

A model for resource-aware load balancing on heterogeneous clusters

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Abstract

We address the problem of partitioning and dynamic load balancing on clusters with heterogeneous hardware resources. We propose DRUM, a model that encapsulates hardware resources and their interconnection topology. DRUM provides monitoring facilities for dynamic evaluation of communication, memory, and processing capabilities. Heterogeneity is quantified by merging the information from the monitors to produce a scalar number called “power.” This power allows DRUM to be used easily by existing load-balancing procedures such as those in the Zoltan Toolkit while placing minimal burden on application programmers. We demonstrate the use of DRUM to guide load balancing in the adaptive solution of a Laplace equation on a heterogeneous cluster. We observed a significant reduction in execution time compared to traditional methods.

1 Introduction

Clusters have gained wide acceptance as a viable alternative to tightly-coupled parallel computers. They provide cost-effective environments for running computationally-intensive parallel and distributed applications. An attractive feature of clusters is the ability to expand

their computational power incrementally by incorporating additional nodes. This expansion often results in heterogeneous environments, as the newly-added nodes often have superior capabilities. Grid technologies such as MPICH-G2 [10] have enabled computation on even more heterogeneous and widely-distributed systems. Internet-connected systems include more heterogeneity and extreme network hierarchy.

Our goal is to improve the efficiency of scientific computations in these heterogeneous environments, while putting as little burden as possible on application programmers. We propose DRUM (Dynamic Resource Utilization Model)¹, a persistent and dynamic model of the execution environment that captures the structure and dynamics of heterogeneous clusters in order to increase the effectiveness of load balancing. The model encapsulates hardware resources, their capabilities and their interconnection topology in a tree structure, and provides a mechanism for dynamic monitoring and evaluation of their utilization. Monitors in DRUM run concurrently with the user application to collect memory, network, and CPU utilization and availability statistics. Since our initial focus is on guiding a resource-aware dynamic load balancing, information from the monitors is distilled to a scalar “power” value, readily used by load-balancing algorithms capable of producing non-uniform partition sizes.

We apply DRUM to applications involving the parallel adaptive solution of partial differential equations. These are among the most demanding computational problems, arising in fields such as fluid dynamics [17], materials science [1], biomechanics [15], and ecology [4]. Adaptive strategies that automatically refine, coarsen, and/or relocate meshes and change the method to obtain a solution with a prescribed level of accuracy as quickly as possible are essential tools to solve modern multi-dimensional transient problems [5]. The usual approach to parallelizing these problems is to distribute a discretization (mesh) of the domain across cooperating processors, and then compute a solution, appraising its accuracy using error estimates at each step. If the solution is accepted, the computation proceeds to the next step. Otherwise, the discretization is refined adaptively, and work is redistributed, if necessary, to correct for any load imbalance introduced by the adaptive step. Thus, dynamic partitioning and load-balancing procedures become necessary. Sandia National Laboratories’

¹<http://www.cs.williams.edu/drum>

*Zoltan*² library [8] provides a common interface to several state-of-the-art partitioners and dynamic load balancers [2, 3, 7, 11, 23]. Most of these procedures seek to achieve an even distribution of computational work, while minimizing interprocess communication and data movement necessary to achieve the new decomposition. However, these procedures do not directly account for heterogeneity in the execution environment. Results herein use power values computed by DRUM to guide Zoltan procedures to produce resource-aware partitions. While our examples will use DRUM with Zoltan, DRUM may also be used as a stand-alone library.

The next section describes some related work. We present the details of DRUM in §3. §4 contains results using DRUM to reduce application run time on a heterogeneous cluster. A discussion of the results and a presentation of plans for future development and experimentation are the subject of §5.

2 Related Work

The popularity of clusters has motivated several recent efforts to study dynamic load balancing for heterogeneous systems. Minyard and Kallinderis [13] use octree structures to conduct partitioning in dynamic execution environments. To account for the dynamic nature of the execution environment, they collect run-time measurements based on the “wait” times of the processors involved in the computation. These “wait” times measure how long each CPU remains idle while all other processors finish the same task. The cells are assigned load factors that are proportional to the “wait” times of their respective owning processes. Each octant load is subsequently computed as the sum of load factors of the cells contained within the octant. The octree algorithm then balances the load factors based on the weight factors of the octants, rather than the number of cells contained within each octant. Walshaw and Cross [24] conduct multilevel mesh partitioning for heterogeneous communication networks. They modify a multilevel algorithm seeking to minimize a cost function based on a model of the heterogeneous communication network. The model, which gives only a static quantification of the network heterogeneity is supplied by the user at run-time as a

²<http://www.cs.sandia.gov/Zoltan>

Network Cost Matrix (NCM). The NCM implements a complete graph representing processor interconnections. Each graph edge is weighted as a function of the length of the path between its corresponding processors. Lowekamp *et al.* [12] present a resource monitoring system called *Remos*. Remos allows applications to collect information about network and host conditions across different network architectures. Sinha and Parashar [20] present a framework for adaptive system-sensitive partitioning and load balancing on heterogeneous and dynamic clusters. They use the Network Weather Service (NWS) [26] to gather information about the state and capabilities of available resources; then they compute the load capacity of each node as a weighted sum of processing, memory, and communications capabilities. Reported experimental results show that system-sensitive partitioning resulted significant decrease of application execution time. Most of these approaches are either related to a specific load-balancing algorithm or rely on information supplied externally at run-time or through instrumentation probes added to the user application. Our proposed system attempts to address some of these issues. As a matter of fact, DRUM does not require a specific load-balancing algorithm and relies on both static and dynamic information to evaluate resource usage.

3 DRUM: Dynamic Resource Utilization Model

We present DRUM, a model that incorporates aggregated information about the capabilities of the network and computing resources composing an execution environment. DRUM can be viewed as an abstract object that (i) encapsulates the details of the execution environment, and (ii) provides a facility for dynamic, modular and minimally-intrusive monitoring of the execution environment.

Unlike the directed-graph hardware model used in the Rensselaer Partition Model [22], DRUM uses a tree structure. DRUM also incorporates a framework that addresses hierarchical clusters (*e.g.*, clusters of clusters, or clusters of multiprocessors) by capturing the underlying interconnection network topology. The inherent structure of DRUM leads naturally to a topology-driven, yet transparent, execution of hierarchical partitioning [21]. The root of the tree represents the total execution environment. The children of the root node

are high level divisions of different networks connected to form the total execution environment. Sub-environments are recursively divided, according to the network hierarchy, with the tree leaves being individual single-processor (SP) nodes or shared-memory multiprocessing (SMP) nodes. *Computation nodes* at the leaves of the tree have data representing their relative computing and communication power. *Network nodes*, representing routers or switches, have an aggregate power calculated as a function of the powers of their children and the network characteristics.

We quantify the heterogeneity of the different components of the execution environment by assessing computational, memory and communication capabilities of each node. The collected data in each node is combined in a single quantity called the node “power.” For load-balancing purposes, we interpret a node’s power as the percentage of overall load it should be assigned based on its capabilities.

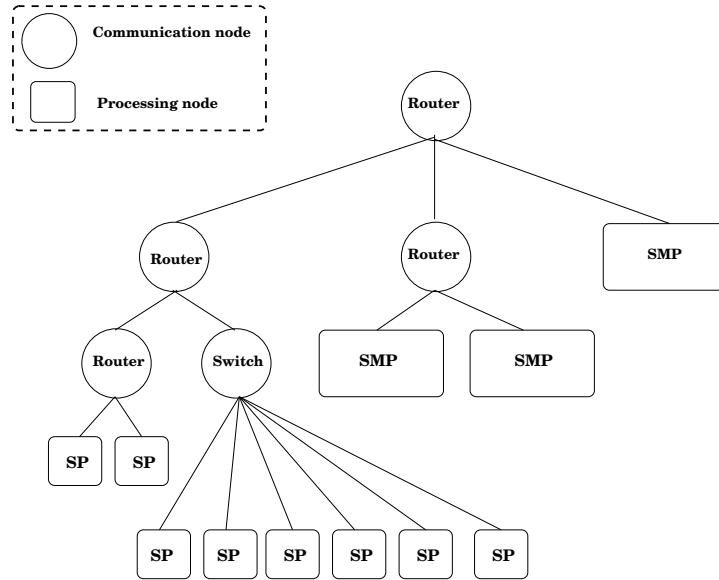


Figure 1: Tree constructed by DRUM to represent a heterogeneous network.

Figure 1 shows an example of a tree constructed by DRUM to represent a heterogeneous cluster. Eight SP nodes and three SMP nodes are connected in a hierarchical network structure consisting of four routers and a network switch.

3.1 Model creation

DRUM's tree model of the execution environment must be created upon initialization. An XML file containing a list of nodes and description of their interconnection topology is used by a configuration tool to generate the initial data structures of DRUM. The configuration tool (Figure 2) provides capabilities including XML file generation using a graphical interface, initial assessment of node capabilities by running distributed benchmarks, and facilities to check availability of network management capabilities such as SNMP (Simple Network Management Protocol) and threading. The configuration tool needs to be re-run only when hardware characteristics of the system have changed.

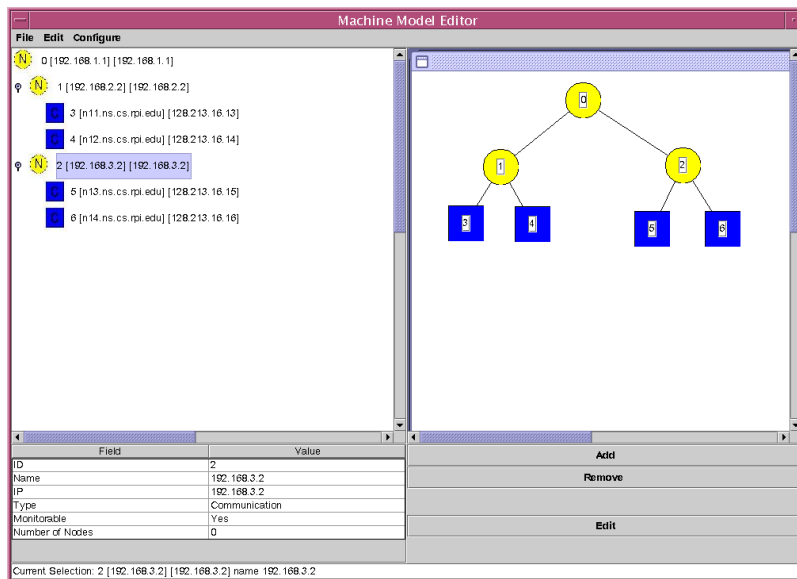


Figure 2: Screen shot of the graphical Java program, used to aid in the creation of the XML machine topology description. Here, a description of a small cluster is being edited.

3.2 Capabilities Assessment

Resource capabilities are assessed initially using benchmarks and are updated dynamically by *agents*: threads that run concurrently with the application to monitor each node. Currently, LINPACK [9] is used as a benchmark to compute a MFLOPS rating for the computation nodes of the cluster. The benchmark may be run from the configuration tool (Figure 2) or

at the command line. The benchmarks may not accurately reflect the characteristics of a particular computation. We intend to allow user-specified callbacks that could be used to calibrate the machine model, likely using the same software that will be used for the actual computation.

An application may use only the static information gathered from the original benchmarks (*e.g.*, if the system does not support threads), or may also use dynamic monitoring. The `DRUM_startMonitoring()` function spawns the agent threads. A call to `DRUM_stopMonitoring()` ends the dynamic monitoring. The monitoring agents contain `DRUM_nic` objects that monitor communication traffic and `DRUM_cpuMem` objects that monitor CPU load and memory usage. Some computation nodes, called *representatives*, are responsible for monitoring one or more network nodes.

A `DRUM_nic` object can be attached to either a computation or a network node. Versions of `DRUM_nic` objects have been implemented using the `net-snmp` library³ and kernel statistics to collect network traffic information at each node.

A `DRUM_cpuMem` object gathers information about the CPU usage and the memory capacity of a computation node using kernel statistics. The statistics are combined with the static benchmark data to obtain a dynamic estimate of the processing power.

After monitoring has been stopped, `DRUM_computePowers()` updates the power of the nodes in the model. These powers are queried with `DRUM_getLocalPartSize()`.

3.3 Node power

DRUM distills the information in the model to a power value for each node, a single number indicating the portion of the total load that should be assigned to that node. This is similar to the Sinha and Parashar approach [20]. Given power values for each node, any partitioning procedure capable of producing variable-sized partitions, including all Zoltan procedures, may be used to achieve an appropriate decomposition. Thus, any applications using a load-balancing procedure capable of producing non-uniform partitions can take advantage of DRUM with little modification. Applications that already use Zoltan can make use of

³<http://www.net-snmp.org>

DRUM simply by setting a Zoltan parameter, with no further changes needed.

The power at each node depends on processing power and communication power. We compute the power of node n as the weighted sum of the processing power p_n and communication power c_n :

$$power_n = w_n^{comm} c_n + w_n^{cpu} p_n, \quad w_n^{comm} + w_n^{cpu} = 1.$$

3.3.1 Processing power

For a computation node n with m CPUs on which k_n application processes are running, we evaluate the processing power $p_{n,j}$ for each process j on node n based on (i) CPU utilization $u_{n,j}$ by process j , (ii) the fraction i_t of time that CPU t is idle, and (iii) the node's static benchmark rating (in MFLOPS) b_n . The overall idle time in node n is $\sum_{t=1}^m i_t$. However, when $k_n < m$, the k_n processes can make use of only k_n processors at any time, so the maximum exploitable total idle time is $k_n - \sum_{j=1}^{k_n} u_{n,j}$. Therefore, the total idle time that the k_n processes could exploit is $\min(k_n - \sum_{j=1}^{k_n} u_{n,j}, \sum_{t=1}^m i_t)$. Since the operating system's CPU scheduler can be expected to give each of the k_n processes the same portion of the time on a node's CPUs, we assign all processes on node n equal power. We compute average CPU usage and idle times per process:

$$\bar{u}_n = \frac{1}{k_n} \sum_{j=1}^{k_n} u_{n,j}, \quad \bar{i}_n = \frac{1}{k_n} \min(k_n - \sum_{j=1}^{k_n} u_{n,j}, \sum_{t=1}^m i_t).$$

Processing power $p_{n,j}$ is estimated as

$$p_{n,j} = b_n(\bar{u}_n + \bar{i}_n), \quad j = 1, 2, \dots, k_n.$$

Since $p_{n,j}$ is the same for all processes j on node n , $p_n = \sum_{j=1}^{k_n} p_{n,j} = k_n p_{n,1}$. On internal nodes, p_n is the sum of the processing powers of the nodes' children.

3.3.2 Communication power

We estimate a node's communication power based on the communication traffic at the node. At each computation and (when possible) network node, agents estimate the average rate of incoming packets λ and outgoing packets μ on each relevant communication interface. We

view a node’s communication power as inversely proportional to the communication activity factor $CAF = \lambda + \mu$ at that node. The CAF provides dynamic information about the traffic passing through a node, the communication traffic at neighboring nodes, and, to some extent, the traffic in the overall system. The communication power of a node with more than one interface (*e.g.*, routers) is computed as the inverse of the average CAF value of its interfaces. Let $CAF_{n,i}$ denote the CAF of interface i of a node n with s interfaces. We estimate the communication power as

$$c_n = \frac{1}{\frac{1}{s} \sum_{i=1}^s CAF_{n,i}}.$$

In practice, software loop-back interfaces and interfaces with $CAF = 0$ are ignored. To compute per-process communication powers for processes $j, j = 1, 2, \dots, k_n$, on node n , we compute c_n and associate $\frac{1}{k_n}c_n$ with each process. For consistency, if at least one non-root network node cannot be probed for communication traffic, all internal nodes are assigned CAF values computed as the sum of their immediate children’s values.

4 Computational results

We present experimental results using DRUM to guide resource-aware load balancing in the adaptive solution of a Laplace equation on the unit square, using Mitchell’s Parallel Hierarchical Adaptive MultiLevel software (PHAML) [14]. After 17 adaptive refinement steps, the mesh has 524,500 nodes. We use a Sun cluster at Williams College consisting of nodes with “fast” (450MHz Sparc UltraII) and “slow” (300/333MHz Sparc UltraII) processors. Fast processors are either in four-way or two-way SMP nodes. Slow processors are in uniprocessor nodes. All nodes are connected by fast (100 Mbit) Ethernet. Benchmark runs indicated that the fast processors have a computation rate of approximately 1.5 times faster than the slow processors. Given an equal distribution of work, the fast processors would be idle one third of the time.

We use the Zoltan’s Hilbert Space Filling Curve (HSFC) procedure for partitioning, though the powers computed by DRUM may be used by any Zoltan procedure and results are similar for other methods. A space-filling curve (SFC) maps n -dimensional space to

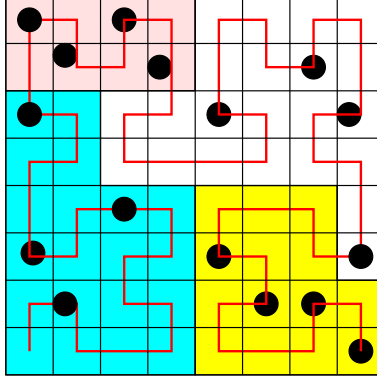


Figure 3: SFC partitioning example. Dots (objects) are ordered along the SFC. Partitions are indicated by shading.

one dimension [19]. In SFC partitioning [25, 16], an object’s coordinates are converted to a SFC key representing the object’s position along a SFC through the physical domain. Sorting the keys gives a linear ordering of the objects (see Figure 3). This ordering is cut into appropriately weighted pieces that are assigned to processors. Zoltan method HSFC [8] replaces the sort with adaptive binning. Based upon their Hilbert SFC keys, objects are assigned to bins associated with partitions. Bin sizes are adjusted adaptively to obtain sufficient granularity for balancing.

4.1 Experiment 1: Heterogeneous cluster with one process per node

The first set of experiments uses combinations of fast and slow processors with one application process being run on each node. Figure 4 shows the run time of the PHAML application on various combinations of fast and slow processors and for communication weights w^{comm} values of 0, 0.1 and 0.25. Runs on only the homogeneous (fast) nodes show very low overhead incurred by the use of DRUM. On heterogeneous configurations, experiments using DRUM’s resource-aware partitions show a clear improvement in execution time compared to those with uniformly sized partitions. Next, we compare the execution time Relative Change (RC) achieved to an “Ideal” Relative Change (RC_{ideal}). RC is the variation in execution

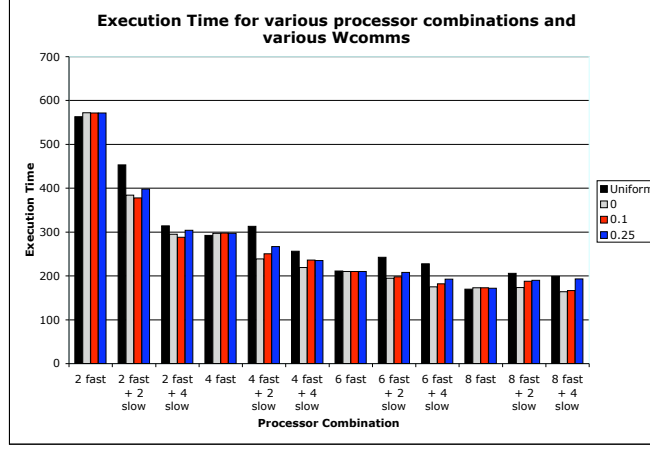


Figure 4: Execution times for PHAML runs when DRUM is used on different combinations of fast and slow processors, with uniform partition sizes and resource-aware partition sizes (with various values for w_{comm}). Here, only one application process is run on each node.

time relative to the original execution time:

$$RC = \frac{t_{uniform} - t_{DRUM}}{t_{uniform}}$$

where $t_{uniform}$ is the execution time of the application without using DRUM and t_{DRUM} is the execution time when DRUM is used. For this example, RC_{ideal} is the relative change that would be achieved if the fast processors were assigned exactly 50% more load than the slow ones and if communication overhead were ignored. In general,

$$RC_{ideal} = \frac{(h-1)f}{(s+h*f)}$$

where s is the number of application processes on the slow nodes, f the number of application processes on the fast nodes and h is the ratio of the fast processors' speed to that of the slow processors. In our case, since the fast nodes are assumed to be 1.5 times faster than the slow ones, $h = 1.5$. The assumption of no communication overhead is not realistic in most adaptive applications and, therefore, RC_{ideal} cannot practically be reached. Figure 5 shows the Relative Change comparisons.

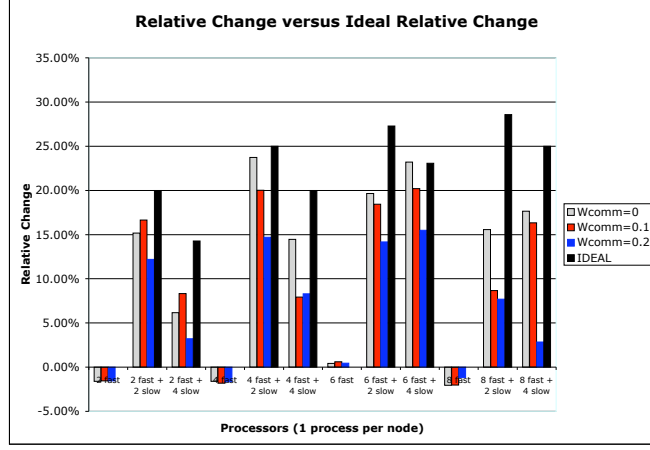


Figure 5: Relative change in CPU times for PHAML runs when DRUM is used on different combinations of fast and slow processors, contrasted with the ideal relative change. Only one application process is running on each node.

4.2 Experiment 2: Communication Weight Study

In order to study the effect of the communication weight w^{comm} in the overall execution time, we repeated the experiment for a wider range of communication weights and processor/process combinations. Here, multiple application processes are running on the SMP nodes. On both the slow and fast nodes, only one (application) process is running per processor. The results are reported in Figure 6. The combination of processes, processors and nodes is reported as:

`#total processes [#fast nodes(#processes per node) + #slow nodes(1)]`

In particular, these results confirm the low overhead of DRUM monitors in the cases of 8 and 16 processor runs, where only fast nodes are used. They also show significant benefits in the case of heterogeneous processor/process combinations. These computations also suggest that a communication weight of more than 0.5 is not appropriate for our test application. This is expected, since this application overlaps computation and communication. Figure 7 shows the best relative change values for each combination of processors and contrasts them

with the ideal overhead.

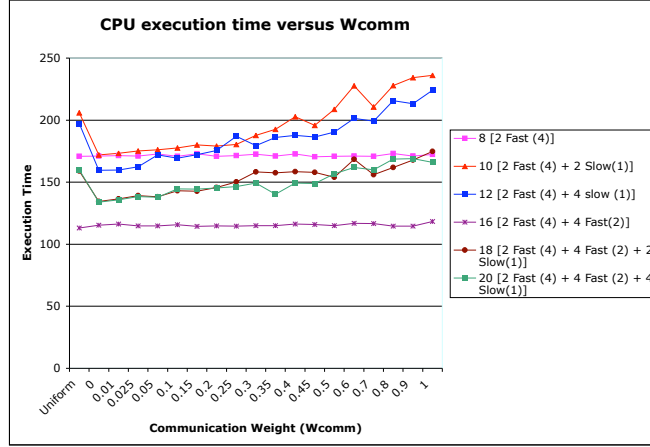


Figure 6: Execution times for PHAML runs when DRUM is used on different combinations of fast and slow processors and with different values for w_n^{comm} .

4.3 Experiment 3: Correlation with Degree of Heterogeneity

The potential improvement from resource-aware load balancing depends to a large extent on the degree of heterogeneity in the system. If the execution environment is nearly homogeneous, very little can be gained by accounting for heterogeneity. In such a situation, the overhead introduced by the dynamic monitoring may even slow the computation slightly. Hence, any measure of improvement should be tied to the degree of heterogeneity of the system.

Xiao, *et al.*, propose metrics for CPU and memory heterogeneity defined as the standard deviation of computing powers and memory capacities among the computation nodes [27]. In particular, they define system CPU heterogeneity for a system with P processors as

$$H_{cpu} = \sqrt{\frac{\sum_{j=1}^P (\bar{W}_{cpu} - W_{cpu}(j))^2}{P}}$$

where $W_{cpu}(j)$ is a measure of the CPU speed relative to the fastest CPU in the system,

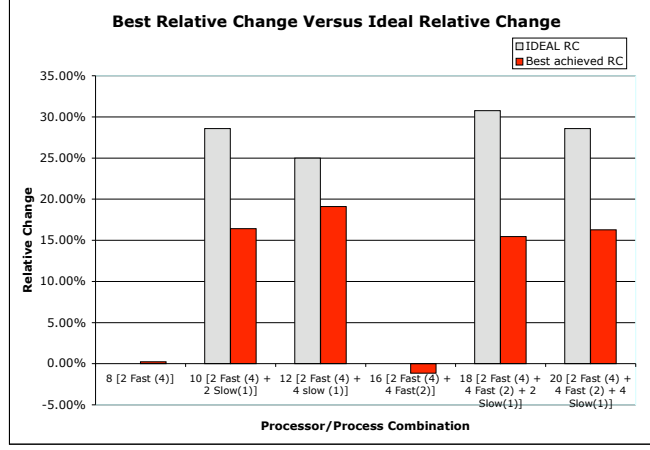


Figure 7: Ideal and best observed relative changes across all values of w_n^{comm} for the timings shown in Figure 6.

computed as

$$W_{cpu}(j) = \frac{V_{cpu}(j)}{\max_{i=1}^P V_{cpu}(i)}$$

and \overline{W}_{cpu} is the average of relative CPU speeds:

$$\overline{W}_{cpu} = \frac{\sum_{j=1}^P W_{cpu}(j)}{P}.$$

$V_{cpu}(i)$ is the MIPS (millions of instructions per second) rating for CPU i . We use the same formulas to measure CPU heterogeneity, substituting the MFLOPS numbers obtained from the benchmark for the MIPS values. MFLOPS provides a more reliable measure of CPU performance than raw MIPS.

Figure 8 shows the evolution of RC as a function of the degree of heterogeneity. As expected, DRUM has a greater impact on the execution time when the heterogeneity of the execution environment is greater.

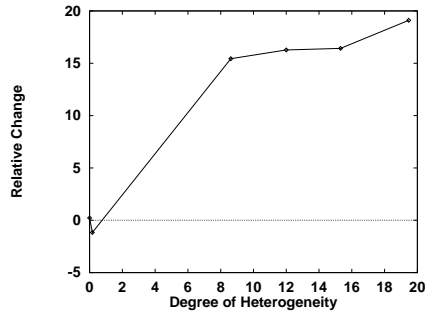


Figure 8: Relative Change in execution time as a function of the degree of heterogeneity.

5 Discussion

Our preliminary results show a clear benefit to resource-aware load balancing. We are currently testing the procedures on a wider variety of heterogeneous systems and for different applications. We have implemented hierarchical balancing procedures that interact with DRUM to tailor partitions to a given network topology [21]. In addition to the ability to produce weighted partitions, this strategy allows different load-balancing algorithms to be used, as appropriate, in different parts of the network hierarchy [21, 22].

DRUM accounts for both static information by using benchmark data and dynamic performance data using monitoring agents. This provides a benefit both on dedicated systems with some heterogeneity that can be captured by the benchmarks, and highly dynamic systems where the execution environment may be shared with other processes. We are currently testing our procedures in these environments.

We are currently integrating tools for data noise filtering and forecasting. This would allow DRUM to produce better estimates for resource utilization and would also permit an implementation of an adaptive probing procedure in the monitors.

DRUM agents monitor the available memory and the total memory on each computation node. Given this limited information, memory utilization should be a factor in the computation of a node's power only when the ratio of available memory to total memory becomes smaller than a specified threshold. More refined memory statistics (*e.g.*, number of cache levels, cache hit ratio, cache and main memory access times) are needed to capture memory

effects more accurately in our model.

Currently, DRUM requires a description of the computing environment, which may be generated by the graphical configuration tool, and uses its own monitoring tools to gather benchmarks and run-time performance statistics. In grid environments, and other situations where this information is available from other tools (*e.g.*, Globus Monitoring and Discovery Service (MDS) [6], the Network Weather Service (NWS) [26], Ganglia [18]), we plan to interface DRUM with these tools.

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