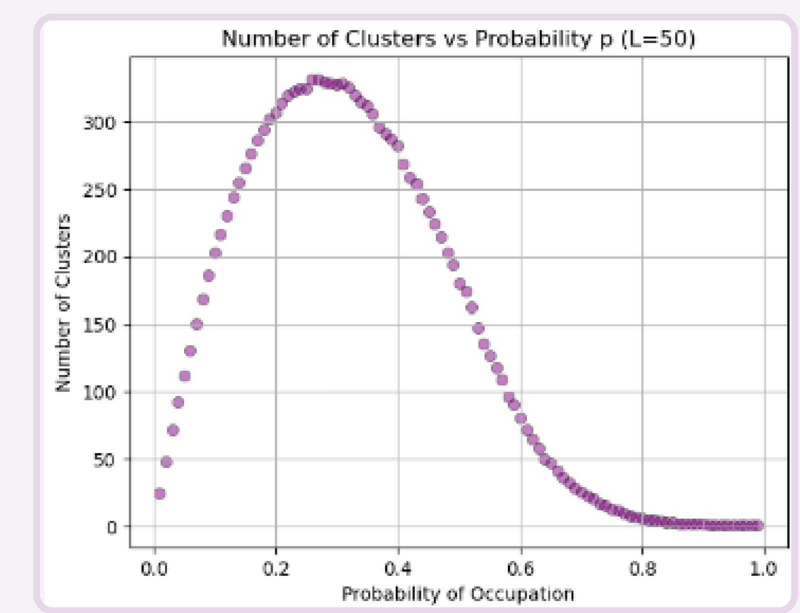


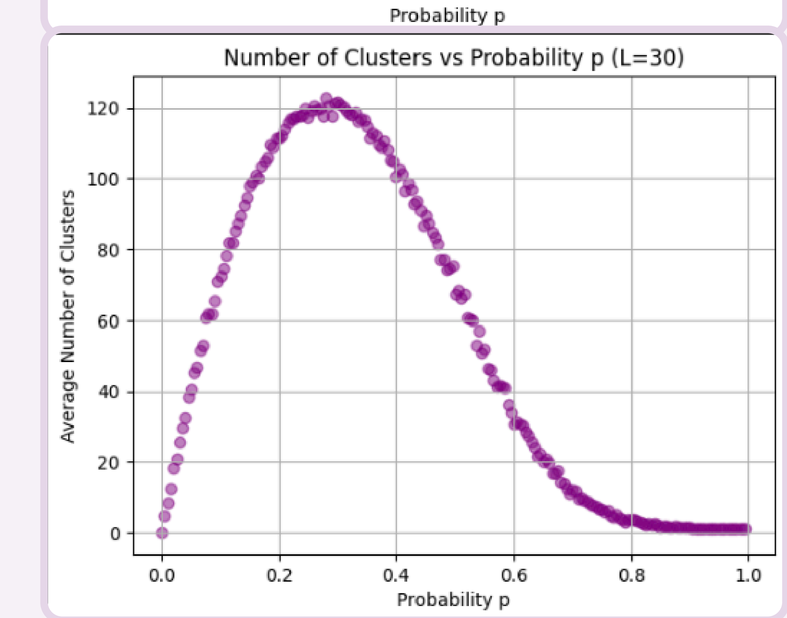


Results (# Clusters vs Probability)

- The number of clusters peaks when P = 30% ± 8%.
- This is a peak and not a phase transition.
- P < 30% there are not enough occupied sites
- P > 30% many clusters merge together to become one cluster.
- This behavior gets more defined as L increases.







Conclusion

- For random walks we measured the linear relationship between mean squared displacement and number of steps for dimensions one through five; our values align with the theoretical value of one for every dimension.
- In non-reversible random walks the theoretical results of mean squared displacement vs steps for dimensions two, three, and four are: 2D: 2, 3D: 1.5, 4D: 1.33[3]; our experimental result: 2D: 2.0331 ± 0.0281, 3D: 1.4887 ± 0.0195, 4D: 1.34 ± 0.02 are quite close to these theoretical measurements.
- In the percolation problem, the theoretical result for what probability the phase transition occurs is 0.59274621 ± 0.00000013 which is within the error bars we got of 0.60 ± 0.05.

References

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