Name: ________________________________

Math 54, Spring 2009, Section 109
Quiz 3

[**1** - (3 pts)] Let V be the vector space $\mathbb{P}_2$ of polynomials of degree at most 2, with bases $\mathcal{B} = \{x^2 + 2x - 2, 2x^2 + x, x^2 + 2x - 1\}$ and $\mathcal{C} = \{x^2, x^2 + x, x - 1\}$. Find the change-of-basis matrices $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ and $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}$. (Hint: try to write elements of $\mathcal{B}$ in terms of the elements of $\mathcal{C}$).

[**2** - (3 pts)] Suppose you solve a non-homogenous system of 8 linear equations in 10 unknowns, and find that in your solution you have exactly 3 free variables. Is it possible to change the right-hand side of the system of equations (i.e. "change $\vec{b}$") and have the resulting system be inconsistent? (No points for correct answer with incorrect reasoning.)

[**3** - (3 pts)] Let V be a vector space with basis $\mathcal{B} = \{\vec{b}_1, \ldots, \vec{b}_n\}$. Prove **exactly one** of the following two things (if you do both, I'll just grade the first):

(a) If $\{\vec{v}_1, \ldots, \vec{v}_k\}$ is linearly independent, then so is $\{[\vec{v}_1]_{\mathcal{B}}, \ldots, [\vec{v}_k]_{\mathcal{B}}\}$.

(b) If $\vec{v} \in \operatorname{Span}\{\vec{v}_1, \ldots, \vec{v}_k\}$, then $[\vec{v}]_{\mathcal{B}} \in \operatorname{Span}\{[\vec{v}_1]_{\mathcal{B}}, \ldots, [\vec{v}_k]_{\mathcal{B}}\}$.