ECON 899 Problem Set 2

Jackson Crawford, Dalya Elmalt, Jon Kroah*

Due 22 Sep 2022

Part 1

Consider the same environment as Huggett (1993, JEDC) except assume that there are enforceable insurance markets regarding the idiosyncratic shocks to earnings and that there are no initial asset holdings. Solve for a competitive equilibrium. What are prices? What is the allocation?

Assume there are complete markets for state-contingent claims. The first welfare theorem holds in this environment, so we can solve the social planner's problem to compute equilibrium allocations. Then, we can use household optimality conditions to compute equilibrium prices for the assets.

Environment

Markov process: $s_t \in S = \{e, u\}$ with transition probabilities $\pi(s_{t+1}|s_t)$

Endowment process: $y(s_t) = \begin{cases} 1, & s_t = e \\ b, & s_t = u \end{cases}$ where $b < 1$

Unit measure of HHs with preferences: $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$ where $\sigma > 1$

Step 1: Social Planner's Problem ⇒ Equilibrium Allocations

Assume the Markov chain s_t has a stationary distribution $\pi(s_t)$. Then in any period t, there is a measure $\pi(e)$ of employed agents and $\pi(u) = 1 - \pi(e)$ of unemployed agents (i.e., the planner faces no aggregate uncertainty). The planner solves:

$$\max_{\{c(s_t)\}_{t,s_t}} \left\{ \sum_{t=0}^{\infty} \sum_{s_t} \beta^t \pi(s_t) u(c(s_t)) \right\}$$

subject to: $\displaystyle\sum_{s_t} \pi(s_t)c(s_t) = \sum_{s_t} \pi(s_t) y(s_t)$ for all t

Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \sum_{s_t} \beta^t \pi(s_t) u(c(s_t)) + \sum_{t=0}^{\infty} \lambda_t \sum_{s_t} \pi(s_t)[y(s_t) - c(s_t)]$$

---

*We also discussed solutions with Sabrina Ben Said, Cache Ellsworth, Serena Xu.

FOCs w.r.t. $c(e)$ and $c(u)$:

$$\begin{aligned}
&\beta^t \pi(e) u'(c(e)) = \lambda_t \pi(e) \\
&\beta^t \pi(u) u'(c(u)) = \lambda_t \pi(u) \\
\Rightarrow &\frac{u'(c(e))}{u'(c(u))} = 1 \\
\iff &\left(\frac{c(e)}{c(u)}\right)^{-\sigma} = 1 \\
\iff &c(e) = c(u) = c^*
\end{aligned}$$

Plugging into the resource constraint for period t (recall that $y(e) = 1, y(u) = b$):

$$\begin{aligned}
&\pi(e)c(e) + \pi(u)c(u) = \pi(e)y(e) + \pi(u)y(u) \\
\iff &\pi(e)c^* + \pi(u)c^* = \pi(e) \cdot 1 + (1 - \pi(e)) \cdot b \\
\iff &c^* = \pi(e) + (1 - \pi(e))b
\end{aligned}$$

So the planner perfectly smoothes consumption and perfectly insures against idiosyncratic risk by redistributing from employed HHs to unemployed HHs. All HHs consume an equal share of the aggregate endowment.

Step 2: Household's Problem ⇒ Equilibrium Prices

Suppose at time t and history $s^t := (s_t, s_{t-1}, \ldots, s_0)$, a household can buy one-period-ahead contingent claims to consumption in state s_{t+1}, denoted $a_{t+1}(s^t, s_{t+1})$, at prices $Q(s_{t+1}|s^t)$.

Each household solves

$$\max_{\{c(s^t), a_{t+1}(s^t)\}_{t,s^t}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c(s^t))\right]$$

$$\begin{aligned}
\text{subject to } c(s^t) + \sum_{s_{t+1}} Q_t(s_{t+1}|s^t) a_{t+1}(s^t, s_{t+1}) &\leq a_t(s^t) + y(s^t) \text{ for all } t, s^t \\
a_{t+1}(s^t, s_{t+1}) &\geq \underline{a} \text{ for all } t, s^t \\
c(s^t) &\geq 0 \text{ for all } t, s^t \\
a_0 &= 0
\end{aligned}$$

Because s_t is a Markov process, we can recursify the problem. The Bellman equation is

$$V(a, s) = \max_{c, \hat{a}} \left\{u(c) + \beta \sum_{s'} \pi(s'|s) V(\hat{a}(s'), s')\right\}$$

$$\text{subject to } c + \sum_{s'} Q(s'|s)\hat{a}(s') \leq y(s) + a$$

Substituting in the BC:

$$V(a, s) = \max_{\hat{a}} \left\{u\Big(y(s) + a - \sum_{s'} Q(s'|s)\hat{a}(s')\Big) + \beta \sum_{s'} \pi(s'|s) V(\hat{a}(s'), s')\right\}$$

The FOC w.r.t. $\hat{a}(s')$ is:

$$u'\Big(y(s) + a - \sum_{s'} Q(s'|s)\hat{a}(s')\Big)Q(s'|s) = \beta\pi(s'|s)\partial_a V(\hat{a}(s'), s')$$

The Envelope condition is:

$$\partial_a V(a, s) = u'\Big(y(s) + a - \sum_{s'} Q(s'|s)\hat{a}(s')\Big)$$
$$\Rightarrow \partial_a V(\hat{a}(s'), s') = u'\Big(y(s') + \hat{a}(s') - \sum_{s''} Q(s''|s')\hat{a}(s'')\Big)$$

Combining and rearranging:

$$u'\Big(y(s) + a - \sum_{s'} Q(s'|s)\hat{a}(s')\Big)Q(s'|s) = \beta\pi(s'|s)u'\Big(y(s') + \hat{a}(s') - \sum_{s''} Q(s''|s')\hat{a}(s'')\Big)$$
$$\iff Q(s'|s) = \beta\pi(s'|s)\frac{u'\Big(y(s') + \hat{a}(s') - \sum_{s''} Q(s''|s')\hat{a}(s'')\Big)}{u'\Big(y(s) + a - \sum_{s'} Q(s'|s)\hat{a}(s')\Big)}$$
$$\iff Q(s_{t+1}|s_t) = \beta\pi(s_{t+1}|s_t)\frac{u'(c(s_{t+1}))}{u'(c(s_t))} = \beta\pi(s_{t+1}|s_t)\left(\frac{c(s_t)}{c(s_{t+1})}\right)^{\sigma}$$

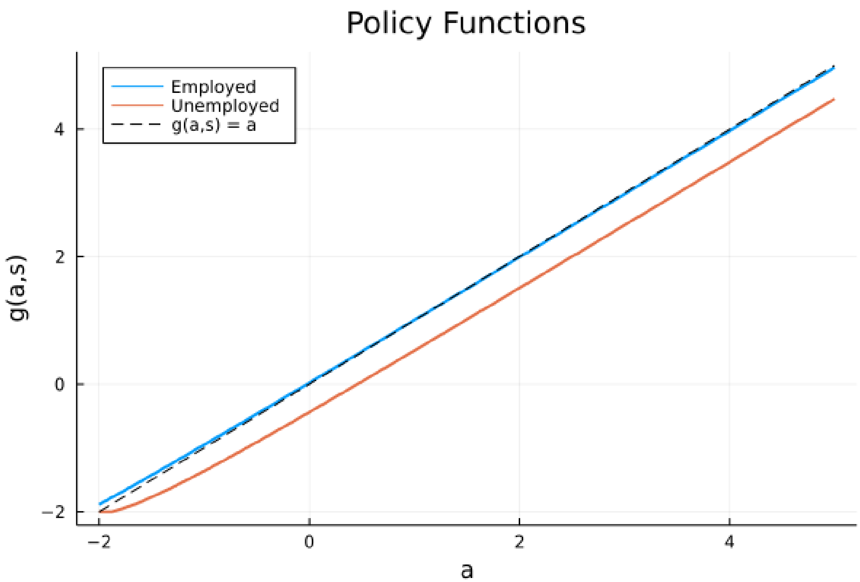
From the planner's solution, we know $c(s_t) = c^*$ for all states s_t. Therefore, $c(s_t) = c(s_{t+1}) = c^*$, and the equation above pins down prices:

$$\boxed{Q(s_{t+1}|s_t) = \beta\pi(s_{t+1}|s_t)} \quad \blacksquare$$

Part 2

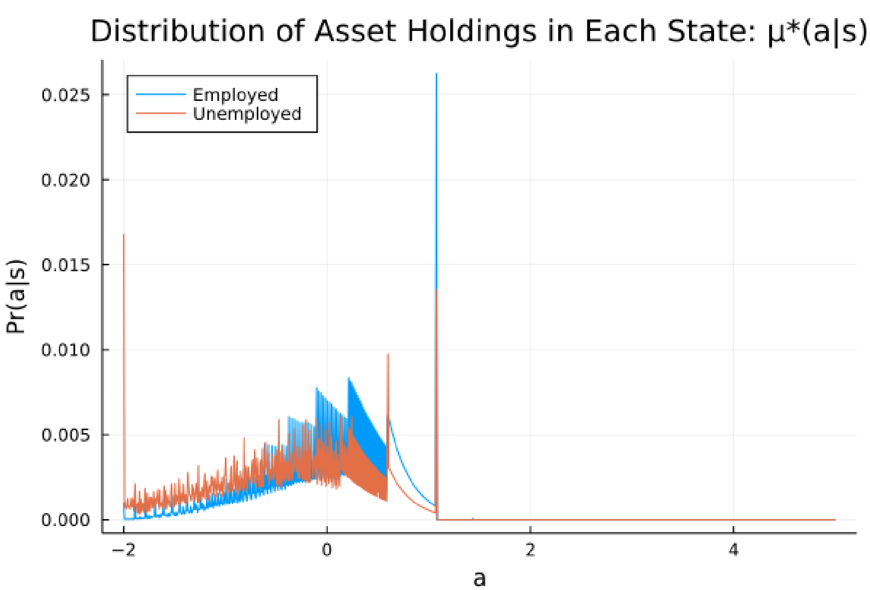
(a) Policy Functions

In the employed state, the policy function starts above the 45-degree line and eventually dips below it. This shows there is a $\hat{a}$ such that $g(\hat{a}, s)$.



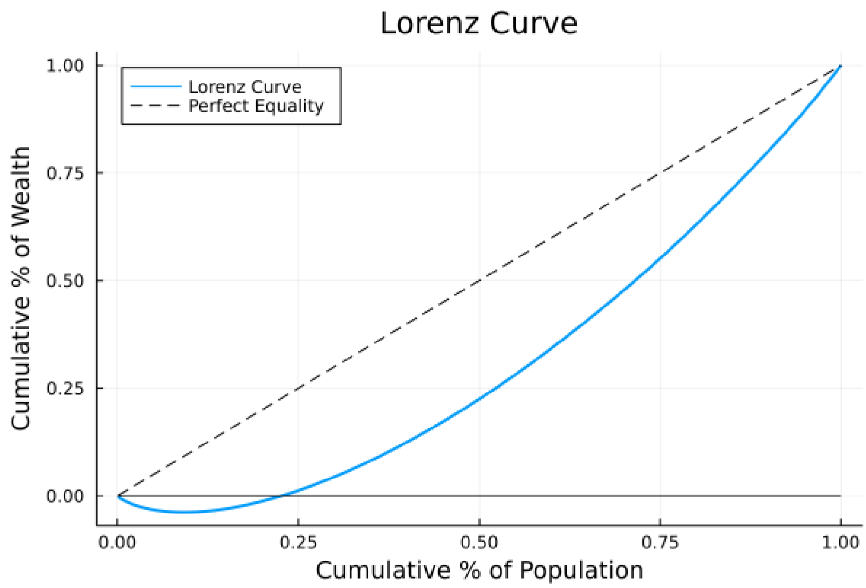
(b) Bond Price and Wealth Distributions

The equilibrium bond price is $q^* \approx \mathbf{0.994272}$, which implies a real interest rate of ≈ 0.00576.



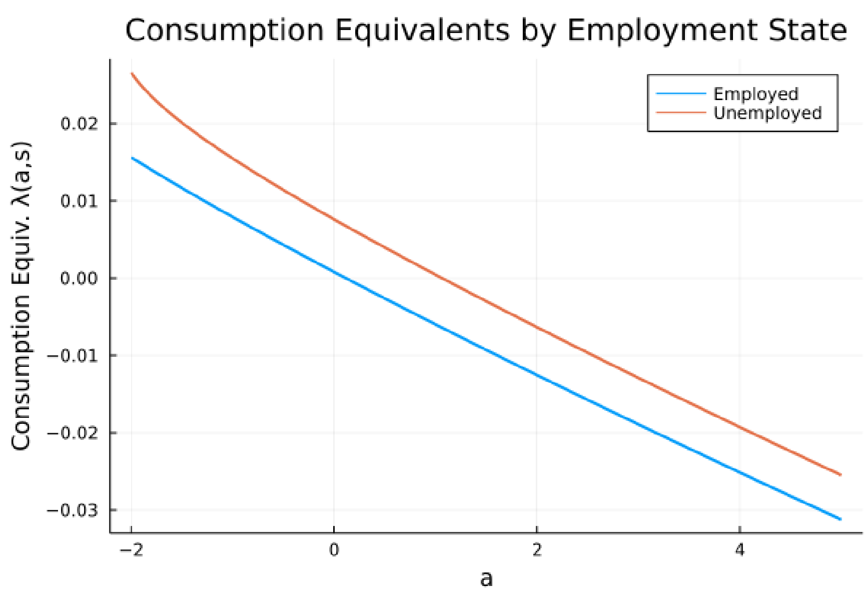
(b) Lorenz Curve and Gini Coefficient

The Gini index is approximately **0.3833**.



Part 3

(a) Plot consumption equivalents $\lambda(a, s)$



(b) Welfare Comparisons

From Part I, in the first-best allocation, all HHs consume $c^* = \pi(e) + (1 - \pi(e))b$ in every period. Therefore, aggregate welfare in the first best is

$$W^{FB} = \sum_{t=0}^{\infty} \beta^t u(c^*) = \frac{c^{*1-\alpha} - 1}{(1-\alpha)(1-\beta)} = \frac{\pi(e) + (1 - \pi(e))b}{1-\beta}$$

Plugging in parameters & equilibrium solutions, we have

$$\begin{aligned} W^{FB} &\approx -4.25252 \\ W^{INC} &\approx -4.44215 \\ WG &\approx 0.001278 \end{aligned}$$

(c) What fraction of the population would vote for complete markets?

Approximately **0.535733** of the population would vote in favor of changing to complete markets.