

Math 54, Spring 2009, Sections 109 and 112  
Worksheet 1: Lay 1.1 - 1.4

(1) Which of the following matrices are in an echelon form? Reduced echelon form? If the matrix is in an echelon form, identify the pivots.

(a)  $\begin{pmatrix} \textcircled{1} & 0 & 1 & 4 \\ 0 & 0 & \textcircled{1} & 0 \\ 0 & 0 & 0 & \textcircled{1} \end{pmatrix}$

Echelon form

(b)  $\begin{pmatrix} \textcircled{2} & 2 & 1 & -2 \\ 0 & 0 & \textcircled{1} & 3 \\ 0 & 0 & 0 & \textcircled{1} \end{pmatrix}$

Echelon form

(c)  $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Neither, row of 0's at the top

(2) (a) Does the following matrix equation have a solution?

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 2 & -4 \\ 1 & 1 & 1 \end{pmatrix} \vec{x} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}.$$

(b) Is  $(4, 0, 2)$  in  $\text{Span}\{(1, 4, 1), (2, 2, 1), (3, -4, 1)\}$ ? (Recall that for convenience we sometimes write vectors horizontally).

(a) Equation has solution if and only if  $\begin{cases} x_1 + 2x_2 + 3x_3 = 4 \\ 4x_1 + 2x_2 - 4x_3 = 0 \\ x_1 + x_2 + x_3 = 2 \end{cases}$  has solution

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & -4 & 0 \\ 1 & 1 & 1 & 2 \end{bmatrix} \xrightarrow{\substack{-4R_1 + R_2 \rightarrow R_2 \\ -R_1 + R_3 \rightarrow R_3}} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -6 & -16 & -16 \\ 0 & -1 & -2 & -2 \end{bmatrix} \xrightarrow{\substack{\text{Flip } R_2/R_3 \\ -1R_2 \rightarrow R_2}} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & -6 & -16 & -16 \end{bmatrix} \rightarrow$$

$$\xrightarrow{\substack{-2R_2 + R_1 \rightarrow R_1 \\ 6R_2 + R_3 \rightarrow R_3}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -4 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

So yes, it has a solution

Can also stop here. Pivot in every row means  $Ax=b$  has sol'n for every  $b$ .

(b) Yes. Recall that  $Ax=b$  has a solution if and only if  $b$  is in the span of the columns of  $A$ .

(3) Consider the set of vectors

$$S = \left\{ \begin{bmatrix} 2x_2 - x_1 \\ \pi x_3 + \sqrt{2}x_1 - x_2 \\ x_1 + x_2 + x_3 \end{bmatrix} : x_1, x_2, x_3 \in \mathbb{R} \right\}.$$

Can you write  $S$  as the span of a collection of vectors?

$$\begin{bmatrix} 2x_2 - x_1 \\ \pi x_3 + \sqrt{2}x_1 - x_2 \\ x_1 + x_2 + x_3 \end{bmatrix} = x_1 \begin{bmatrix} -1 \\ \sqrt{2} \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ \pi \\ 1 \end{bmatrix}$$

so  $S = \text{span} \{$

$\}$  Later, we'll see that these choices aren't unique.

(4) Suppose  $\vec{v}_1, \dots, \vec{v}_p$  are vectors in  $\mathbb{R}^n$  and let  $V = \text{Span}\{\vec{v}_1, \dots, \vec{v}_p\}$ . Prove that if  $\vec{x} \in V$  and  $\vec{y} \in V$ , then  $\vec{x} + \vec{y} \in V$  as well.

Since  $\vec{x} \in V$ , there are numbers  $c_1, \dots, c_p$  such that

$$\vec{x} = c_1 \vec{v}_1 + \dots + c_p \vec{v}_p. \quad \text{Similarly, there are numbers } d_1, \dots, d_p$$

such that  $\vec{y} = d_1 \vec{v}_1 + \dots + d_p \vec{v}_p$ . So

$$\vec{x} + \vec{y} = (c_1 + d_1) \vec{v}_1 + \dots + (c_p + d_p) \vec{v}_p, \quad \text{so by definition}$$

$$\vec{x} + \vec{y} \in V.$$