Math 54, Spring 2009, Sections 109 and 112
Worksheet 3 (Lay 2.8-2.9)

(1) True or False? If true, justify. If false, give a counterexample.

(a) A set of 3 vectors in $\mathbb{R}^4$ must be linearly independent.

(b) A set of 3 vectors in $\mathbb{R}^4$ cannot span $\mathbb{R}^4$.

(c) A set of 5 vectors in $\mathbb{R}^4$ must be linearly dependent.

(d) A set of 5 vectors in $\mathbb{R}^4$ must span $\mathbb{R}^4$.

(a) False. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

(b) True. $\dim \operatorname{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} \leq 3$, so $\operatorname{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} \neq \mathbb{R}^4$

(c) True. 5 vectors, 4 entries in each

(d) False. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}, \dots, \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix} \right\}$. Their span is 1-dimensional, so not $\mathbb{R}^4$.

(2) Let $H \subseteq \mathbb{R}^n$ be a subspace. What is the definition of a basis for H? What is the definition of the dimension of H?

See p. 170, p. 177.

(Thm 13, p. 172)

(3) Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -2 & 4 \\ 1 & 4 & -2 \end{bmatrix}$. Find a basis for Col A. What is dim Col A? What is dim Nul A? (Hint: you don't need to do any extra work to answer the last part.) Need to find pivot cols.

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & -2 & 4 \\ 1 & 4 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & -2 \\ 1 & -2 & 4 \\ 2 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & -2 \\ 0 & -6 & 6 \\ 0 & -7 & 7 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

pivot colms.

So a basis for Col A is $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} \right\}$, and $\dim \text{Col } A = 2$.

By Rank theorem (p. 178), $\dim \text{Nul } A = \#\text{ colms} - \dim \text{Col } A = 1$.

(4) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and let $H = \text{Span}\{v_1, v_2\} \subseteq \mathbb{R}^3$. Find the coordinates of $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ with respect to the basis $\mathfrak{B} = \{v_1, v_2\}$ for H.

Want c_1, c_2 such that $c_1\vec{v}_1 + c_2\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, i.e. $\begin{bmatrix} 1 & 1 \\ 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$.

So we solve:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ -1 & 0 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix},$$

so $c_1 = 3$, $c_2 = -2$ is the (unique) solution. Thus

$$\left[\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \right]_{\mathcal{B}} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$