

# Discrete Sliding Mode control of small UAS in tight formation flight under information constraints

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**Abstract:** This paper is concerned with a new control strategy based on discrete sliding mode control of small Unmanned Aerial Systems (UAS) in tight formation flight under information constraints. It addresses the problem of managing and coordinating several UAS to fly in a tight control formation. Since the formation of  $n$  UAS to be controlled is considered to fly in an arbitrary pattern in this work, a high-performance relative position control becomes an important issue. Indeed, a robust control strategy based on the sliding mode approach is proposed to achieve the desired flight performances while assuming realistic information constraints imposed by limited communication bandwidth and availability of relative localization sensors and preserving scalability with respect to formation size.

Also, this paper presents a meaningful study and comparison of the discrete sliding mode control design (DSMC) and time sampling continuous sliding mode control (TSCSMC). Indeed, it is important to study the performance differences between the two approaches that allow to choose the best strategy to be used to solve the considered problem. Here, the comparison focuses on the effect of the sampling on the control error, the mesh stability analytically and the mesh stability in simulation.

Simulation results of a flight scenario with two different sampling frequencies proves the efficiency of the proposed control strategy and shows clearly the effect of the discretization and the sampling on the formation flight behaviour of the UAS obtained by the chosen considered control.

*Keywords:* Flight formation, Aerospace unmanned vehicle, sliding mode control.

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## 1. INTRODUCTION

Tight formation flight of aircraft can enable energy savings thanks to aerodynamic performance gains. The benefits of tight formation flight for bird-sized aircraft have been studied theoretically as early as in Hummel (1982). Both later theoretical (?), wind-tunnel results (?) and flight tests with manned aircraft consistently confirm energy savings of the order of 10%.

Numerous other applications such as distributed aerial imaging benefit from accurate relative position holding.

Enabling tight formation flight for small UAS is interesting for a number of reasons. Firstly, small UAS can serve as an inexpensive testbed for larger aircraft. Secondly,

small electrically driven UAS suffer from poor operational ranges.

Riding the wake of another aircraft, the follower is subject to large unknown wake disturbances. Sliding mode control provides robustness against large unknown perturbations.

Sliding mode control has successfully been applied to ground vehicle platooning (?). Higher order sliding mode has been proposed for UAS formation flight by Galzi and Shtessel (2006).

, DSMC, ACC on HOSMC for formations, applications of SMC for vehicle platoons

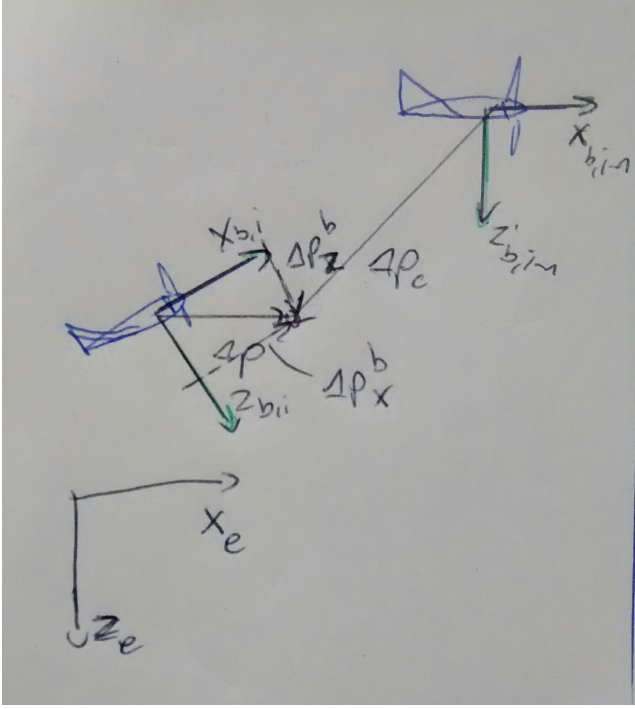


Fig. 1. Predecessor-follower geometry longitudinal

## 2. MODEL

### 2.1 Coordinate frames

The system to be controlled is a formation of  $n$  UAS flying in an arbitrary pattern. It is assumed that load factors are controlled in each vehicle's body frame (index  $b$ ). Therefore separation errors are transformed into the body frame for control purposes, decoupling the three axes. The dynamics of each vehicle are defined in a local inertial North-East-Down frame (index  $e$ ).

To dispose of a common reference frame for multiple members of the formation, the **Leader Speed Frame** (index  $ls$ ) is defined. Its  $x$  axis is aligned with the formation leader's NED speed projected on the horizontal plane, its  $z$  axis is aligned with the NED frame's  $z$  axis, and its  $y$  axis completes a right-handed Cartesian coordinate system. It is used to define the separation vectors that make up the formation.

### 2.2 Vehicle Model

The continuous time vehicle position dynamics w.r.t. the local inertial frame are given by

$$\mathbf{x} = (\mathbf{p} \ \mathbf{v})^T \quad (1)$$

$$\dot{\mathbf{p}} = \mathbf{v} \quad (2)$$

$$\dot{\mathbf{v}} = \mathbf{a}_c + \mathbf{a}_w + \mathbf{g} \quad (3)$$

where  $\mathbf{p} \in \mathbb{R}^3$  is the vehicle position,  $\mathbf{v} \in \mathbb{R}^3$  is its velocity w.r.t. to the local inertial frame,  $\mathbf{a}_w \in \mathbb{R}^3$  are accelerations induced by exogenous disturbances such as turbulence and another aircraft's wake,  $\mathbf{a}_c \in \mathbb{R}^3$  are commanded accelerations and  $\mathbf{g} \in \mathbb{R}^3$  is the gravity vector in the local inertial frame.

It is assumed that load factors  $\mathbf{n}_c$  are tracked by fast inner loop controllers in the vehicle body frame, leading to actual load factors  $\mathbf{n} = \mathbf{n}_c + \mathbf{n}_w$  where  $\mathbf{n}_w = \frac{1}{|\mathbf{g}|} \mathbf{a}_w$  are parasitic load factors introduced by exogenous disturbances and imperfect tracking. This leads to

$$\dot{\mathbf{v}} = \mathbf{R}_{eb} |\mathbf{g}| \mathbf{n}_c + \mathbf{a}_w + \mathbf{g} \quad (4)$$

where  $\mathbf{R}_{eb} \in \mathbb{R}^{3 \times 3}$  is the rotation matrix from the body frame to the local inertial frame and  $\mathbf{n}_c = \frac{1}{|\mathbf{g}|} \mathbf{a}_c$  are commanded load factors. To simplify notation, it is assumed that the vehicle is trimmed, i.e. the nominal gravitational acceleration is compensated for by a trim control input  $\mathbf{n}_{c,0} = \mathbf{R}_{be} (0 \ 0 \ -1)^T$  and a virtual control input is defined as

$$\mathbf{u} = \mathbf{R}_{eb} |\mathbf{g}| (\mathbf{n}_c - \mathbf{n}_{c,0}) \quad (5)$$

leading to

$$\dot{\mathbf{v}} = \mathbf{R}_{eb} |\mathbf{g}| \left( \frac{1}{|\mathbf{g}|} (\mathbf{R}_{be} \mathbf{u} + \mathbf{n}_{c,0}) \right) + \mathbf{a}_w + \mathbf{g} \quad (6)$$

$$\dot{\mathbf{v}} = \mathbf{u} + \mathbf{a}_w \quad (7)$$

where now  $\mathbf{a}_w$  also includes the small effects of imperfect knowledge of local gravitation. ???doubledefinition

Saturations on the virtual control input (27) can be derived from (15) as

$$|\mathbf{u}'| \leq \mathbf{U}' \quad (8)$$

$$\mathbf{U}' = \mathbf{G} \mathbf{B} \mathbf{R}_{eb} |\mathbf{g}| (\mathbf{N} - \mathbf{n}_{c,0}) \quad (9)$$

Considering two UAS  $i$  and  $i-1$ , this leads to relative position error dynamics

$$\Delta \mathbf{p} = \mathbf{p}_i - \mathbf{p}_{i-1} - \Delta \mathbf{p}_c \quad (10)$$

$$\Delta \dot{\mathbf{p}} = \mathbf{v}_i - \mathbf{v}_{i-1} - \Delta \dot{\mathbf{p}}_c \quad (11)$$

$$\Delta \dot{\mathbf{v}} = \mathbf{a}_i - \mathbf{a}_{i-1} - \Delta \dot{\mathbf{p}}_c \quad (12)$$

$$= \mathbf{a}_{c,i} + \mathbf{a}_{w,i} - \mathbf{a}_{i-1} - \Delta \ddot{\mathbf{p}}_c \quad (13)$$

$$= \mathbf{u} + \mathbf{a}_{w,i} - \mathbf{a}_{i-1} - \Delta \ddot{\mathbf{p}}_c \quad (14)$$

where  $\Delta \mathbf{p}$  is the relative position error between UAS  $i$  and its predecessor  $i-1$ ,  $\Delta \mathbf{v}$  is the corresponding relative velocity error,  $\Delta \mathbf{p}_c$  is the desired relative position to the predecessor,  $\mathbf{a}$  are accelerations.

The presented model is essentially of the same type as that used in Galzi and Shtessel (2006) and provides the benefit of being vehicle-agnostic, as the specific vehicle dynamics are covered by the inner loop load factor controllers. On the other hand, perturbations  $\mathbf{a}_w$  and control input saturations are specific to a given vehicle and mission environment.

## 3. CONTROL DESIGN

To benefit from significant aerodynamic performance gains, a follower aircraft needs to stay within a narrow



$$\Phi_k = \Delta \ddot{\mathbf{p}}_c \quad (26)$$

further defining

$$\mathbf{u}' = \mathbf{G}\mathbf{B}\mathbf{u} \quad (27)$$

$$\Phi'_k = \mathbf{G}\Phi_k \quad (28)$$

$$\Phi'_u = \mathbf{G}\mathbf{B}\Phi_u \quad (29)$$

Note that all perturbations are assumed to satisfy the matching condition. The three axes are considered decoupled by the inner load factor controllers, allowing for SISO design.

Saturations on the virtual control input (27) can be derived from (15) as

$$|\mathbf{u}'| \leq \mathbf{U}' \quad (30)$$

$$\mathbf{U}' = \mathbf{G}\mathbf{B}\mathbf{R}_{eb}|\mathbf{g}|(\mathbf{N} - \mathbf{n}_{c,0}) \quad (31)$$

Since the inner loop load factor controllers cannot track discontinuous reference signals, a continuous control signal is mandatory. The system (23) is of relative degree  $\mathbf{r} = (1 \ 1 \ 1)^T$ , thus continuous-time Super-Twisting Sliding Mode controllers (CSTSMC, see e.g. Shtessel et al. (2014)) can be applied, providing continuous control signals. We apply the controller presented in Galzi and Shtessel (2006) extending it trivially from 2D to 3D tracking. The CSTSMC controller is then given by

$$u'_p = \alpha_p |\sigma_p|^{1/2} \text{sign}(\sigma_p) + \beta_p \int \text{sign}(\sigma_p) dt \quad (32)$$

where  $p = 1 \dots 3$  indicates the three decoupled axes. Adding a term that eliminates the known disturbances  $\Phi'_{k,i}$  as in Galzi and Shtessel (2006)

$$u'_p = \alpha_p |\sigma_p|^{1/2} \text{sign}(\sigma_p) + \beta_p \int \text{sign}(\sigma_p) dt - \Phi'_{k,p} \quad (33)$$

leads to closed loop  $\sigma$  dynamics of

$$\dot{\sigma}_p = \alpha_p |\sigma_p|^{1/2} \text{sign}(\sigma_p) + \beta_p \int \text{sign}(\sigma_p) dt + \Phi'_{u,p} \quad (34)$$

Provided the disturbances  $\Phi'_{u,p}$  are bounded by  $\Phi'_{u,p} \leq L_p$  controller parameters that fulfill

$$\alpha_p = 1.5\sqrt{L_p} \quad (35)$$

$$\beta_p = 1.1L_p \quad (36)$$

drive the system into 2-sliding mode, i.e.  $\dot{\sigma} = \sigma = 0$  in finite time. The reaching time is bounded by  $t_p^* \leq \frac{7.6\sigma_p(0)}{\beta_p - L_p}$ , see Galzi and Shtessel (2006).

The actual control input  $\mathbf{n}_c$  is computed from (27, 5) as

$$\mathbf{n}_c = \frac{1}{|\mathbf{g}|} \mathbf{R}_{be}(\mathbf{G}\mathbf{B})^{-1} \mathbf{u}' + \mathbf{n}_{c,0} \quad (37)$$

using  $\mathbf{R}_{eb}^{-1} = \mathbf{R}_{eb}^T = \mathbf{R}_{be}$

*Discretization* For implementation, the CSTSMC is sampled with a zero-order hold scheme. There are results for homogeneous sliding controllers such as the CSTSMC (see Shtessel et al. (2014)) stating that the error introduced by discrete sampling is quadratically proportional to the sampling time, i.e.

$$\sigma = O(T^2) \quad (38)$$

### 3.4 DSMC

Designing a sliding mode controller in the discrete time domain allows to take sampling time effects into account right from the beginning.

The  $\sigma$  dynamics assuming forward Euler discretization are

$$\sigma(k+1) = \sigma(k) + T(\Phi'_k(k) + \Phi'_u(k) + \mathbf{u}'(k)) \quad (39)$$

Since a discrete controller has no control over what happens to the continuous system between sampling instants, ideal sliding mode is not achievable. It is however possible to drive the system into so-called quasi-sliding mode, defined by the control objective

$$|\sigma_i(k)| \leq \epsilon_i \quad (40)$$

for  $i = 1 \dots 3$ , for all  $k \geq k^*$  where  $\epsilon$  is the width of the quasi-sliding mode boundary layer and  $k^*$  is the first sample for which eq. 40 is satisfied, i.e. when the system transitions from the reaching phase into quasi-sliding mode. The proposed DSMC is based on ideas presented by the authors of Monsees and Scherpen (2001). The following simple linear reaching law (proposed e.g. by Spurgeon (1992)) ensures asymptotic convergence to the sliding surface

$$\sigma(k+1) = \Psi \sigma(k) \quad (41)$$

with a diagonal  $\Psi \in \mathbb{R}^{3 \times 3}$ ,  $0 < \Psi_{i,i} < 1$  for  $i = 1 \dots 3$ . The choice of  $\Psi$  allows to trade off control effort and reaching time. Since (41) is equivalent to

$$|\sigma(k+1)| = \Psi |\sigma(k)| \quad (42)$$

the norm of the sliding variable decreases with every time step, indicating convergence to the sliding surface.

The control input  $\mathbf{u}(k)$  required to drive the system (39) according to the reaching law (41) can be computed from the open-loop  $\sigma$  dynamics to

$$\mathbf{u}'(k) = \frac{1}{T} \sigma(k)(\Psi - \mathbf{I}) - \Phi'_k - \Phi'_u \quad (43)$$

Note that  $\mathbf{u}'(k)$  contains the unknown perturbations. Assuming an estimate of the unknown perturbation  $\tilde{\Phi}'_u(k) = \Phi'_u(k) + \Delta \Phi'_u(k)$  and closing the loop, one obtains

$$\sigma(k+1) = \Psi \sigma(k) + T(\Delta \Phi'_u(k)) \quad (44)$$

Note that the reaching law (41) can not be followed due to the estimation error  $\Delta\Phi'_u(k)$ . Instead, assuming that initially the system is outside the quasi-sliding mode band, it will approach the sliding surface at least as long as

$$\Psi\sigma(k) \geq T(\Delta\Phi_u(k)) \quad (45)$$

This defines the maximum boundary layer thickness as

$$\epsilon = \Psi^{-1}T(\Delta\Phi_u(k)) \quad (46)$$

A simple way to obtain an estimate of the unknown disturbances is from the previous sample by

$$\Phi_u(k-1) = \sigma(k) - \sigma(k-1) - T(\Phi'_k(k-1) + \mathbf{u}'(k-1)) \quad (47)$$

assuming that the disturbance rate is bounded by  $|\frac{d\Phi'}{dt}| \leq \delta\Phi'$ , assuming  $\Phi'(k) = \Phi'(k-1)$  and a first-order approximation introduces an error  $\Delta\Phi'_u(k)$  that is bounded by  $|\Delta\Phi'_u(k)| \leq T\delta\Phi'$ .

Note that with (46), the boundary layer thickness depends quadratically on the sampling time.

#### 4. SIMULATIONS

Both controllers have been evaluated in a simulation environment. In all cases, the vehicle dynamics have been integrated with a forward Euler scheme at a sampling time of  $t_{sim} = 10^{-4} s$ , while the controller sampling time has been varied. While three-dimensional predecessor tracking is performed, only the vertical channel  $\Delta p_3$  is considered here for clarity.

##### 4.1 Controller parameters

For the CSTSMC, the controller gains are computed according to (35, 36), requiring the bounds  $L_i$  on the maximum unknown perturbations  $\Phi'_{u,i}$  which are driven by exogenous disturbances  $a_{w,i}$  and acceleration of the predecessor  $a_{k-1,i}$ .

These being bounded by

$$|a_{w,p}| \leq A_{w,p} \quad (48)$$

$$|a_{i-1,p}| \leq A_{i-1,p} \quad (49)$$

for  $i = 1...3$ , the bounds on  $|a_{w,i}|$  have been obtained empirically from a time series representing 1h of simulated Dryden turbulence at the position of maximum incremental lift in the predecessor's wake filtered by closed loop inner load factor controllers.

The bounds  $A_{k-1,i}$  are less obvious, since they depend on the closed-loop behavior of the predecessor. We assume that the control inputs of the predecessor stay within their saturation limits, i.e.

$$-U_i < \mathbf{u}_{i-1,p} < U_i \quad (50)$$

Now since

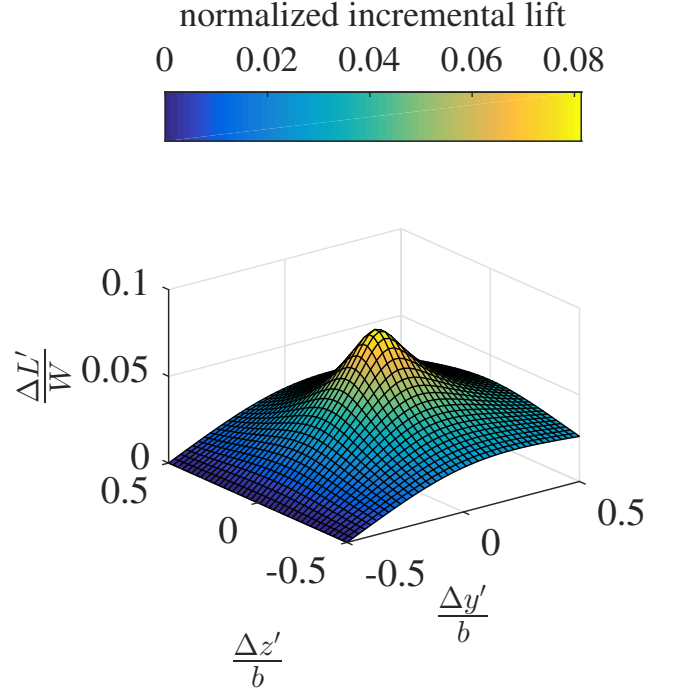


Fig. 3. Normalized incremental lift predicted by modified Horseshoe Vortex Model for  $\Delta x' = 2b$ . Note that  $\Delta x', \Delta y', \Delta z'$  are wingtip to wingtip separations

$$\mathbf{a}_{i-1} = \mathbf{u}_{i-1} + \mathbf{a}_{w,i-1} \quad (51)$$

$\mathbf{a}_{i-1}$  is bounded by

$$|a_{i-1,p}| \leq U_p + A_{w,p} \quad (52)$$

leading to

$$L_p = U_p + 2A_{w,p} \quad (53)$$

**Remark:** From (53) it becomes clear that no guarantees can be given for the system to stay in sliding mode at all times under the assumption 50. This is due to the fact that the assumption 50 allows the predecessor  $i-1$  to apply maximum accelerations that, if at the same time disturbances  $\mathbf{w}, \mathbf{p}$  act on the predecessor, cannot be followed by the follower  $i$ . Formally this can be checked by computing the equivalent control that results from the sliding condition  $\sigma = \mathbf{0}$  as

For the DSMC, the entries of the reaching rate matrix  $\Psi$  are selected on line to stay within control input limits given the initial error state.

##### 4.2 Disturbance models

**Wake vortex disturbances** A variety of approaches has been proposed to simulate the effects of wake trailing vortices on following UAS, mostly based on modified Horseshoe Vortex Models (HVM) (citethemall) or Vortex Lattice methods (VLM) (citethemall). For the purpose of this work, a HVM with modified core model presented in (Dogan et al. (2005)) is used. It is reported to provide predictions that are in good agreement of both VLM



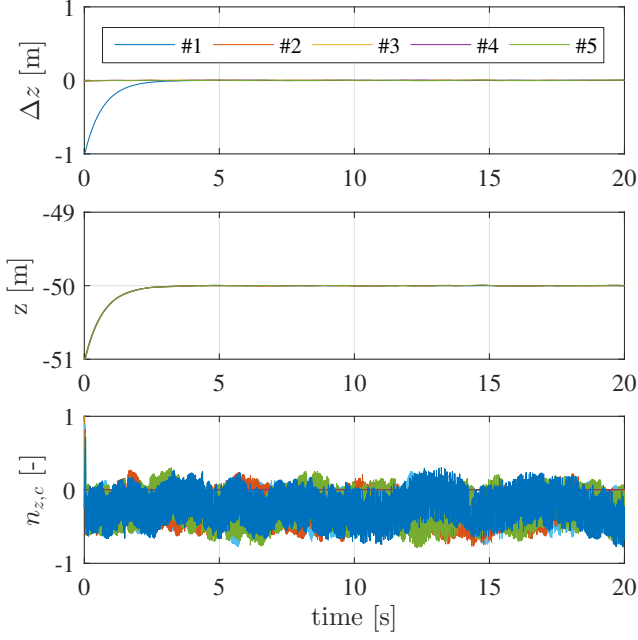


Fig. 4. CSMC controller  $10^{-3}s$  sampling time

models and wind tunnel measurements while being of great simplicity. In the vertical channel, the model predicts incremental aerodynamic lift perturbations as a function of the separation vector between a UAS and its predecessor, see fig. 3. A variety of approaches has been proposed to simulate the effects of wake trailing vortices on following UAS, mostly based on Horseshoe Vortex Models (HVM) (!!!cite) or Vortex Lattice methods (VLM) (!!!cite). For the purpose of this work, a HVM with modified core model (MHVM) presented in (Dogan et al. (2005)) is used. It is reported to provide predictions that are in good agreement of both VLM models and wind tunnel measurements while being of great simplicity. The model predicts incremental aerodynamic lift perturbations as a function of the separation vector between a UAS and its predecessor, see fig. 3.

*Atmospheric turbulence* Atmospheric turbulence time series are generated according to the Dryden turbulence spectrum. The induced velocities are filtered by transfer functions corresponding to LQR load factor controllers designed for a small UAS ( $b = 2.6m$ ).

*Maneuver*

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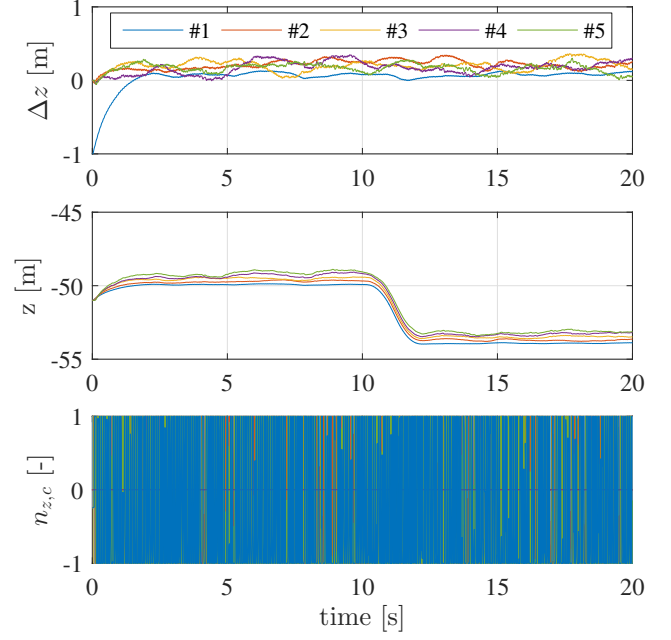


Fig. 5. CSMC controller  $10^{-2}s$  sampling time

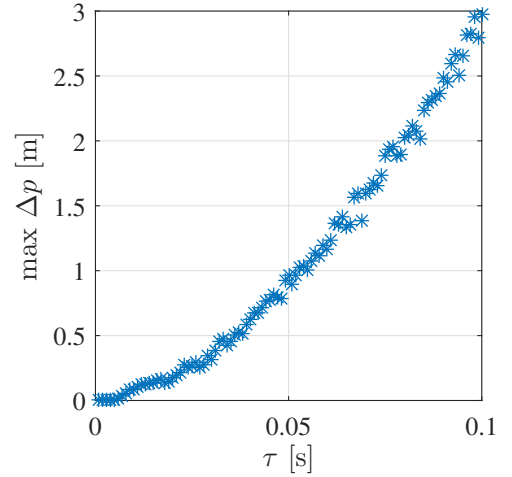


Fig. 6. Maximum vertical position error vs. CSMC controller sampling time

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