Name: Solution

Math 54, Spring 2009, Section 112
Quiz 1

(1) Find the general solution:

$$\begin{array}{rcrcrcr} x & - & 2y & + & 8z & = & -5 \\ & & 2y & + & 6z & = & -8 \\ -2x & + & 8y & - & 4z & = & -6 \end{array}$$

$$\begin{bmatrix} 1 & -2 & 8 & -5 \\ 0 & 2 & 6 & -8 \\ -2 & 8 & -4 & -6 \end{bmatrix} \xrightarrow[R_3]{2R_1+R_3\to} \begin{bmatrix} 1 & -2 & 8 & -5 \\ 0 & 2 & 6 & -8 \\ 0 & 4 & 12 & -16 \end{bmatrix} \xrightarrow[-2R_2+R_3\to R_3]{\frac{1}{2}R_2\to R_2} \begin{bmatrix} 1 & -2 & 8 & -5 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & 14 & -13 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x_3 free

$x_1 = -13 - 14x_3$

$x_2 = -4 - 3x_3$

or $\begin{bmatrix} -13 \\ -4 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -14 \\ -3 \\ 1 \end{bmatrix}$

(2) (a) State the definition of a set $\{\vec{v}_1, \ldots, \vec{v}_p\}$ being linearly dependent.

The set is called linearly dependent if there are scalars $x_1, \ldots, x_p \in \mathbb{R}$, not all 0, such that $x_1\vec{v}_1 + \ldots + x_p\vec{v}_p = \vec{0}$.

(b) Can you have a set of two linearly dependent vectors in $\mathbb{R}^4$? Give an example, or say why it is not possible.

Yes. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

(3) For which values of h does the following vector equation have one solution? No solutions? Many solutions?

$$x_1 \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ h \\ 1 \end{bmatrix}.$$

This vector equation is equivalent to

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 3 \\ 1 & -2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ h \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 3 & -1 & 3 & h \\ 1 & -2 & -4 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & -1 & -3 & h-3 \\ 0 & -2 & -6 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 3-h \\ 0 & 0 & 0 & 6-2h \end{bmatrix}$$

There is a free variable (x_3), so there will never be just one solution. If $h=3$, the system is consistent and there ~~is exactly one~~ are infinitely many solutions.

If $h \neq 3$, there are no solutions.