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In [ ]: pip install matplotlib
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In [6]: import numpy as np
import matplotlib.pyplot as plt

class EquationSolver:
    def __init__(self, f, df=None):
        self.f = f
        self.df = df
        self.history = []

    def bisection(self, a, b, tol=1e-10, max_iter=100):
        self.history = []
        fa, fb = self.f(a), self.f(b)
        if fa * fb > 0:
            raise ValueError("La fonction doit changer de signe sur [a,b].")
        for k in range(max_iter):
            m = (a + b) / 2
            fm = self.f(m)
            self.history.append(m)
            if abs(fm) < tol or (b - a) / 2 < tol:
                return m
            if fa * fm < 0:
                b, fb = m, fm
            else:
                a, fa = m, fm
        return (a + b) / 2

    def fixed_point(self, g, x0, tol=1e-10, max_iter=100):
        self.history = []
        x = x0
        for k in range(max_iter):
            self.history.append(x)
            x_new = g(x)
            if abs(x_new - x) < tol:
                return x_new
            x = x_new
        return x

    def newton_raphson(self, x0, tol=1e-10, max_iter=100):
        if self.df is None:
            raise ValueError("Il faut fournir la dérivée pour Newton-Raphson")
        self.history = []
        x = x0
        for k in range(max_iter):
            self.history.append(x)
            fx, dfx = self.f(x), self.df(x)
            if dfx == 0:
                raise ZeroDivisionError("Dérivée nulle, Newton échoue")
            x_new = x - fx / dfx
            if abs(x_new - x) < tol:
                return x_new
            x = x_new
        return x
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def plot_function(self, a, b, title=""):
    X = np.linspace(a, b, 400)
    Y = [self.f(x) for x in X]
    plt.axhline(0, color="black")
    plt.plot(X, Y, label="f(x)")
    plt.title(title)
    plt.grid()
    plt.legend()
    plt.show()

def plot_convergence(self, true_root=None, method=""):
    if not self.history:
        return
    errors = [abs(x - true_root) for x in self.history] if true_root else None
    plt.plot(range(len(self.history)), errors if errors else self.history, mark
    plt.xlabel("Itération")
    plt.ylabel("Erreur" if true_root else "Approximation")
    plt.title(f"Convergence - {method}")
    plt.yscale("log") if true_root else None
    plt.grid()
    plt.show()

# =====
# Définition des fonctions
# =====

functions = {
    "f1": {
        "f": lambda x: x**3 - x - 1,
        "df": lambda x: 3*x**2 - 1,
        "interval": (1, 2),
        "g": lambda x: (x + 1)**(1/3) # reformulation point fixe
    },
    "f2": {
        "f": lambda x: np.exp(x) - 2*x - 1,
        "df": lambda x: np.exp(x) - 2,
        "interval": (0, 2),
        "g": lambda x: (np.exp(x) - 1)/2
    },
    "f3": {
        "f": lambda x: np.cos(x) - x,
        "df": lambda x: -np.sin(x) - 1,
        "interval": (0, 1),
        "g": lambda x: np.cos(x)
    }
}

# =====
# Application
# =====

for name, data in functions.items():
    print(f"\n=== {name} ===")
    solver = EquationSolver(data["f"], data["df"])

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a, b = data["interval"]

# Tracer fonction
solver.plot_function(a, b, title=f"{name}: f(x)")

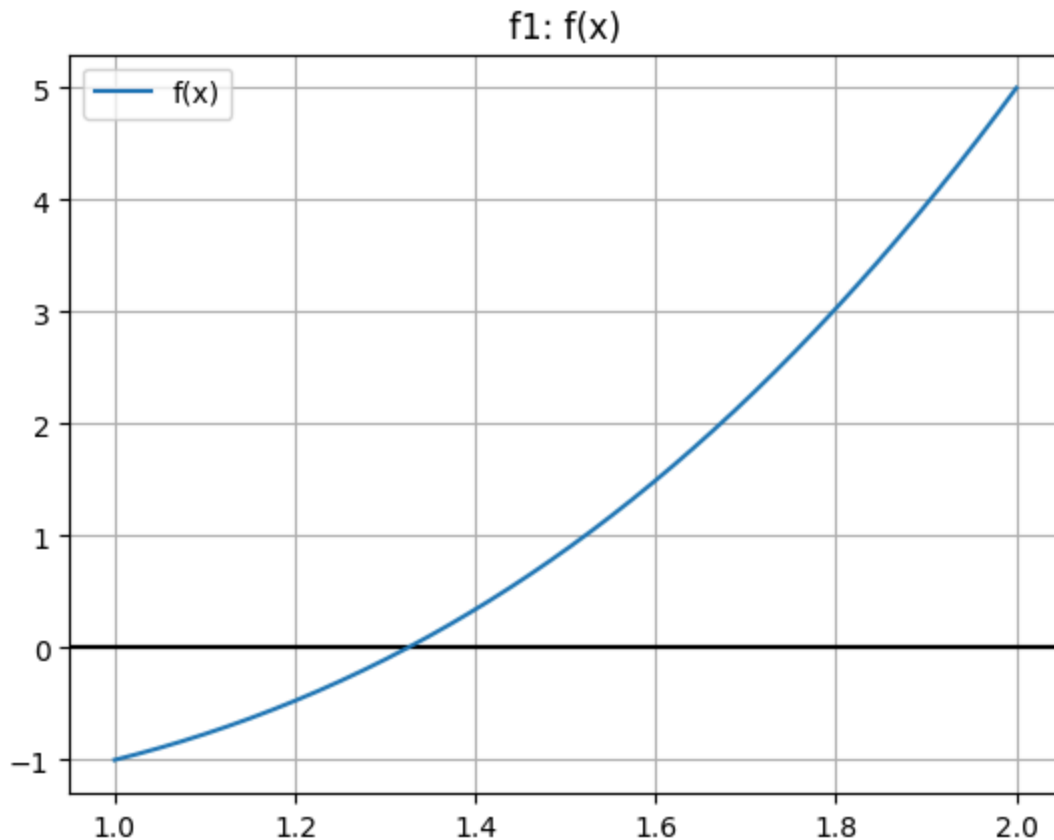
# Bisection
root_bisec = solver.bisection(a, b)
solver.plot_convergence(true_root=root_bisec, method=f"{name} - Bisection")
print("Bisection root =", root_bisec)

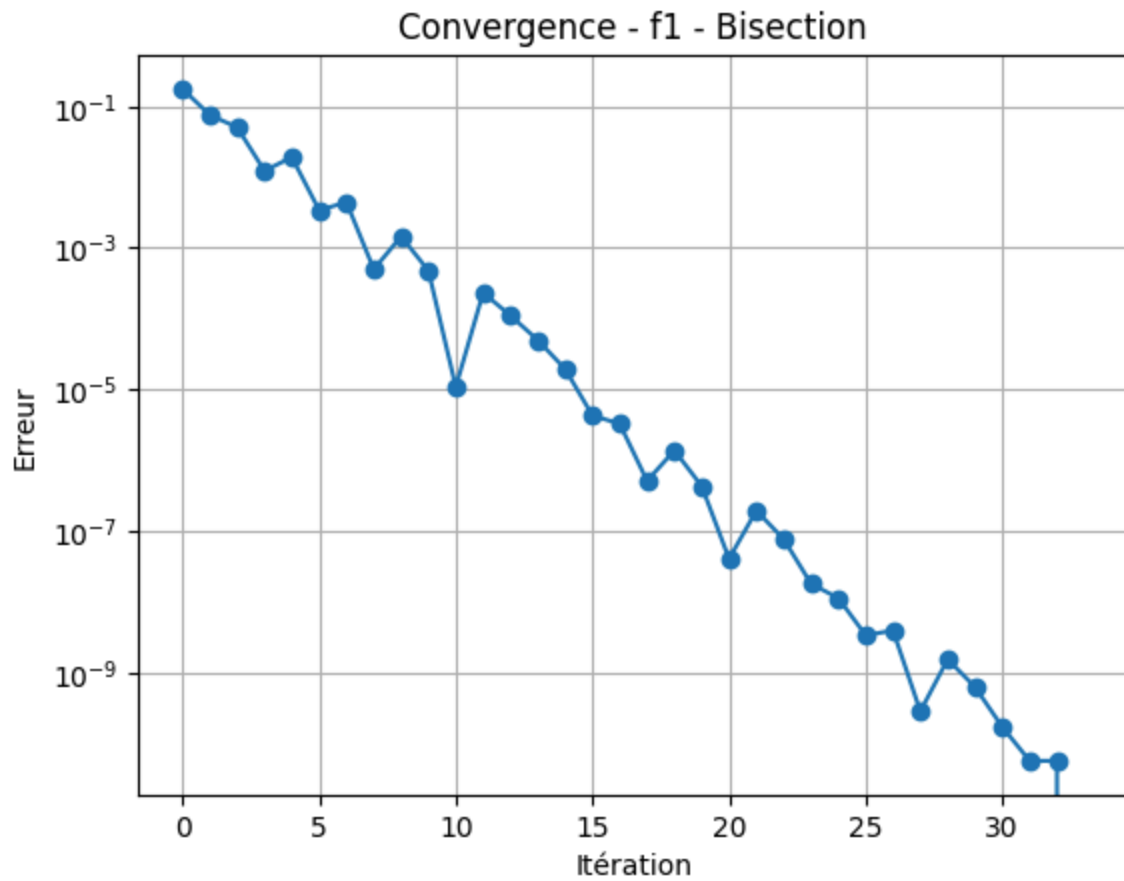
# Point fixe
root_fixed = solver.fixed_point(data["g"], (a+b)/2)
solver.plot_convergence(true_root=root_bisec, method=f"{name} - Point Fixe")
print("Fixed point root =", root_fixed)

# Newton-Raphson (différents points de départ)
for x0 in [a, (a+b)/2, b]:
    solver = EquationSolver(data["f"], data["df"])
    try:
        root_newton = solver.newton_raphson(x0)
        solver.plot_convergence(true_root=root_bisec, method=f"{name} - Newton")
        print(f"Newton-Raphson root (x0={x0}) =", root_newton)
    except Exception as e:
        print(f"Newton échoue pour x0={x0}: {e}")

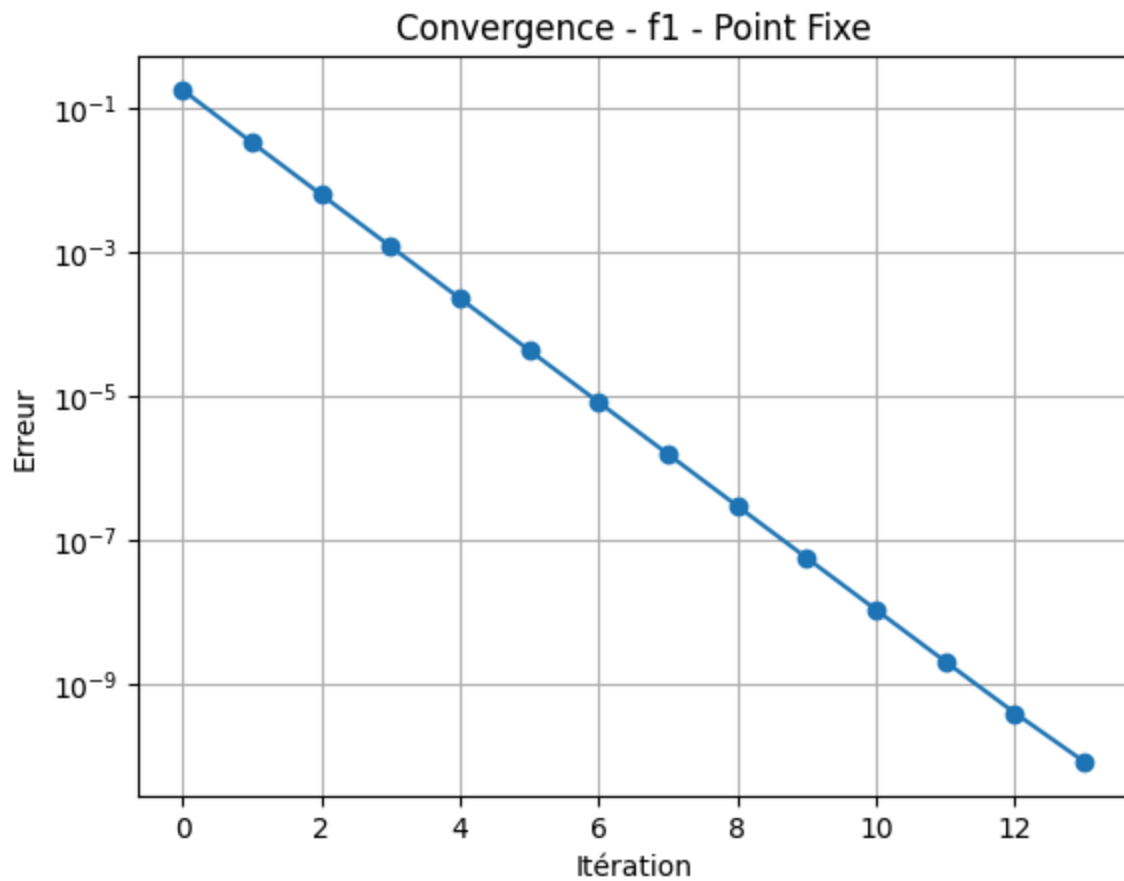
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=== f1 ===

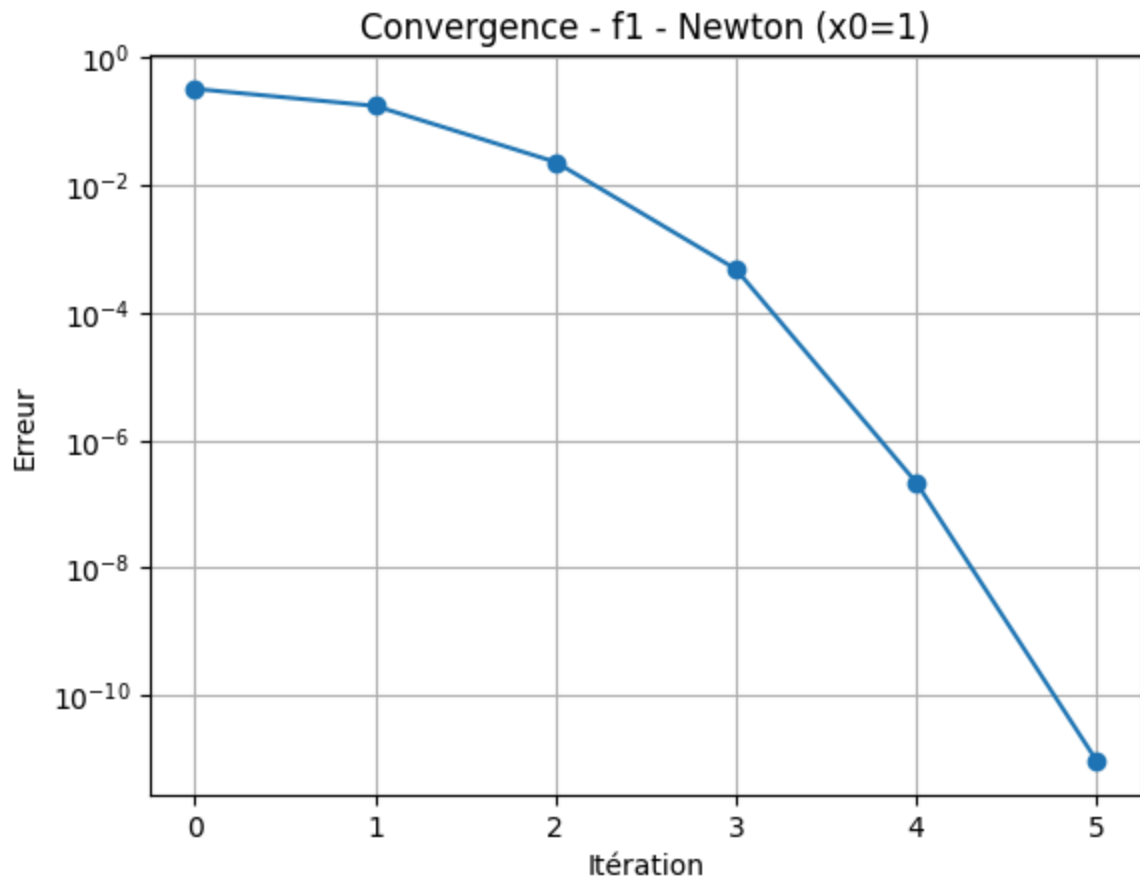




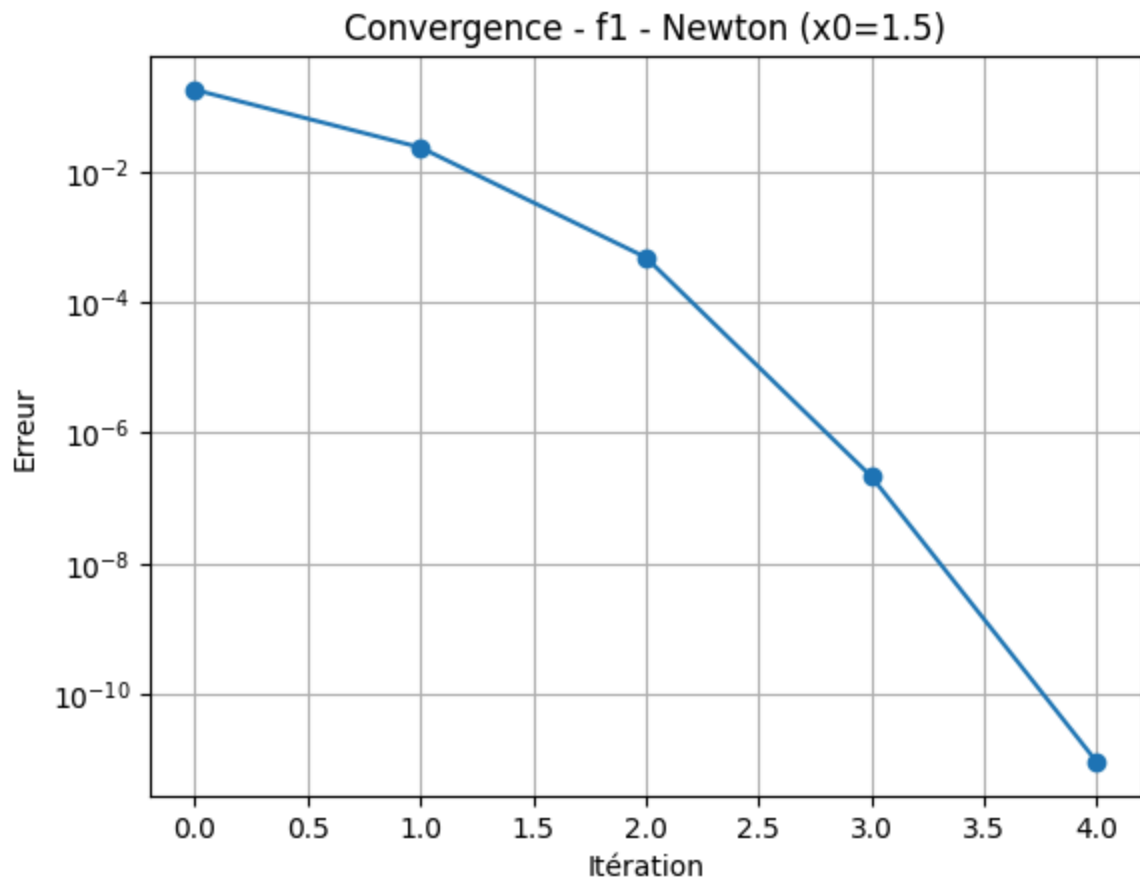
Bisection root = 1.324717957235407



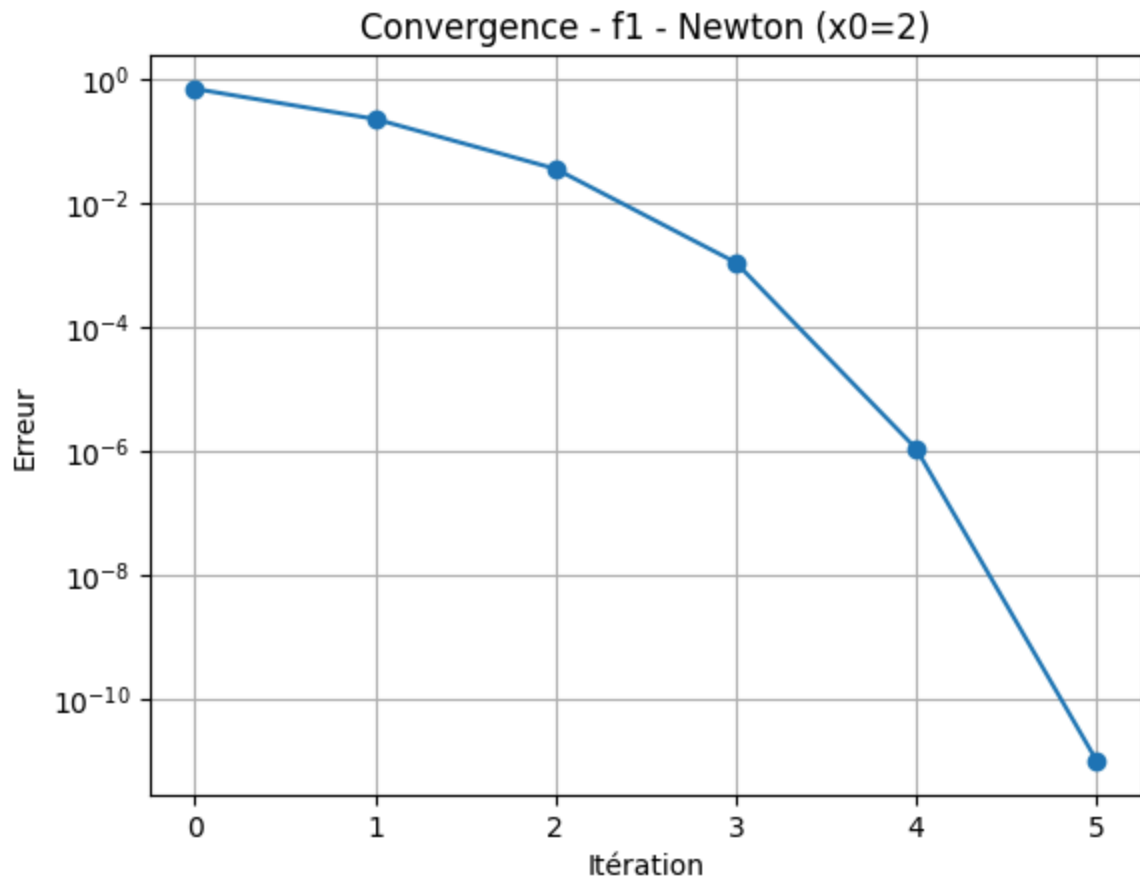
Fixed point root = 1.3247179572582821



Newton-Raphson root (x0=1) = 1.324717957244746

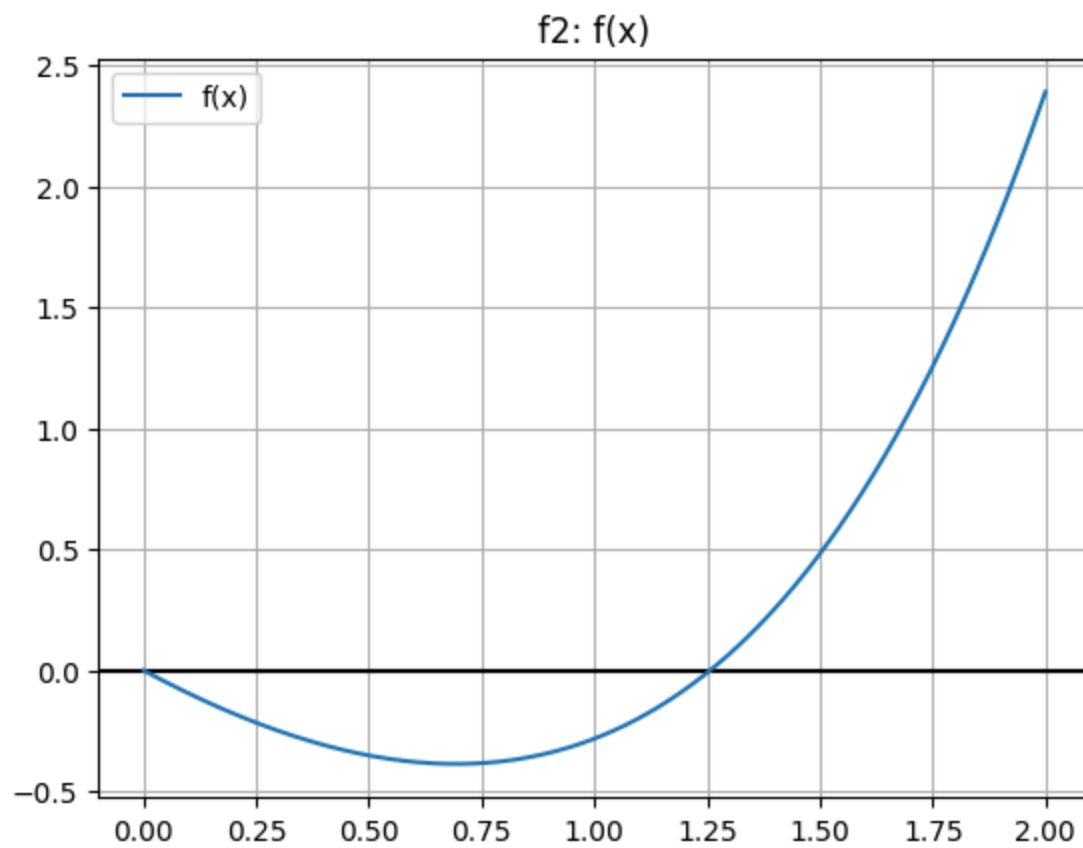


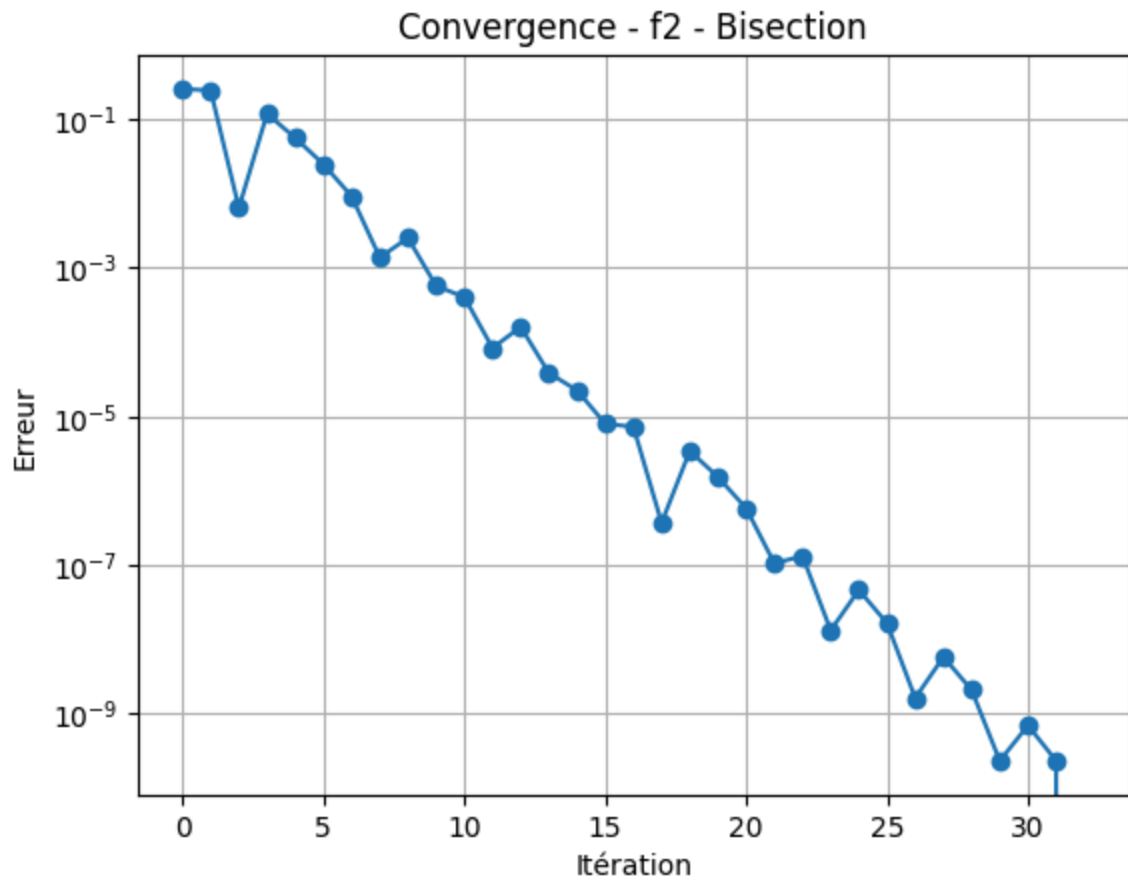
Newton-Raphson root (x0=1.5) = 1.324717957244746



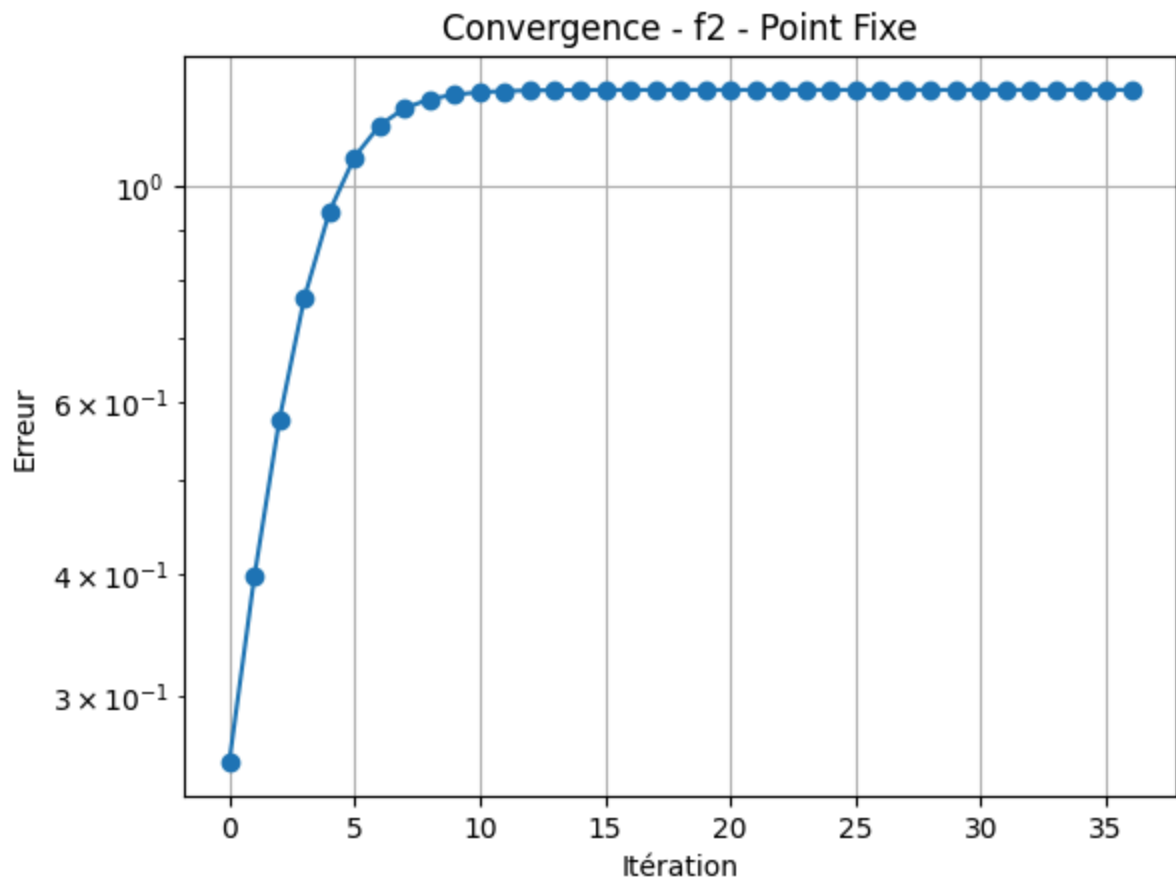
Newton-Raphson root ( $x_0=2$ ) = 1.324717957244746

=== f2 ===

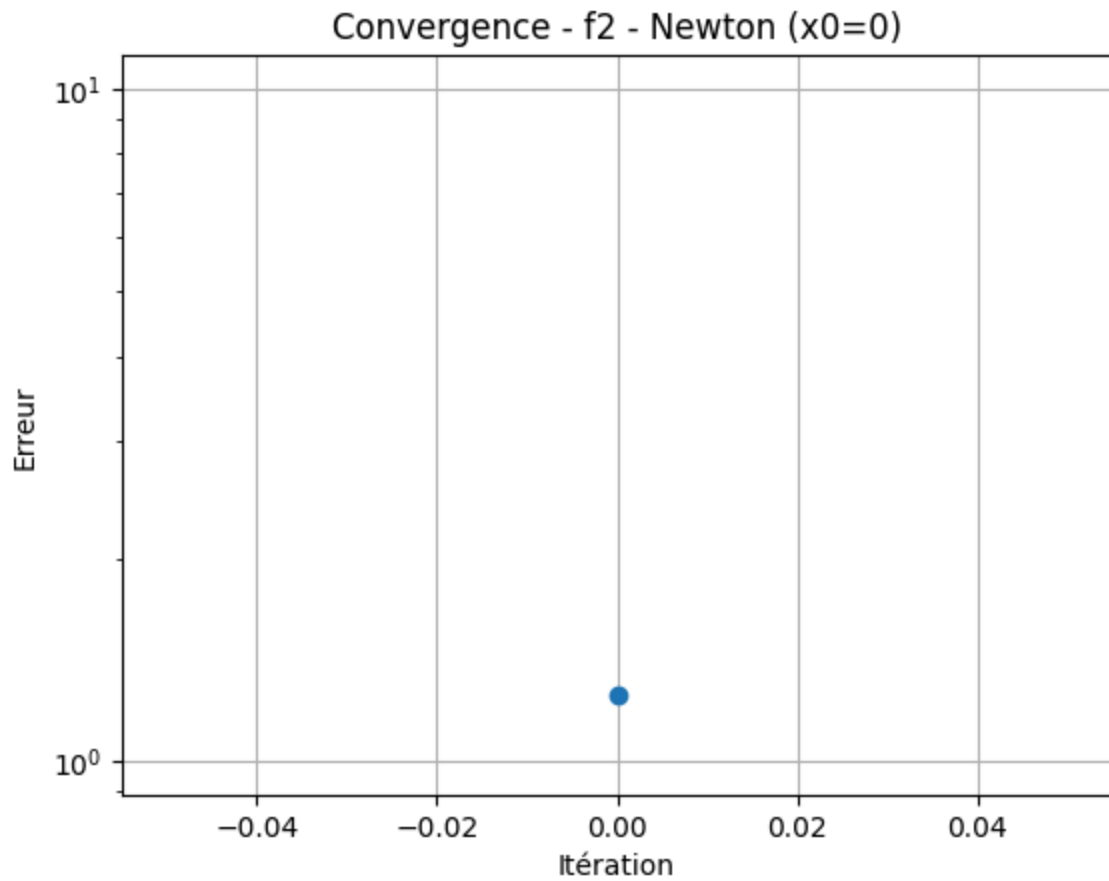




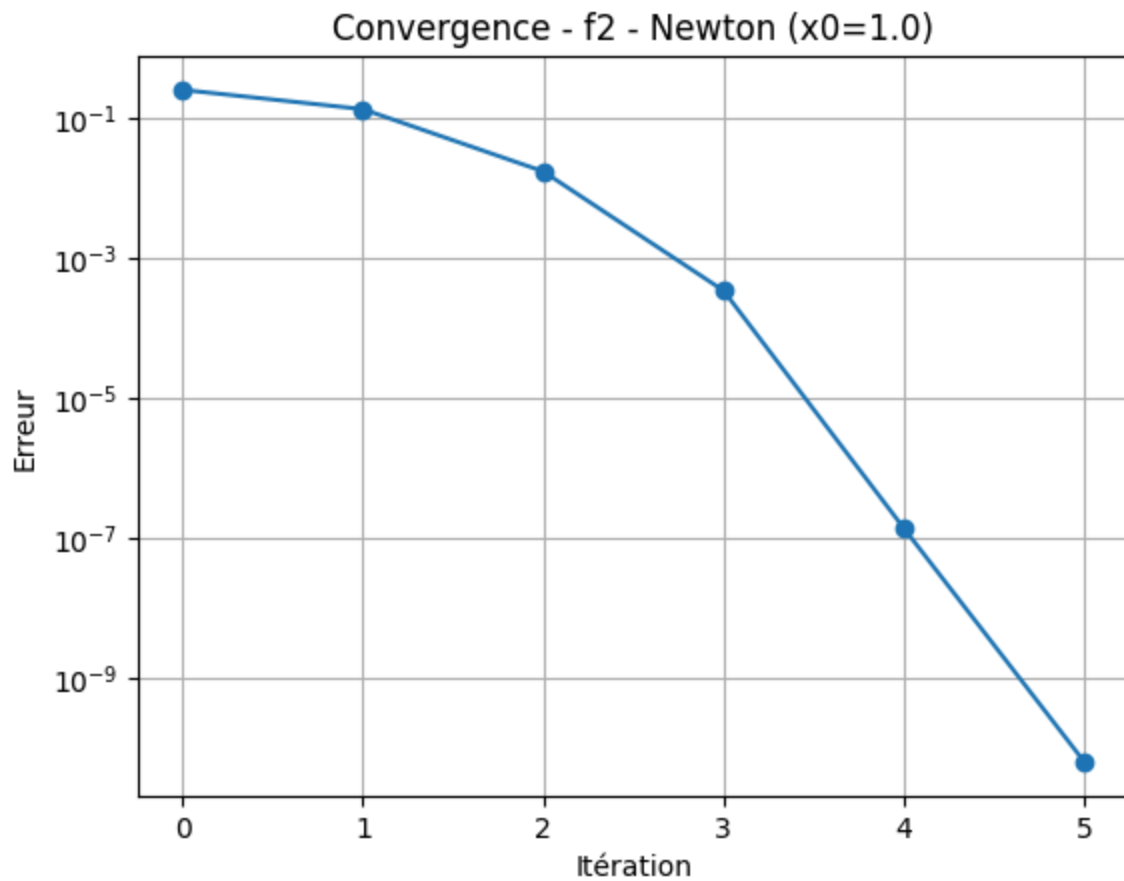
Bisection root = 1.2564312086906284



Fixed point root = 5.243483425232398e-11

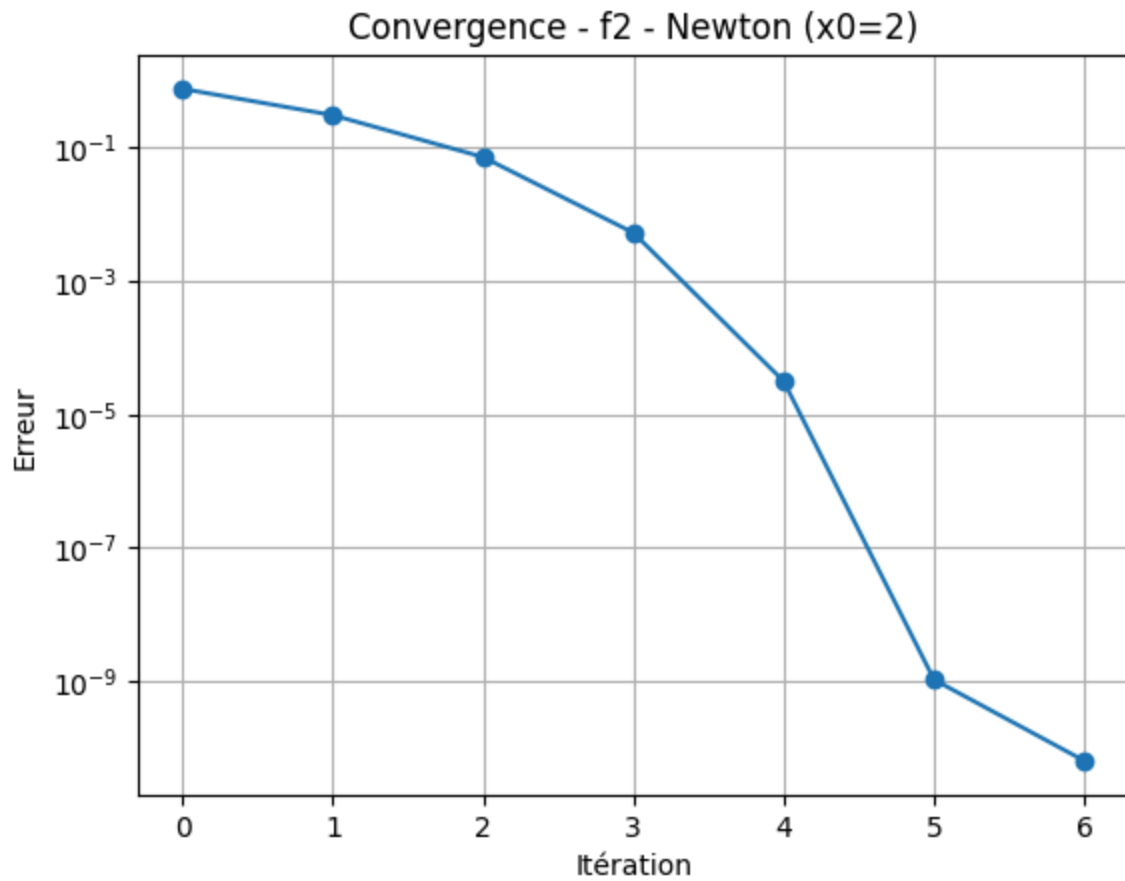


Newton-Raphson root ( $x_0=0$ ) = 0.0



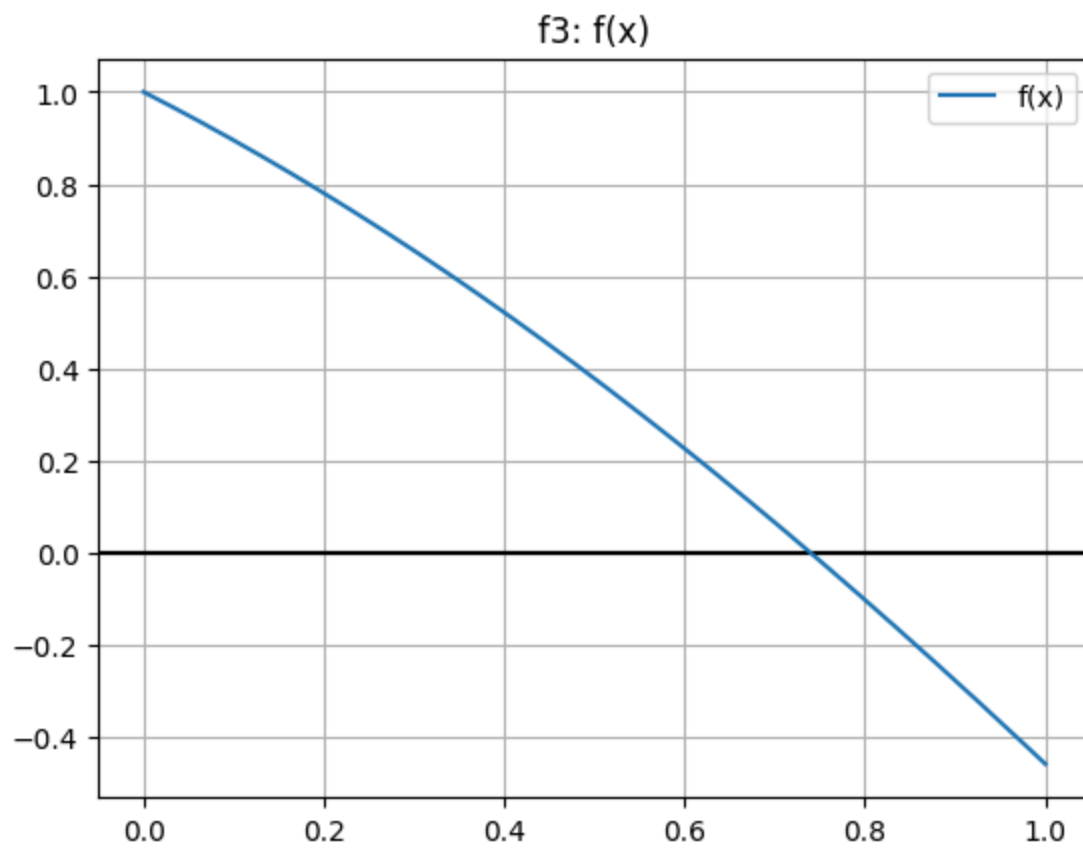
Newton-Raphson root ( $x_0=1.0$ ) = 1.2564312086261695

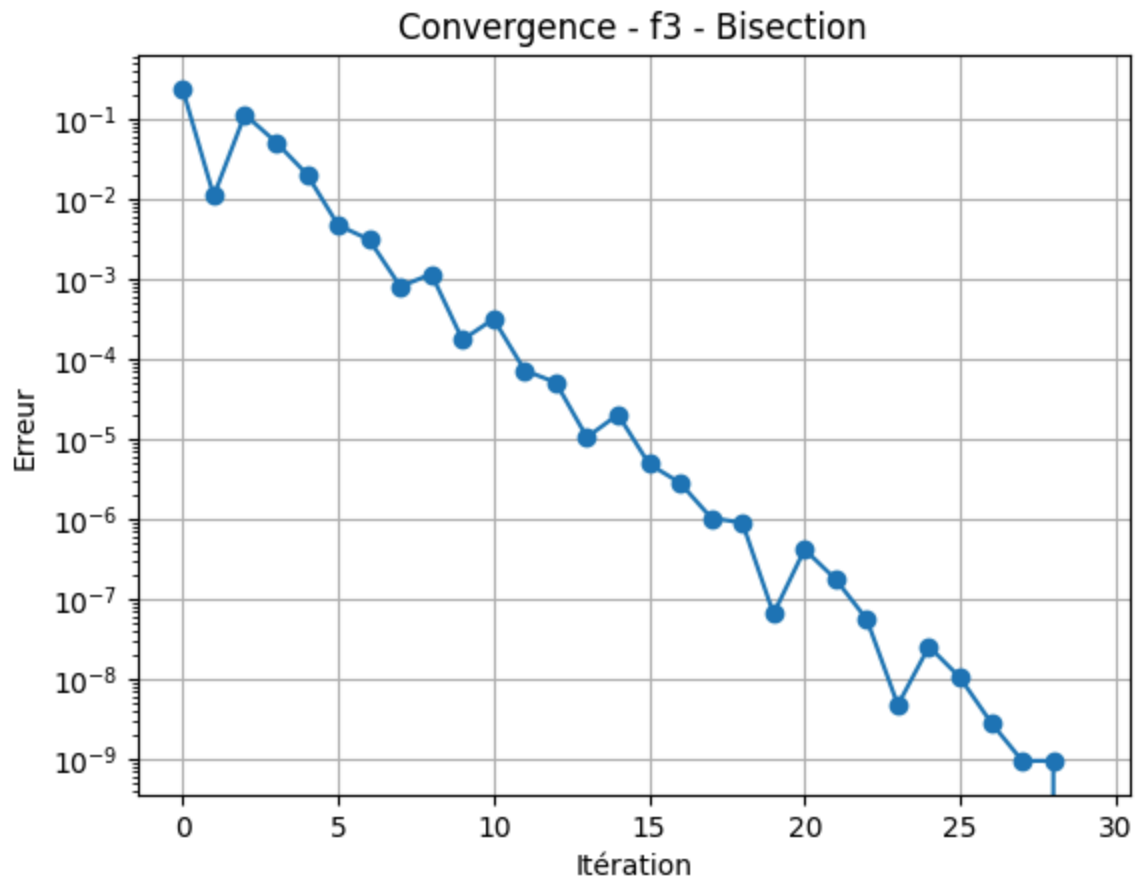




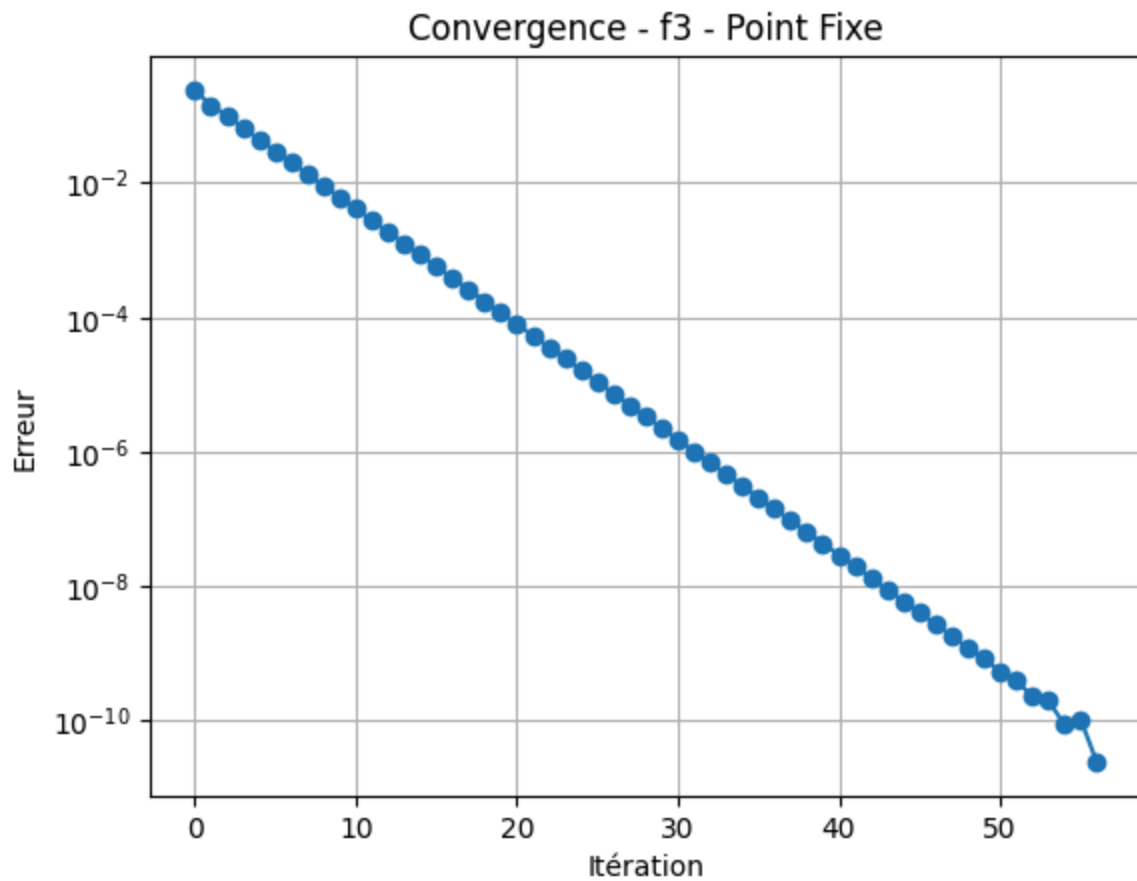
Newton-Raphson root ( $x_0=2$ ) = 1.2564312086261697

=== f3 ===

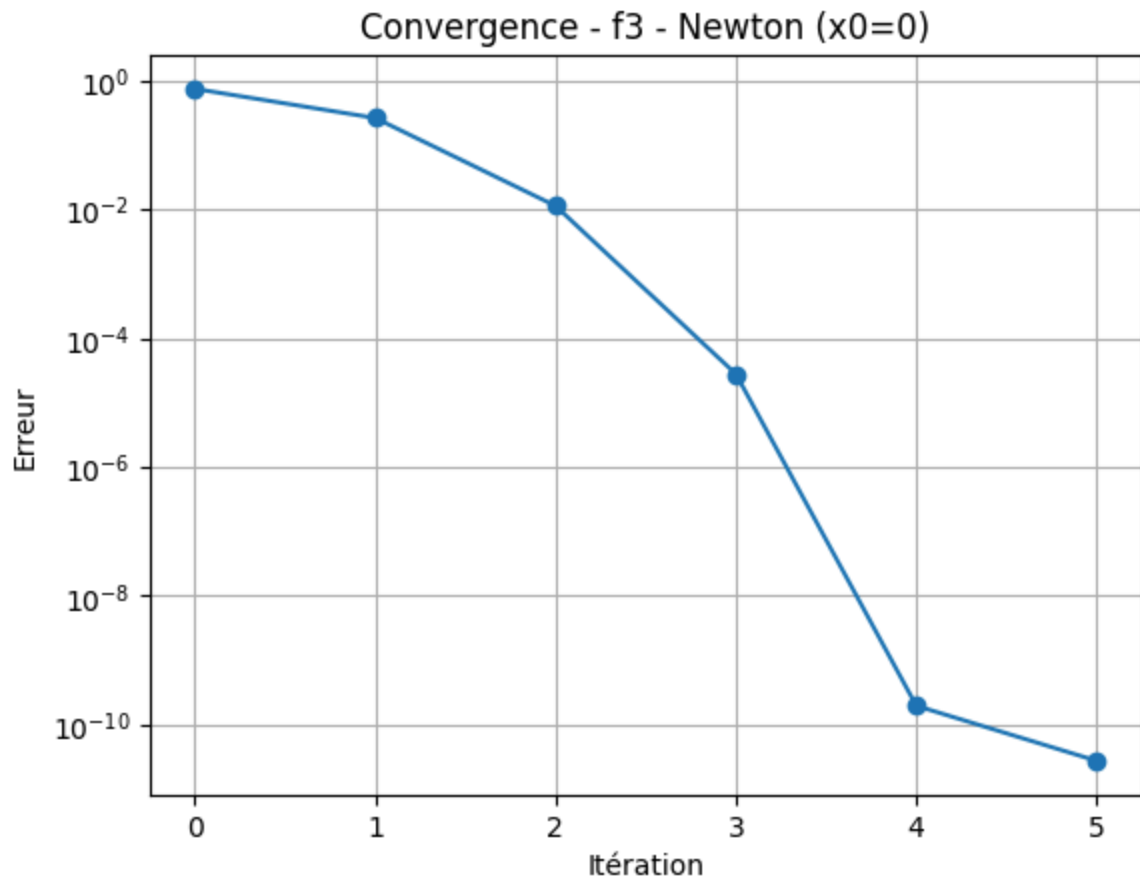




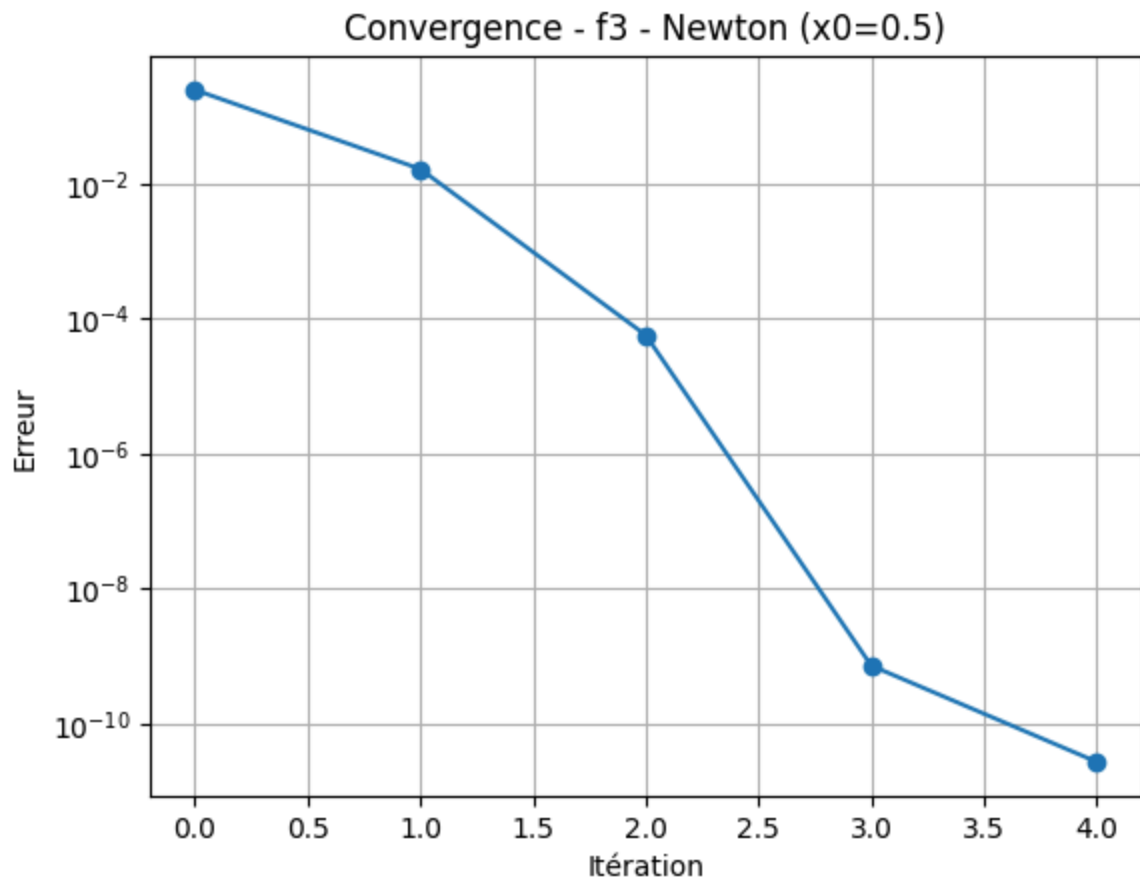
Bisection root = 0.7390851331874728



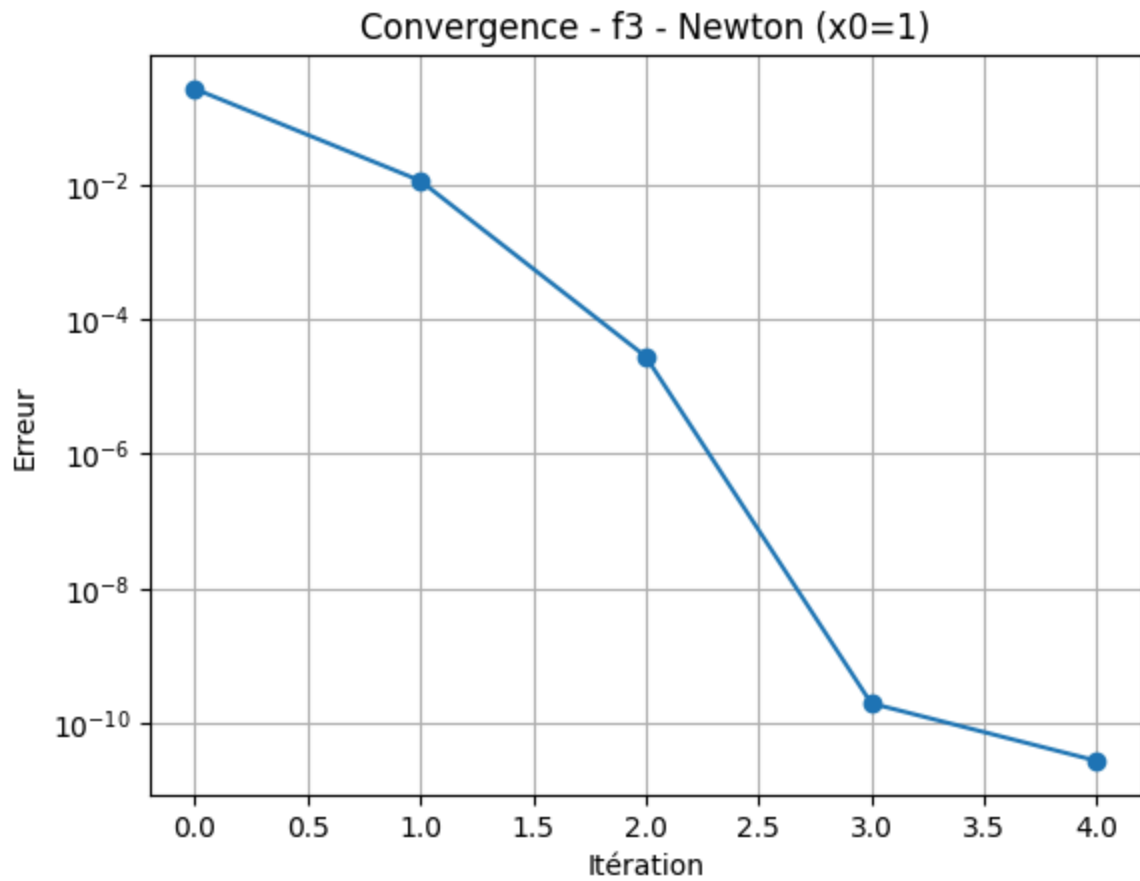
Fixed point root = 0.7390851332502528



Newton-Raphson root (x0=0) = 0.7390851332151607



Newton-Raphson root (x0=0.5) = 0.7390851332151607



Newton-Raphson root ( $x_0=1$ ) = 0.7390851332151607