Chapter 1

Concepts and principles

Classical ciphers

Cryptology, Cryptography, Cryptoanalysis

Cryptology = science of data encryption

Cryptography

- developing cryptoalgorithms

Cryptoanalysis

- testing and breaking encryption algorithms

Finland has also some significant industries in this field. **SSH** is a widely used encryption software developed by Finnish Tatu Ylönen. SSH software is used by f.e EU and NASA.

Internet Engineering Task Force (IETF) has defined **information security services** as listed below. The implementation of these services requires cryptographic methods.

Service	Implementation
1. **Confidentiality ***	- encryption of data and messages
2. **Integrity ***	- cryptographic hash functions
3. **Authentication ***	- user authentication in TLS
4. **Non-repudiation ***	- digital signatures
5. **access control**	- password authentication, nowadays more often authentication with public key cryptography
6. **availability**	

Explanation of terminology:

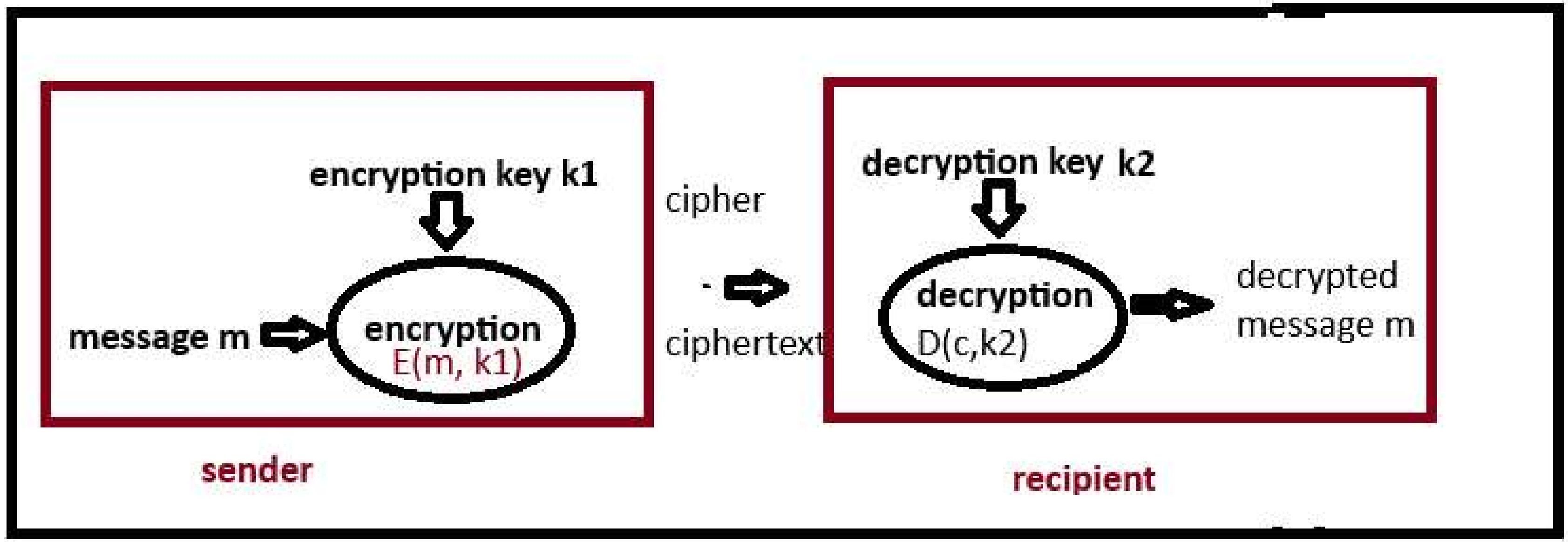
Confidentiality: Only those who are entitled to information can access to it
Integrity: Information is not changed during transfer. Sender is authenticated
Authentication: The sender is provably identified
Non-repudiation: A communicating party cannot deny the contents of his/her messages or deny having participated in the interaction.

Basic concepts and principles

Topics:

1. Encryption of messages as a diagram
2. Symmetric and asymmetric encryption
3. Kerckhoff principle
4. Key space, effective key space
5. Mathematical concepts

1. Encryption : diagram presentation



Message m (called also plaintext) is encrypted using encryption function E(m, k1), where the second argument is encryption key k1. Output of the function is called cipher or ciphertext c .

Decryption is done with decryption function D(c, k2) with arguments ciphertext c and decryption key k2. Output is the original plaintext m

2. SYMMETRIC AND ASYMMETRIC ENCRYPTION

SYMMETRIC ENCRYPTION: Encryption key k1 and decryption key k2 are identical.

* The parties have to agree the session key before they start communicating (Usually agreeing on symmetric key is done using ”key exchange protocol” (f.e DH, RSA or ECDHE)

ASYMMETRIC ENCRYPTION: Encryption key k1 and decryption key k2 are different.

- Most common form of asymmetric encryption is public key encryption (PKI), in which **every user has two keys** : **Public key** , which is used to encrypt messages to the user, and its pair the **private key**, which only the user knows and uses for decryption of received messages.

3. Kerckhoff principle

Kerckhoff principle:
"A cryptosystem should be secure even if all its details are public".
Only the key is secret.

(Auguste Kerckhoff 1835 - 1903 was a Dutch linguistic and cryptographer)

In history DES encryption algorithm and also GSM mobile network algorithms were tried to kept secret. Both algorithms were leaked out.

More recent algorithms are completely transparent and open for everyone who wants to find their weaknesses and backfdoors.

4. Key space

A good cryptosystem should not have weaknesses or backdoors.

The best option for cryptoanalysis to break a good cryptosystem is to use **Brute Force attack**: trying to find the key among all possible keys by trial and error or alternatively systematic key search.

The most important security factor against brute force is **the number of all possible keys**, which is called the **key space**. Key space is usally measured in **bits**.

Absolute minimum against brute force is 80 bits, which means that there are $2^{80} = 1.2*10^{24}$ possible keys.

Keys vary from 00...0 to 11...1 (80 bit binary numbers)

Practical minimum for key space is 128 bits.

Effective key space

For most cryptosystems it is possible to find methods of breaking the key using less searches than the **theoretical key space** would indicate.

Effective key space means the average number of steps needed to break the key using best known methods of cryptanalysis. Effective key space is smaller than the theoretical key space. In good cryptosystems the difference of theoretical and effective key spaces is small .

Most common symmetric cipher is AES

AES128	key space 128 bits, effective 126.1
AES256	key space 256 bits, effective 254.4

In some older algorihtms difference is bigger

DES	key space 64 bits, effective 54
3DES	key space 168 bits, effective 112

Key space size applied to password lenght

The 80 bit security minimum $2^{80} = 1.2 \cdot 10^{24}$ can be applied also to security of passwords against Brute Force – attack.
Following examples show how to calculate secure password lengths.

1. Password length = 8 characters, only english alphabet allowed. No distinction between uppercase and lower case letters. (=26 characters)

Password space = $26^{8} = 2 \cdot 10^{11}$ which is not secure

2. Password length = 11 characters, alphabet = english lower case and upper case letter + numbers 0…9: (= 62 characters)
Password space = $62^{11} = 5 \cdot 10^{19}$ still not secure

3. Password length = 13 characters, alphabet = english lower case and upper case letter + numbers 0…9 + ten special characters: (=72 characters)
Password space $72^{13} = 1.4 \cdot 10^{24}$ provides adequate security

5. Mathematical concepts

HARD PROBLEM

= **mathematical problem which is hard** and time consuming to solve **due to its complexity**. The security of cryptoalgorithms is often based on some known ”hard problem” . F.e RSA algorithm is based on the difficulty of factoring large integers.
Diffie-Hellman key exchange is based on Discrete Logarithm Problem (DLP).

ONE WAY FUNCTIONS

= function y = f(x), that is fast and easy to calculate if argument x is known, but the inverse function which calculates x for known y is difficult or impossible withou extra knowledge.

BACKDOOR

= Additional knowledge which makes the calculation of inverse function of an one way function possible

In RSA calculation of the decryption key from the public key of the user is practically impossible. The public key is a large integer. However **knowledge of the factors of public key changes the game and is a backdoor to RSA**: calculation of decryption key becomes trivial.

Sometimes a cryptoalgorithm has an intentional backdoor, which the intelligense authorities can use to listen to assumably secure communication.

Classical ciphers

* cipher = encryption algorithm, encryption method

Mono- and polyalphabetic ciphers

Some classical ciphers:

1. Caesar cipher
   - Cryptanalysis with frequency analysis
2. Affine encryption
3. Hill cipher (Matrix cipher)
4. Enigma
5. Simple substitution
6. Vigenere cipher with key phrase
7. Autokey cipher
8. One Time Pad: ”the only unbreakable cipher”

Mono- and polyalphabetic ciphers

A) In a **monoalphabetic cipher** message is encrypted character by character. Every character has always the same image character. Examples of monoalphabetic ciphers are Caesar -cipher and simple substitution.
Frequency analysis is effective in breaking any monoalphabetic cipher.

B) **Polyaphabetic cipher** is a system in which the same character can have different image characters at different points of the message. Example of polyalphabetic cipher is Vigenère cipher. Frequency analysis is less effective against this cipher.

Another division of ciphers based on the block size

In **Monographic ciphers** message is encrypted in blocks of one character at a time.
In **Polygraphic ciphers** message is divided to blocks of several characters and encryption is applied on one block at the time. Modern ciphers are polygraphic.

1. Caesar cipher

- Encryption is based on **rotation of alphabet**. Key is the amount of rotation

The ciphertext of message “AAMU” is “NNZH” when the disc of the picture is used

Key space size = 25 (the number of possible rotations of english alphabet)

Cipher is easily broken with *Brute Force attack* (trying all 25 keys)
With *frequency analysis* breaking is even faster.

Frequency analysis of monoalphabetic ciphers

- Frequency analysis can used to break many classical ciphers.
- It is most effective in breaking monoalphabetic ciphers.
- It is also a basic tool of cryptanalysis of polyalphabetic ciphers in all cases, when the characters of cipher text are not evenly distributed.

Breaking monoalphabetic ciphers (of messages in English) the table of relative frequencies of most common characters in english texts is used

e	*t*	*a*	*o*	*n*	*i*	*s*	*r*	*h*
12.3	9.6	8.1	7.9	7.2	7.2	6.6	6.0	5.1

Letter **e** is clearly most common character of english lanquage.
In monoalphabetic cipher the character with biggest frequency is very probably the image of **e** (can also be t or a)

For other languages similar tables are found in Wikipedia.
In Finnish language the 6 most common characters in orded are a, i , t , n , e , s

Example: frequency analysis of Caecar cipher

Following ciphertext is obtained with Caesar cipher.

```
cqnujcnbcvxernjkxdclahycxpajyqhrbwxfrwcqnjcnabrbcnuubjkxdcbnlxwmfxaumfjajwm
   cqnjccnvycbvjmnrwnwpujwmcxkajtnpnavjwnwrpvjlryqna
```

Cryptanalysis: Calculate the frequencies of characters of the ciphertext

n : 15 , c : 14, j : 13 , w : 9 , a : 8 , x : 8 , ...

Character n has biggest frequency. Make hypothesis**:** n is the image of e**.** The amount of shift from e to n , number **9** is a candidate for encryption key.

Test the hypothesis by shifting the cipher characters 9 steps in opposite direction. Result is the following meaningful english message:

```
thelatestmovieaboutcryptographyisnowintheatersistellsaboutsecondworldwarand
   theattemptsmadeinenglandtobrakegermanenigmacipher
```

2. Affine cipher

1. Characters are coded to numbers 0 – 25 using the coding below

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Encryption

2. Let m = character of the message, c = image character of m. Key is the pair (a,b) of Z_{26}

Encryption formula : **$c = a\,m + b \pmod{26}$**

3. Ciphertext is a sequence of numbers c, which are decoded using the table above

Decryption

4. Multiplication encryption formula by a^{-1} (= inverse of a mod 26) gives $a^{-1}c = (a^{-1}a)m + a^{-1}b$. Rearranging gives:

Decryption formula: **$m = a^{-1}\,c - a^{-1}\,b \pmod{26}$**

5

- There are several ways of calculating the inverse of a.
- 1. Extended Euclid's algorithm
- 2. Formula a^{-1} = a^(ϕ(26)-1) mod 26 = a^11 mod 26 (based on Euler's theorem)
- 3. WolframAlpha calculator accepts: a^-1 mod 26

The key space size is 26*ϕ(26) = 26*12 = 312
(only those values of a can be keys, which are coprime with 26; gcd(a,26) = 1)

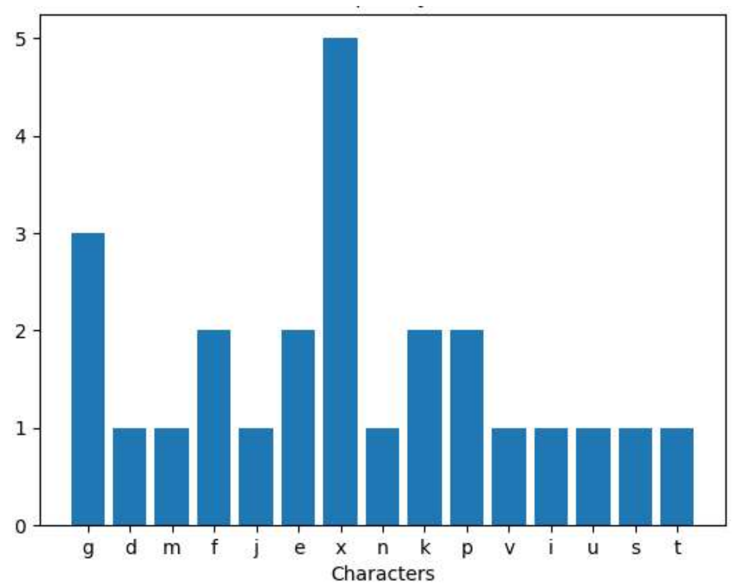
Example of affine cipher

Encrypt message ”kemi” using key (a = 11 , b = 3)
1. Coding message gives m = (10, 4 , 12 , 8)
2. Encryption formula gives cipher
$c = (11*10 + 3 , 11*4 + 3, 11*12 + 3, 11*8 + 3) \mod 26$
$= (9 , 21 , 5 , 13)$, which decodes to ”jvfn”

Decryption: **$m = a^{-1} c - a^{-1} b \pmod{26}$**
1. Calculate inverse $a^{-1} = 11^{-1} \mod 26 = 19$ *)WolframAlpha
2. Decrypted cipher is
$(19*9 - 19*3 , 19*21 - 19*3 , 19*5 - 19*3 , 19*13 - 19*3) \mod 26 =$
$(10 , 4 , 12 , 8) =$ ”kemi”

***)WolframAlpha.com calculator calculates inverse easily: 11^-1 mod 26.**

Cryptanalysis example



Break affine cipher ”gdmfjgexnxfgexkpvikxxupst”

Frequency diagram on the right show that **x** and **g** have biggest frequencies:

Hypothesis E(e)=x and E(t)=g gives
$a*4 + b \equiv 23$ and $a*19 + b \equiv 6 \pmod{26}$
Subtract (1) from (2) gives
$15*a \equiv -17 \equiv 9 \Rightarrow$
$a \equiv 15^{-1}*9 \equiv 7*9 \equiv \mathbf{11} \pmod{26}$
Substituting a=11 gives $b \equiv 23-11*4 \equiv \mathbf{5} \pmod{26}$

Test hypothesis (a,b) = (11,5)
Inverse $a^{-1} \equiv 11^{-1} \equiv 19$.

Using Excel ”classical ciphers” in Moodle with these values we get sensible plaintext

”todaytheweatherisfreezing”

3. Hill Cipher (Matrix cipher)

Next development after affine cipher is Hill cipher (1929) whics encrypts message blocks instead of single letters. Hill cipher is polyalphabetic: a character of message has several image characters.   Hill cipher uses matrix multiplication for encryption.

Encrypt block ”act” using matrix key ((G,Y,B),(N,Q,K),(U,R,P)) as multiplier

$$\begin{pmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 19 \end{pmatrix} = \begin{pmatrix} 67 \\ 222 \\ 319 \end{pmatrix} \equiv \begin{pmatrix} 15 \\ 14 \\ 7 \end{pmatrix} \pmod{26}$$ = ”poh”

Decryption of ”poh” uses inverse matrix of the key matrix

$$\begin{pmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{pmatrix}^{-1} \pmod{26} \equiv \begin{pmatrix} 8 & 5 & 10 \\ 21 & 8 & 21 \\ 21 & 12 & 8 \end{pmatrix}$$

Taking the previous example ciphertext of 'POH', we get:

$$\begin{pmatrix} 8 & 5 & 10 \\ 21 & 8 & 21 \\ 21 & 12 & 8 \end{pmatrix} \begin{pmatrix} 15 \\ 14 \\ 7 \end{pmatrix} = \begin{pmatrix} 260 \\ 574 \\ 539 \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 2 \\ 19 \end{pmatrix} \pmod{26}$$ = ”act”

Hill cipher achieves **Shannon’s diffusion principle**: (discussed later in context with block ciphers)

”Each character of the ciphertext should depend on several characters of the message, obscuring the connections between the two.”

- For block size of 4 letters, Hill cipher uses 4x4 matrices, e.t.c
- If the block size is 3 letters, the key space is 26^9 around 10^{12}
-
- First versions of Enigma cipher machine use by Germans in WW2 were based of Hill cipher’s ideas.

4. Encryption machine Enigma

In World War II the Germans used Enigma encryption machine in communication with submarines.

The first computers were made in England for breaking the encrypted messages of Enigma. (”Tummy – machine”)

Videos: 15 min

https://www.youtube.com/watch?v=GBsfWSQVtYA Lorentz machine

https://www.youtube.com/watch?v=b4WBINgRMTY Tummy machine

The era of classical ciphers ends to the invention of computers and communication networks.

Modern encryption algorithms appear in business and administration in 1970’s

5. Simple substitution cipher

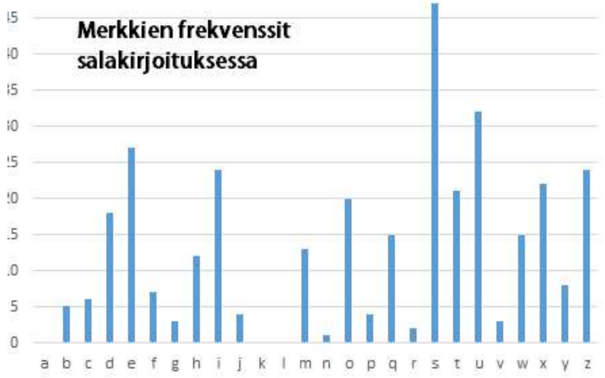
Encryption key is a permutation of alphabet (e.e. a->k, b->z, c->q,…).

Key space = nr. of permutations = 26! = $4*10^{26}$. Brute force is powerless.
With frequency analysis breaking of simple substition cipher is easier and can be compared to solving a difficult cross words puzzle.

CIpher:

u	o	s	w	e	i	u	s	m	m	s	q	u	z	r	s	w
s	u	o	e	b	e	m	q	d	c	v	u	e	x	t	x	h
c	i	z	i	e	m	j	x	d	u	z	w	s	q	z	v	o
s	d	i	z	i	m	d	s	p	f	s	t	q	c	x	t	x
h	c	i	z	i	u	o	z	i	w	s	u	o	e	b	d	s
p	f	z	d	s	i	n	t	e	j	h	s	b	y	s	x	g
e	f	u	u	o	s	d	s	h	x	u	z	r	s	m	d	s
p	f	s	t	q	z	s	i	e	m	q	o	x	d	x	q	u
s	d	i	e	m	u	o	s	h	x	t	y	f	x	y	s	z
t	u	o	s	q	x	i	s	e	m	s	t	y	h	z	i	o
h	x	t	y	f	x	y	s	u	o	s	w	e	i	u	q	e
w	w	e	t	h	s	u	u	s	d	z	i	s	u	o	s	e
u	o	s	d	h	s	u	u	s	d	i	q	e	w	s	m	x
d	g	s	o	z	t	b	x	t	e	u	o	s	d	j	x	c
e	m	z	w	v	h	s	w	s	t	u	z	t	y	u	o	z
i	w	s	u	o	e	b	z	i	u	e	x	t	x	h	c	i
s	m	d	s	p	f	s	t	q	z	s	i	e	m	u	j	e
q	o	x	d	x	q	u	s	d	q	e	w	g	z	t	x	u
z	e	t	i	z	t	s	t	y	h	z	i	o	u	o	z	i
w	e	i	u	q	e	w	w	e	t							

Frequency analysis



Alla englanninkielen tekstin merkkien suht. frekvenssit

e	t	a	o	n	i	s	r	h
12.3	9.6	8.1	7.9	7.2	7.2	6.6	6.0	5.1

Initial phase of cryptanalysis

"RISTISANATEHTÄVÄ"

t	h	e				t	e			e		t			e	
e	t	h									t					h
									t			e				
e							e			e	t	h				
					t	h				e	t					e
				e							e			e		
		t	t	h	e		e			t			e			e
		e				e										t
e					t	h	e								e	
	t	h	e				e			e						h
							e	t	h	e				t		
					e	t	t	e				e	t	h	e	
t	h	e			e	t	t	e						e		
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						e		e		t				t	h	
		e	t	h					t							
e			e			e				e						
	h						e									t
						e						h	t	h		
			t													

*) The table on the right is based on assumptions, that the character s with the highest frequency s is the image of e, and u of cipher is image of t. In addition the middle character of s_u of chiper (t_e in original) is h, because cobination ”the” is most common in english language (appears in ”the”, ”these”, ”they”, ”them”, ….). This helps a lot in solving the ”cross words puzzle”.

5. Vigenère cipher using key word or phrase

Blaise de Vigenère 1523 -1596 was a french linguistic and cryptographer

- Encryption key is a password
- The whole key is obtained by copying password to the length of the message
- In encryption the characters of message and key are added using the table shown in the next slide

- Vigenere encryption is vulnerable if the password is moderately short. In 1800's prussian officer Kasiski broke it. Kasiski's cryptanalysis is based on frequency analysis of 2- 3 character long "substrings" in ciphertext

Vigenere encryption with a table

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Encrypt message "helsinki" with password "oulu"

H E L S I N K I

O U L U O U L U

=============

V Y W M W H V C

Key space is 25^n ,where n = password length. If f.e the password length n = 20, the key space = $9*10^{27}$ (large enough against Brute Force)

6. Autokey cipher: improvement of Vigenère

- Encryption primary key is a password
- Message is divided into blocks of length of the password
- First cipher block is calculated adding first message block with the message
- After that the new key used for encryption of next block is always the previous cipher block
- This procedure can be called CBC : ”cipher block chaining”.

Encrypt ”konferenssi” with key ”lumi”

message	k	o	n	f	e	r	e	n	s	s	i
key	l	u	m	i	**v**	**i**	**z**	**n**	z	z	d
cipher	**v**	**i**	**z**	**n**	z	z	d	a	r	r	l

Decrypt ”viznzzdaffl” with key ”lumi”

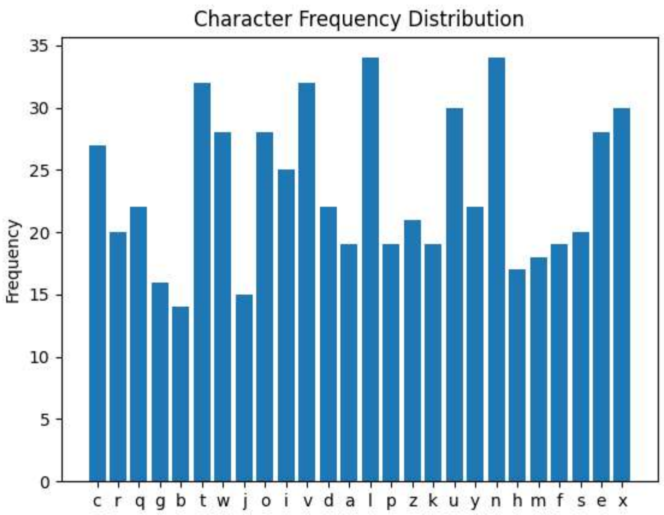
”vizn” – ”lumi” = ”konf”
”zzdz”-”vizn” = ”eren”
”rrl”-”zzs” = ”ssi”

Why inprovement: Key has no periodicity, which makes cryptanalysis more complex. (Breaking Vigenere started by finding the length of the keyword)

7. One time pad the unbreakable cipher

- If Vigenère encryption is used with a random, one-time password of same lenght as the message, encryption is impossible to break. (true even with quantum computers)

- This is obvious, because for each cipher text c and each possible message m there is a key k which encrypts m to c. It is impossible to distinguish the right message from all other possible messages.



Frequency analysis of ciphers produced by One Time Pad encryption show nearly even distribution of characters. The longer is the message, the smaller are the differences of frequencies.

A basic property of secure encryption algorithm is that no information about the message or key can be retrieved from frequencies.

7. Binary One Time Pad (Vernam 1919)

A binary version of One Time Pad is Vernam cipher (1919)

Message m and a random key k of same lentgh are binary sequences.
In encryption m and k are added using XOR addition.

Decryption is done adding the same key k to the cipher.

XOR addition : 0 + 0 = 0, 1 + 1 = 0 , 1 + 0 = 1, 0 + 1 = 1

Encryption:

Message	1	0	1	1	0	0	1	0
Key	0	1	1	0	1	0	1	1
Cipher	1	1	0	1	1	0	0	1

Decryption:

Cipher	1	1	0	1	1	0	0	1
Key	0	1	1	0	1	0	1	1
Decrypted m	1	0	1	1	0	0	1	0

Why One Time Pad encryption is not widely used even though it is unbreakable.
The problem is that both communicating parties must have similar ”codebooks” containing very long one-time passwords.