Name: Solution

Math 54, Spring 2009, Section 112
Quiz 2

(1) (3 pts) Find det $\begin{bmatrix} 3 & -2 & 1 & 3 \\ 0 & 2 & 0 & 4 \\ 1 & 4 & 6 & 5 \\ 0 & 0 & 0 & -1 \end{bmatrix}$.

$$\begin{vmatrix} 3 & -2 & 1 & 3 \\ 0 & 2 & 0 & 4 \\ 1 & 4 & 6 & 5 \\ 0 & 0 & 0 & -1 \end{vmatrix} \overset{\text{expand across last row}}{=} -1 \begin{vmatrix} 3 & -2 & 1 \\ 0 & 2 & 0 \\ 1 & 4 & 6 \end{vmatrix} \overset{\text{expand across second row}}{=} -2 \begin{vmatrix} 3 & 1 \\ 1 & 6 \end{vmatrix} = -34$$

(2) (a) (2 pts) What is the definition of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ that is one-to-one? What is the definition of a linear transformation that is onto?

T is called one-to-one if whenever $\vec{x} \neq \vec{y}$, we have $T(\vec{x}) \neq T(\vec{y})$.

T is called onto if for every $\vec{y} \in \mathbb{R}^m$ there is some $\vec{x} \in \mathbb{R}^n$ such that $T(\vec{x}) = \vec{y}$.

(b) (1 pt) Choose one of the above properties (one-to-one, or onto), and state an equivalent property of the standard matrix of T. (e.g. "T is one-to-one if and only if the standard matrix of T has 47 rows," but something true...)

Let A be the standard matrix of T.

T is one-to-one $\Leftrightarrow$ the columns of A are linearly independent $\Leftrightarrow$ A has a pivot in every column $\Leftrightarrow$ ...

T is onto $\Leftrightarrow$ $A\vec{x} = \vec{b}$ has a solution for every $\vec{b}$ $\Leftrightarrow$ A has a pivot in every row $\Leftrightarrow$ ...

(3) (a) (2pts) Let H be the subspace of $\mathbb{R}^3$ with ordered basis $\mathfrak{B} = \left\{ \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$. Given that $\vec{x} = \begin{bmatrix} 1 \\ -4 \\ -6 \end{bmatrix}$ is in H, find the coordinates of $\vec{x}$ with respect to $\mathfrak{B}$.

Want c_1, c_2 such that $\vec{x} = c_1 \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, i.e. $\begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ -6 \end{bmatrix}$

so row reduce $\begin{bmatrix} -1 & 1 & 1 \\ 2 & 2 & -4 \\ 3 & 3 & -6 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & -1 \\ 0 & 4 & -2 \\ 0 & 6 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -\frac{3}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$

so $c_1 = -\frac{3}{2}$ and $c_2 = -\frac{1}{2}$. Thus $[x]_B = \begin{bmatrix} -3/2 \\ -1/2 \end{bmatrix}$

(b) (1 pt) Suppose $T : \mathbb{R}^3 \to \mathbb{R}$ is a linear transformation with

$$T\left(\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}\right) = -2, \qquad T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = 2.$$

Find $T\left(\begin{bmatrix} 1 \\ -4 \\ -6 \end{bmatrix}\right)$.

$$T\left(\begin{bmatrix} 1 \\ -4 \\ -6 \end{bmatrix}\right) = T\left(-\frac{3}{2}\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = -\frac{3}{2} T\left(\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}\right) + -\frac{1}{2} T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right).$$

$$= -\frac{3}{2} \cdot -2 + -\frac{1}{2} \cdot 2 = 2$$