Midterm 1

Math 1B, Fall 2008

Wilkening

0	1	
1	5	
2	5	
3	5	
4	7	
5	6	
6	3	
total	32	

0. (1 point) write your name, your GSI's name, and your section number on your exam.

1. (5 points) Evaluate the integral: $\displaystyle\int x \sec^2 x \tan x \, dx$

2. (5 points) Find a and b: $\displaystyle\int_1^{\sqrt{2}} \frac{\sqrt{x^2 - 1}}{x}\, dx = a - \frac{\pi}{b}$

3. (5 points) Evaluate the integral: $\displaystyle\int \frac{x^4 + 3}{x^3 - x}\, dx$

4a. (2 points) Write down the equation of the parabola $p(x)$ such that

$$p(1) = 0, \qquad p(2) = 0, \qquad p(3) = 1.$$

4b. (2 points) With $p(x)$ as above, evaluate $\displaystyle\int_1^3 p(x)\,dx$

4c. (3 points) Let $f(x) = (x+1)e^{-x}$. Find K_2 such that $|f''(x)| \leq K_2$ for $1 \leq x \leq 3$.

5a. (3 points) Prove that

$$\cos x \geq 1 - \frac{2}{\pi}x \quad \text{for} \quad 0 \leq x \leq \frac{\pi}{2}$$

5b. (3 points) Use part (a) to show that the following integeral is convergent:

$$\int_0^{\pi/2} \frac{1}{\sqrt{\cos x}} \, dx$$

6. (3 points for correct answer, -1 for wrong answer, 0 for blank)

Let $F(x) = \int_0^x e^{-t^2}\, dt$. Evaluate $\int_0^\infty F(x)\, dx$.

(a) -1/2
(b) 1/2
(c) 1
(d) ∞
(e) cannot be expressed in terms of elementary functions