Name: ______________________________

Math 32, Spring 2010, Section 101
Quiz 9 Solutions

(1) (3 pts) Use half-angle identities (given below) to compute (a) $\sin 105\,^\circ$ and (b) $\cos 105\,^\circ$. Don't worry too much about simplification. Recall that "$\pm$" in half-angle formulas doesn't mean plus *and* minus, it means plus *or* minus, and you have to figure out which one.

Identities: $\boxed{\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}, \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}, \quad \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}.}$

(a) We'll use $\theta = 210°$, so

$$\sin 105° = \sqrt{\frac{1-\cos 210°}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}.$$

We chose $+$ out of the $\pm$ because $105°$ is in the second quadrant, and therefore $\sin 105°$ is positive. (b) Similarly, $\cos 105°$ should be negative, so we get

$$\cos 105° = -\sqrt{\frac{1+\cos 210°}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\frac{\sqrt{2-\sqrt{3}}}{2}.$$

(2) (3 pts) Evaluate each of the quantities that is defined. If a quantity is undefined, say so.

(a) $\sin^{-1}(\sqrt{3}/2)$ (b) $\cos(\cos^{-1}(\frac{3}{4}))$ (c) $\arccos(\cos(2\pi))$

(a) The answer is the number x such that $-\pi/2 \le x \le \pi/2$ and $\sin x = \sqrt{3}/2$. From our unit circle knowledge, we know that this number is $\pi/3$.

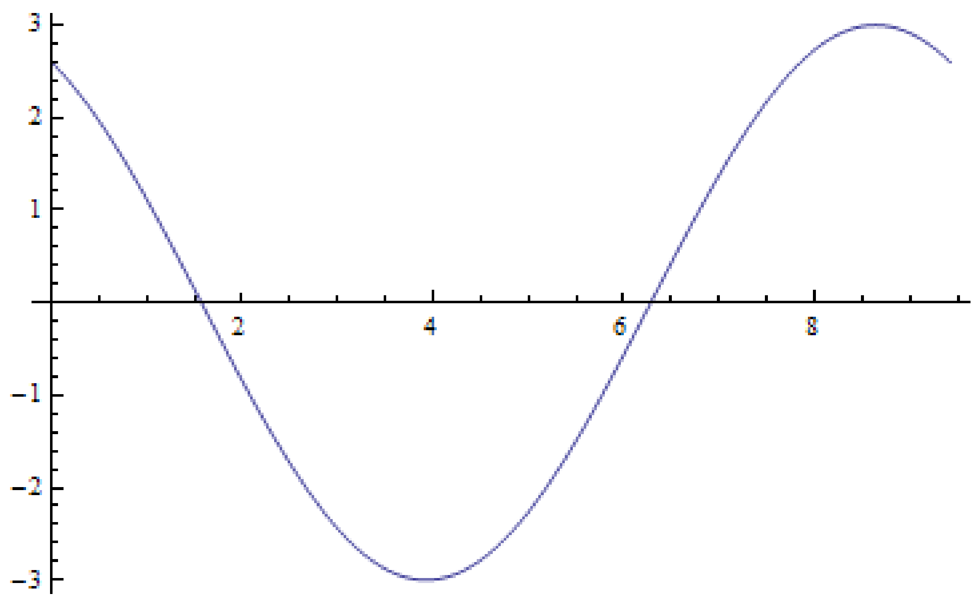
(b) Note that $\frac{3}{4}$ is in the domain of $\cos^{-1}$. Thus $\cos^{-1}(\frac{3}{4})$ is some number whose cose in $\frac{3}{4}$. So $\cos(\cos^{-1}(\frac{3}{4})) = \frac{3}{4}$.

(c) $\arccos(\cos(2\pi)) = \arccos(1) = 0$.

(3) (4 pts) Determine the amplitude, period , and phase shift for the function

$$y = 3\cos\left(\frac{2x}{3} + \frac{\pi}{6}\right).$$

Graph the function over one period. Indicate the x-intercepts and the x-coordinates of the highest and lowest points on the graph.



The amplitude is 3, the period is 3π, and the shift is $\pi/4$ to the left. The x-intercepts in the graph are $\pi/2$ and 2π. The highest point is at $x = 11\pi/4$ and the lowest point is at $x = 5\pi/4$. These last two points could be different if you started the period of your graph somewhere else (e.g. $x = -\pi/4$).